## Parsing, PCFGs, and the CKY Algorithm

Yoav Goldberg

(with slides by Michael Collins, Julia Hockenmaier)

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## What is grammar?





## Natural Language Parsing

- Sentences in natural language have structure.
- Linguists create Linguistic Theories for defining this structure.
- The parsing problem is recovering that structure.

## What is grammar?

Grammar formalisms

(= linguists' programming languages)

A precise way to define and describe

the structure of sentences.

(N.B.: There are many different formalisms out there, which each define their own data structures and operations)

Specific grammars

(= linguists' programs)

Implementations (in a particular formalism) for a particular language (English, Chinese,....)

Can we define a program that generates all English sentences?

## The number of sentences is infinite. But we need our program to be finite.



## **Basic sentence structure**



## A finite-state-automaton (FSA)



## A Hidden Markov Model (HMM)



## Words take arguments

I eat sushi. ✓ I eat sushi you. ??? I sleep sushi ??? I give sushi ??? I drink sushi ?

#### Subcategorization:

Intransitive verbs (sleep) take only a subject. Transitive verbs (eat) take also one (direct) object. Ditransitive verbs (give) take also one (indirect) object.

#### **Selectional preferences:**

The object of eat should be edible.

CS498JH: Introduction to NLP



## Natural Language Parsing

#### Previously

- Structure is a sequence.
- Each item can be tagged.
- We can mark some spans.

## Natural Language Parsing

#### Previously

- Structure is a sequence.
- Each item can be tagged.
- We can mark some spans.

#### Today

Hierarchical Structure.

## **Hierarchical Structure?**

## Language is recursive

#### the ball the big ball the big, red ball the big, red, heavy ball

Adjectives can **modify** nouns.

The **number of modifiers/adjuncts** a word can have is (in theory) **unlimited**.



## Recursion can be more complex

the ball the ball in the garden the ball in the garden behind the house the ball in the garden behind the house next to the school

. . . .

## What is the structure of a sentence?

Sentence structure is hierarchical:

A sentence consists of **words** (*I, eat, sushi, with, tuna*) ...which form phrases or **constituents**: "*sushi with tuna*"

Sentence structure defines dependencies between words or phrases:



# Strong vs. weak generative capacity

Formal language theory:

- defines language as string sets
- is only concerned with generating these strings (weak generative capacity)

Formal/Theoretical syntax (in linguistics):

- defines language as sets of strings with (hidden) structure
- is also concerned with generating the right structures (strong generative capacity)

## Structure

Example 1: math



## Structure

Example 1: math



#### Structure Example 1: math

ADD ADD MUL MUL

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## Constituency (Phrase Structure) Trees

 Phrase structure organizes words into nested constituents



## Constituency (Phrase Structure) Trees

- Phrase structure organizes words into nested constituents
- Linguists can, and do, argue about details



## Constituency (Phrase Structure) Trees

- Phrase structure organizes words into nested constituents
- Linguists can, and do, argue about details



Programming Languages?

They also have structure. How does it differ from human's language structure?

### Structure

Example 2: Language Data

Fruit flies like a banana

### Structure

Example 2: Language Data







I heard that the lawyer questioned the witness under oath yestarday





Dependency representation is very common. We will return to it in the future.

#### Structure

#### **Constituency Structure**

In this class we concentrate on Constituency Parsing: mapping from sentences to trees with labeled nodes and the sentence words at the leaves.

## Why is Parsing Interesting?

- It's a first step towards understanding a text.
- Many other language tasks use sentence structure as their input.

## Some things that are done with parse trees

#### Grammar Checking

- Word Clustering
- Information Extraction
- Question Answering
- Sentence Simplification
- Machine Translation
- ...and more

#### Word can fixed grammar mistakes

Word can fixed grammar mistake	Ignore All
Suggestions:	
fix	Change

## Some things that are done with parse trees

#### Grammar Checking

- Word Clustering
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## Words in similar grammatical relations share meanings.

