Ordering Frechet-Uryshon Filters

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Contenido

FU-spaces

Our spaces will be completely regular and Hausdorff.

Definition

A space X is called *Frechét-Uryshon* if $x \in cl(A)$, then there is a sequence $(x_n)_{n\in\mathbb{N}}$ in A such that $x_n \to x$.

The countable FAN is a *FU*-space non-metrizable.

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A countable space with just one accumulation point will be denoted by $\xi(\mathcal{F}) = \omega \cup \{\mathcal{F}\}$ where ω is discrete and $\mathcal{F}$ is a free filter on ω.

$\xi(\mathcal{F})$ is a FU-space iff $\xi(\mathcal{F})$ is sequential.

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The Frechét filter $\mathcal{F}_r$ and the *FAN*-filter are *FU*-filters.

Proposition

A filter $\mathcal{F}$ is a *FU*-filter if $\forall F \in \mathcal{F}^+ \exists A \in [F]^\omega (A \to \mathcal{F})$.

Notation

$[F]^\omega$ = infinite subsets of F, $\mathcal{F}^+ = \{A \in [\omega]^\omega : \forall F \in \mathcal{F}(A \cap F \neq \emptyset)\}$ and, for $A \in [\omega]^\omega$, $A \to \mathcal{F}$ means that $\forall F \in \mathcal{F}(|A \setminus F| < \omega)$.
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A family $\mathcal{A} \subseteq [\omega]^\omega$ is called *almost disjoint* (*AD*-family) if it is infinite and $\forall A, B \in \mathcal{A}(|A \cap B| < \omega)$.

Given an infinite $\mathcal{B} \subseteq [\omega]