(train (desc 94476.5) (sims recruit 0.150024 educate 0.144399 employ 0.143866 hire 0.139511 teach 0.128722 prepare 0.125677 involve 0.124054

(vodka (desc 16439.9) (sims rum 0.234184 whiskey 0.22456 tequila 0.223834 Cognac 0.221328 whisky 0.210289 bourbon 0.203362 liqueur 0.185826

(train (desc 129401) (sims bus 0.255581 "passenger train" 0.199742 "freight train" 0.182971 truck 0.175791 "express train" 0.163199 plane 0.160376 ferry 0.153743 car 0.152089 boat 0.149375

> (pizza (desc 38638) (sims sandwich 0.217842 salad 0.178421 "hot dog" 0.176298 Hamburger 0.175821 pasta 0.169979 steak 0.167151 soup 0.165197 bread 0.164151 "french fries" 0.164104 chicken 0.158617 pie 0.158087
- Grammar Checking
- Word Clustering
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#### Extract factual relations from text

Factz from Wikipedia: we found the following about CIA

CIA	trained :	exiles, agent, fighters, army, issues, Farm, facili
	used :	buildings, dealers, seal, missile, methods, proce
	provided :	proof, lists, leftists, communists, Factbook, train

#### Answer questions

 Powerset

 Wikipedia Articles

 when did earthquakes hit tokyo

 Wikipedia Articles

 Image: Tokyo Tokyo was hit by powerful earthquakes in 1703, 1782, 1812, 1855 and 1923.

 Image: Tokyo Tokyo was hit by powerful earthquakes in 1703, 1782, 1812, 1855 and 1923.

 Image: Sweden, whose death toll was 543, ... The deadliest earthquakes since 1900 were the

Sweden, whose death toll was 543. ... The deadliest **earthquakes** since 1900 were the Tangshan, China earthquake of 1976, in which at least 255,000 were killed; the earthquake of 1927 in Xining, Qinghai, China (200,000); the Great Kanto **earthquake** which **struck Tokyo** in **1923** (143,000); and the Gansu, China, earthquake of 1920 (200,000).

- Grammar Checking
- Word Clustering
- Information Extraction
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- ...and more

The first new product, ATF Protype, is a line of digital postscript typefaces that will be sold in packages of up to six fonts.



ATF Protype is a line of digital postscript typefaces will be sold in packages of up to six fonts.

Reorder VB VB PRP VB2 VB1 PRP VB1 VB2 adores He adores VB VB listenina listening TO NN ΤÔ ΝN to music to music 1. Original Parse Tree 2. After Reorder Insert VB PRP VB2 VB1 He ha adores desu ga VB ГО listening no TO NN music to Translate VB PRP VB1 VB2 kare ha daisuki desu ga VB kiku no ΤO NN wo ongaku < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > E

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- Grammar Checking
- Word Clustering
- Information Extraction
- Question Answering
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- Machine Translation
- ...and more

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### Constituency (Phrase Structure) Trees

 Phrase structure organizes words into nested constituents

#### What is a constituent? How do we know they exist?



# Context-free grammars (CFGs) capture recursion

Language has complex constituents ("the garden behind the house")

Syntactically, these constituents behave just like simple ones.

("behind the house" can always be omitted)

CFGs define nonterminal categories to capture equivalent constituents.

#### The black dog sat

is "dog" a constituent?

is "The black dog" a constituent?

is "the black" a constituent?

is "dog sat" a constituent?

why?

Every word is a constituent.

If we can substitute a sequence of words by a single word, it is likely to be a constituent.

In particular, pronouns.

- (1) a. <u>The little boy fed the cat</u>.  $\rightarrow$  <u>He fed her</u>.
  - b. <u>Black cats</u> detest green peas.  $\rightarrow$  <u>They</u> detest <u>them</u>.

- (2) a. <u>The little boy</u> from next door fed <u>the cat</u> without a tail.
  - b. These <u>black cats</u> detest those green peas.

- (2) a. <u>The little boy</u> from next door fed <u>the cat</u> without a tail.b. These <u>black cats</u> detest those <u>green peas</u>.
  - $\rightarrow$  \* <u>He</u> from next door fed <u>her</u> without a tail.  $\rightarrow$  \* These <u>they</u> detest those <u>them</u>.

- (3) a. <u>The little boy from next door fed the cat without a tail</u>.
  - b. <u>These black cats</u> detest <u>those green peas</u>.
    - $\rightarrow$  <u>He</u> fed <u>her</u>.
      - $\rightarrow$  <u>They</u> detest <u>them</u>.

adverbs (here/there) can also be used.

- (4) a. Put it <u>on the table</u>.
  - b. Put it over <u>on the table</u>.
  - c. Put it over on the table.
- Put it <u>on the table</u> that's by the door.  $\rightarrow$  \* Put it <u>there</u> that's by the door. (5) a.

- Put it <u>there</u>.
- Put it over there.
- $\rightarrow$  Put it <u>there</u>.
- b. Put it over <u>on the table</u> that's by the door.  $\rightarrow$  \* Put it over <u>there</u> that's by the door.
- c. Put it <u>over on the table</u> that's by the door.  $\rightarrow$  \* Put it <u>there</u> that's by the door.

### Substitution Test "so", In a comparison.

(6) a. I am very happy,

b. I am very fond of Lukas,

and Linda is <u>so</u>, too.

and Linda is <u>so</u>, too.

c. I am very fond of my nephew, \* and Linda is so of her niece.

'so', 'it', instead of a subordinate clause.

(7) a. I { know, suspect } <u>that they're invited</u>. → I { know, suspect } <u>it</u>.
b. I { imagine, think } <u>that they're invited</u>. → I { imagine, think } <u>so</u>.

Substitution tests are a good indicator of constituency.

But they may fail, also for real constituents.

There are other tests: movement, short answers, it clefts

We will briefly cover movement. See more in the Santorini's chapter.

Can we move the sequence to a different position in the sentence?

- (8) a. I fed the cats.
  - b. I fed the cats with long, fluffy tails.

- $\rightarrow$  <u>The cats</u>, I fed \_\_\_\_. (The dogs, I didn't.)
- $\rightarrow$  <u>The cats with long, fluffy tails</u>, I fed \_\_\_\_. (The other cats, I didn't.)

The cat strolled across the porch with a
confident air.
Ali Baba returned from his travels wiser than
<u>before</u> .
They arrived at the concert hall more quickly
than they had expected.

- → With a confident air, the cat strolled across the porch \_\_\_\_.
- → <u>Wiser than before</u>, Ali Baba returned from his travels \_\_\_\_.
- → More quickly than they had expected, they arrived at the concert hall \_\_\_\_.

- (10) a. I fed the cats with long, fluffy tails.
  - b. The cat strolled across the porch with a confident air.
  - c. Ali Baba returned from his travels wiser than before.
  - d. They arrived at the concert hall <u>more quickly than</u> <u>they</u> had expected.

- (10) a. I fed the cats with long, fluffy tails.
  - b. The cat strolled across the porch with a confident air.
  - c. Ali Baba returned from his travels wiser than before.
  - d. They arrived at the concert hall <u>more quickly than</u> <u>they</u> had expected.
    - $\rightarrow$  \* <u>The cats</u>, I fed <u>with long</u>, fluffy tails.<sup>1</sup>
    - → \* <u>With a</u>, the cat strolled across the porch \_\_\_\_ confident air.
    - → \* <u>Wiser than</u>, Ali Baba returned from his travels \_\_\_\_\_ before.
    - → \* More quickly than they, they arrived at the concert hall \_\_\_\_ had expected.

See more tests in the notes.

I hope that you are convinced that constituents are "real".

You need to know how to identify them.

Parsing: recovering the constituents of a sentence.

Fat people eat candy





#### Fat people eat accumulates



#### Fat people eat accumulates



### Why is parsing hard?

Real Sentences are long...

"Former Beatle Paul McCartney today was ordered to pay nearly \$50M to his estranged wife as their bitter divorce battle came to an end . "

"Welcome to our Columbus hotels guide, where you'll find honest, concise hotel reviews, all discounts, a lowest rate guarantee, and no booking fees." Let's learn how to parse

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Let's learn how to parse . . . but first lets review some stuff we learned at Automata and Formal Languages.

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### **Context Free Grammars**

A context free grammar  $G = (N, \Sigma, R, S)$  where:

- N is a set of non-terminal symbols
- Σ is a set of terminal symbols
- *R* is a set of rules of the form *X* → *Y*<sub>1</sub> *Y*<sub>2</sub> · · · *Y<sub>n</sub>* for *n* ≥ 0, *X* ∈ *N*, *Y<sub>i</sub>* ∈ (*N* ∪ Σ)
- $S \in N$  is a special start symbol

### **Context Free Grammars**

```
a simple grammar
N = \{S, NP, VP, Adj, Det, Vb, Noun\}
\Sigma = \{fruit, flies, like, a, banana, tomato, angry\}
S = S'
R =
   S \rightarrow NP VP
   NP \rightarrow Adj Noun
   NP \rightarrow Det Noun
   VP \rightarrow Vb NP
   Adj \rightarrow fruit
   Noun \rightarrow flies
   Vb \rightarrow like
   \text{Det} \rightarrow \text{a}
   Noun \rightarrow banana
   Noun \rightarrow tomato
   Adj \rightarrow angry
```

#### Left-most derivations

Left-most derivation is a sequence of strings  $s_1, \dots, s_n$  where

- $s_1 = S$  the start symbol
- ►  $s_n \in \Sigma^*$ , meaning  $s_n$  is only terminal symbols
- Each  $s_i$  for  $i = 2 \cdots n$  is derived from  $s_{i-1}$  by picking the left-most non-terminal X in  $s_{i-1}$  and replacing it by some  $\beta$  where  $X \rightarrow \beta$  is a rule in R.

For example: [S],[NP VP],[Adj Noun VP], [fruit Noun VP], [fruit flies VP],[fruit flies Vb NP],[fruit flies like NP], [fruit flies like Det Noun], [fruit flies like a], [fruit flies like a banana]

#### Left-most derivation example

S

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### Left-most derivation example

 $\ensuremath{\mathsf{S}}$   $$\mathsf{NP}$\ensuremath{\mathsf{VP}}$$ 

### Left-most derivation example

S NP VP Adj Noun VP

 $\mathsf{NP}\to\mathsf{Adj}\:\mathsf{Noun}$
S NP VP Adj Noun VP fruit Noun VP

### $\text{Adj} \rightarrow \text{fruit}$

S NP VP Adj Noun VP fruit Noun VP fruit flies VP

 $\text{Noun} \to \text{flies}$ 

S NP VP Adj Noun VP fruit Noun VP fruit flies VP fruit flies Vb NP

 $\text{VP} \rightarrow \text{Vb} \; \text{NP}$ 

S NP VP Adj Noun VP fruit Noun VP fruit flies VP fruit flies Vb NP fruit flies like NP

 $\text{Vb} \rightarrow \text{like}$ 

S NP VP Adj Noun VP fruit Noun VP fruit flies VP fruit flies Vb NP fruit flies like NP fruit flies like Det Noun

 $NP \rightarrow Det Noun$ 

S NP VP Adj Noun VP fruit Noun VP fruit flies VP fruit flies Vb NP fruit flies like NP fruit flies like Det Noun fruit flies like a Noun

 $\text{Det} \to a$ 

S NP VP Adj Noun VP fruit Noun VP fruit flies VP fruit flies Vb NP fruit flies like NP fruit flies like Det Noun fruit flies like a Noun fruit flies like a banana

Noun  $\rightarrow$  banana

S NP VP Adj Noun VP fruit Noun VP fruit flies VP fruit flies Vb NP fruit flies like NP fruit flies like Det Noun fruit flies like a Noun fruit flies like a banana

The resulting derivation can be written as a tree.

S NP VP Adj Noun VP fruit Noun VP fruit flies VP fruit flies Vb NP fruit flies like NP fruit flies like Det Noun fruit flies like a Noun fruit flies like a banana

- The resulting derivation can be written as a tree.
- Many trees can be generated.

a simple grammar  $S \rightarrow NP VP$  $NP \rightarrow Adj Noun$  $NP \rightarrow Det Noun$  $VP \rightarrow Vb NP$  $Adj \rightarrow fruit$ Noun  $\rightarrow$  flies  $Vb \rightarrow like$  $\text{Det} \rightarrow \text{a}$ Noun  $\rightarrow$  banana Noun  $\rightarrow$  tomato  $Adj \rightarrow angry$ 

• • •

a simple grammar  $S \rightarrow NP VP$  $NP \rightarrow Adj Noun$  $NP \rightarrow Det Noun$  $VP \rightarrow Vb NP$  $Adj \rightarrow fruit$ Noun  $\rightarrow$  flies  $Vb \rightarrow like$  $\text{Det} \rightarrow \text{a}$ Noun  $\rightarrow$  banana Noun  $\rightarrow$  tomato  $Adj \rightarrow angry$ 

. . .



a simple grammar  $S \rightarrow NP VP$  $NP \rightarrow Adj Noun$  $NP \rightarrow Det Noun$  $VP \rightarrow Vb NP$  $Adj \rightarrow fruit$ Noun  $\rightarrow$  flies  $Vb \rightarrow like$  $\text{Det} \rightarrow \text{a}$ Noun  $\rightarrow$  banana Noun  $\rightarrow$  tomato  $Adj \rightarrow angry$ 











A Brief Introduction to English Syntax



Product Details (from Amazon) Hardcover: 1779 pages Publisher: Longman; 2nd Revised edition Language: English ISBN-10: 0582517346 ISBN-13: 978-0582517349 Product Dimensions: 8.4 x 2.4 x 10 inches Shipping Weight: 4.6 pounds

## A Brief Overview of English Syntax

### Parts of Speech (tags from the Brown corpus):

- Nouns
  - NN = singular noun e.g., man, dog, park
  - NNS = plural noun e.g., telescopes, houses, buildings
  - NNP = proper noun e.g., Smith, Gates, IBM
- Determiners

DT = determiner e.g., the, a, some, every

Adjectives

JJ = adjective e.g., red, green, large, idealistic

### A Fragment of a Noun Phrase Grammar



fast

metal

 $\Rightarrow$ 

## Prepositions, and Prepositional Phrases

Prepositions

IN = preposition e.g., of, in, out, beside, as

An Extended Grammar

N	$\Rightarrow$	NN	
Ñ	$\Rightarrow$	NN	Ñ
Ñ	$\Rightarrow$	JJ	Ñ
Ñ	$\Rightarrow$	Ñ	Ñ
NP	$\Rightarrow$	DT	Ñ
PP	$\Rightarrow$	IN	NP
Ñ	$\Rightarrow$	Ñ	PP
			I

## An Extended Grammar



#### **Generates:**

in a box, under the box, the fast car mechanic under the pigeon in the box,  $\ldots$ 

### Verbs, Verb Phrases, and Sentences

- Basic Verb Types
  Vi = Intransitive verb
  Vt = Transitive verb
  Vd = Ditransitive verb
  e.g., sees, saw, likes
  e.g., gave
- Basic VP Rules
  VP  $\rightarrow$  Vi
  VP  $\rightarrow$  Vt NP
  VP  $\rightarrow$  Vd NP NP
- Basic S Rule
  - $\mathsf{S} \ \rightarrow \ \mathsf{NP} \ \mathsf{VP}$

#### Examples of VP:

sleeps, walks, likes the mechanic, gave the mechanic the fast car **Examples of S**:

the man sleeps, the dog walks, the dog gave the mechanic the fast car

### PPs Modifying Verb Phrases

A new rule: VP  $\rightarrow$  VP PP

#### New examples of VP:

sleeps in the car, walks like the mechanic, gave the mechanic the fast car on Tuesday, ...

### Complementizers, and SBARs

- Complementizers COMP = complementizer e.g., that
- ► SBAR
  - $\mathsf{SBAR} \ \rightarrow \ \mathsf{COMP} \ \mathsf{S}$

#### **Examples:**

that the man sleeps, that the mechanic saw the dog ...

### More Verbs

- New Verb Types
  V[5] e.g., said, reported
  V[6] e.g., told, informed
  V[7] e.g., bet
- New VP Rules
  VP  $\rightarrow$  V[5] SBAR
  VP  $\rightarrow$  V[6] NP SBAR
  VP  $\rightarrow$  V[7] NP NP SBAR

#### **Examples of New VPs:**

said that the man sleeps told the dog that the mechanic likes the pigeon bet the pigeon \$50 that the mechanic owns a fast car

### Coordination

- A New Part-of-Speech: CC = Coordinator e.g., and, or, but

### We've Only Scratched the Surface...

Agreement

The dogs laugh vs. The dog laughs

Wh-movement

The dog that the cat liked \_\_\_\_

Active vs. passive

The dog saw the cat *vs.* The cat was seen by the dog

If you're interested in reading more:

Syntactic Theory: A Formal Introduction, 2nd Edition. Ivan A. Sag, Thomas Wasow, and Emily M. Bender.

# CFGs and center embedding

The mouse ate the corn. The mouse that the snake ate ate the corn. The mouse that the snake that the hawk ate ate the corn.

### These sentences are all grammatical. They can be generated by a CFG:

### Linguists distinguish between a speaker's

- competence (grammatical knowledge) and
- performance (processing and memory limitations)

### Let's assume...

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## Problem

Natural Language is NOT generated by a CFG.

## Solution

We assume really hard that it is.

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We don't have the grammar.

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## Solution

We'll ask a genius linguist to write it!
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#### Problem

How do we find the chain of derivations?

#### Solution

With dynamic programming! (soon)

#### Let's assume...

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- Then parsing is easy: given a sentence, find the chain of derivations starting from S that generates it.

#### Problem

Real grammar: hundreds of possible derivations per sentence.

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- ... and let's assume we have the grammar.
- Then parsing is easy: given a sentence, find the chain of derivations starting from S that generates it.

#### Problem

Real grammar: hundreds of possible derivations per sentence.

#### Solution

No problem! We'll choose the best one. (sooner)

## Obtaining a Grammar

Let a genius linguist write it

- Hard. Many rules, many complex interactions.
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- Ask a linguist to annotate sentences with tree structure.
- (This need not be a genius Smart is enough.)
- Then extract the rules from the annotated trees.

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#### Treebanks

- English Treebank: 40k sentences, manually annotated with tree structure.
- Hebrew Treebank: about 5k sentences

# **Treebank Sentence Example**

```
( (S
(NP-SBJ
  (NP (NNP Pierre) (NNP Vinken) )
  (,,)
  (ADJP
    (NP (CD 61) (NNS years) )
    (JJ old) )
  (,,)
(VP (MD will)
  (VP (VB join)
    (NP (DT the) (NN board) )
    (PP-CLR (IN as)
      (NP (DT a) (JJ nonexecutive) (NN director
    (NP-TMP (NNP Nov.) (CD 29) )))
(. .) ))
```

#### Supervised Learning from a Treebank



- The leafs of the trees define Σ
- ► The internal nodes of the trees define N
- Add a special S symbol on top of all trees
- Each node an its children is a rule in *R*

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**Extracting Rules** 



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**Extracting Rules** 



 $\begin{array}{l} S \rightarrow \mathsf{NP} \; \mathsf{VP} \\ \textbf{NP} \rightarrow \textbf{Adj Noun} \end{array}$ 

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**Extracting Rules** 



 $\begin{array}{l} S \rightarrow NP \ VP \\ NP \rightarrow Adj \ Noun \\ \textbf{Adj} \rightarrow \textbf{fruit} \end{array}$ 

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 $tree = \underset{tree \in trees(sent)}{arg \max} P(tree|sent) = \underset{tree \in trees(sent)}{arg \max} P(tree, sent)$ 

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a simple PCFG  $1.0 \text{ S} \rightarrow \text{NP VP}$  $0.3 \text{ NP} \rightarrow \text{Adj Noun}$  $0.7 \text{ NP} \rightarrow \text{Det Noun}$ 1.0 VP  $\rightarrow$  Vb NP 0.2 Adj  $\rightarrow$  fruit 0.2 Noun  $\rightarrow$  flies 1.0 Vb  $\rightarrow$  like 1.0 Det  $\rightarrow$  a 0.4 Noun  $\rightarrow$  banana 0.4 Noun  $\rightarrow$  tomato 0.8 Adj  $\rightarrow$  angry



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- Dealing with unknown words like in the HMM

The CKY algorithm

# The Problem

#### Input

- Sentence (a list of words)
  - n sentence length
- CFG Grammar (with weights on rules)
  - ► *g* number of non-terminal symbols

#### Output

A parse tree / the best parse tree

#### But...

Exponentially many possible parse trees!

#### Solution

Dynamic Programming!

#### Cocke Kasami Younger

# Cocke Kasami Younger 196?

#### Cocke Kasami Younger 196? 1965

#### Cocke Kasami Younger 196? 1965 1967

- Recognition
- Parsing
- Disambiguation

- Recognition
  - Can this string be generated by the grammar?
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CKY can do all of these in polynomial time
# **3 Interesting Problems**

- Recognition
  - Can this string be generated by the grammar?
- Parsing
  - Show me a possible derivation...
- Disambiguation
  - Show me THE BEST derivation

#### CKY can do all of these in polynomial time

For any CNF grammar

#### **CNF** Chomsky Normal Form

#### Definition

A CFG is in CNF form if it only has rules like:

- $\blacktriangleright A \rightarrow B C$
- $\mathbf{A} \rightarrow \alpha$

A,B,C are non terminal symbols

 $\alpha$  is a terminal symbol (a word...)

- All terminal symbols are RHS of unary rules
- All non terminal symbols are RHS of binary rules

CKY can be easily extended to handle also unary rules:  $A \rightarrow B$ 

Fact

Any CFG grammar can be converted to CNF form

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Speficifally for Natural Language grammars

 $\blacktriangleright \text{ We already have } \mathbf{A} \to \alpha$ 

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Speficifally for Natural Language grammars

- $\blacktriangleright$  We already have  ${\it A} \rightarrow \alpha$ 
  - ( $A \rightarrow \alpha \beta$  is also easy to handle)

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#### **Binarization**

 $S \rightarrow NP NP|PP.VP.PP$   $NP|PP.VP.PP \rightarrow PP NP.PP|VP.PP$  $NP.PP|VP.PP \rightarrow VP NP.PP.VP|PP$ 

# Finally, CKY

#### Recognition

- Main idea:
  - Build parse tree from bottom up
  - Combine built trees to form bigger trees using grammar rules
  - ► When left with a single tree, verify root is *S*
- Exponentially many possible trees...
  - Search over all of them in polynomial time using DP
  - Shared structure smaller trees

### Main Idea

If we know:

- $w_i \dots w_j$  is an NP
- $w_{j+1} \dots w_k$  is a VP

and grammar has rule:

►  $S \rightarrow NP VP$ 

Then we know:

• S can derive  $w_i \ldots w_k$ 

#### (Half a) two dimensional array $(n \times n)$



#### On its side



Each cell: all nonterminals than can derive word *i* to word *j* 



Each cell: all nonterminals than can derive word *i* to word *j* imagine each cell as a *g* dimensional array



### Filling the table



# Handling Unary rules?



#### Which Order?



•  $n^2 g$  cells to fill

- $n^2 g$  cells to fill
- $g^2 n$  ways to fill each one

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# $O(g^3n^3)$

### A Note on Implementation

Smart implementation can reduce the runtime:

- Worst case is still  $O(g^3n^3)$ , but it helps in practice
- No need to check all grammar rules  $A \rightarrow BC$  at each location:
  - only those compatible with B or C of current split
  - prune binarized symbols which are too long for current position
  - once you found 1 way to derive A can break out of loop
  - order grammar rules from frequent to infrequent
- Need both efficient random access and iteration over possible symbols
  - Keep both hash and list, implemented as arrays

#### Finding a parse

Parsing – we want to actually find a parse tree

Easy: also keep a possible split point for each NT



### **PCFG** Parsing and Disambiguation

#### **Disambiguation** – we want THE BEST parse tree

Easy: for each NT, keep best split point, and score.



# **Implementation Tricks**

#1: sum instead of product

#### As in the HMM - Multiplying probabilities is evil

- keeping the product of many floating point numbers is dangerous, because product get really small
  - either grow in runtime
  - or loose precision (overflowing to 0)
  - either way, multiplying floats is expensive

#### Solution: use sum of logs instead

- remember: log(p1 \* p2) = log(p1) + log(p2)
- $\Rightarrow$  Use log probabilities instead of probabilities
- $\Rightarrow$  add instead of multiply

### **Implementation Tricks**

#2: Two passes

#### Some recognition speedup tricks are no longer possible

- need the best way to derive a symbol, not just one way
  - $\Rightarrow$  can't abort of loops early...

#### Solution: two passes

- pass 1: just recognition
- pass 2: disambiguation, with many pruned symbols at each cell

# Summary

- Hierarchical Structure
- Constituent / Phrase-based Parsing
- CFGs, Derivations
- (Some) English Syntax
- PCFG
- CKY

# Parsing with PCFG

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- This can be done quite efficiently with dynamic programming (the CKY algorithm)

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- We can also add smoothing and backoff, as before.
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# The big question

Does this work?

#### Evaluation

- Let's assume we have a parser, how do we know how good it is?
- $\Rightarrow$  Compare output trees to gold trees.

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  - But how do we compare trees?
  - Credit of 1 if tree is correct and 0 otherwise, is too harsh.
  - Represent each tree as a set of labeled spans.
    - ► NP from word 1 to word 5.
    - VP from word 3 to word 4.
    - S from word 1 to word 23.
    - ▶ ...
  - Measure Precision, Recall and F<sub>1</sub> over these spans, as in the segmentation case.

#### **Evaluation: Representing Trees as Constituents**



(by Mike Collins)

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#### Precision and Recall

Lahel	Start Point	End Point	] ,		
Laber				Label	Start Point
NP NP NP PP NP VP	1 4 4 6 7 3	2 5 8 8 8 8		NP NP PP NP VP S	1 4 6 7 3 1
5	1	ŏ	l i		

• G = number of constituents in gold standard = 7

- P = number in parse output = 6
- C = number correct = 6

$$\text{Recall} = 100\% \times \frac{C}{G} = 100\% \times \frac{6}{7} \qquad \text{Precision} = 100\% \times \frac{C}{P} = 100\% \times \frac{6}{6}$$

(by Mike Collins)

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End Point

2

5

8

8

8

8

Is this a good measure?

Why? Why not?


## **Parsing Evaluation**

How well does the PCFG parser we learned do?

Not very well: about 73%  $F_1$  score.



Problems with PCFGs



Weaknesses of Probabilistic Context-Free Grammars

Michael Collins, Columbia University

(by Mike Collins)

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Weaknesses of PCFGs

- Lack of sensitivity to lexical information
- Lack of sensitivity to structural frequencies



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$$p(t) = q(S \rightarrow NP VP) \times \\ \times q(VP \rightarrow V NP) \times \\ \times q(NP \rightarrow NNP) \times \\ \times q(NP \rightarrow NNP) \times \\ \end{pmatrix}$$

$$\begin{array}{l} \times q(\mathsf{NNP} \to IBM) \\ \times q(\mathsf{Vt} \to bought) \\ \times q(\mathsf{NNP} \to Lotus) \end{array}$$

(by Mike Collins)

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## Another Case of PP Attachment Ambiguity



(by Mike Collins)

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If  $q(NP \rightarrow NP PP) > q(VP \rightarrow VP PP)$  then (b) is more probable, else (a) is more probable. Attachment decision is completely independent of the words

(by Mike Collins)

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## A Case of Coordination Ambiguity



(by Mike Collins)

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Here the two parses have identical rules, and therefore have identical probability under any assignment of PCFG rule probabilities

(by Mike Collins)

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 $\mathcal{O} \mathcal{Q} (\mathcal{P})$ 

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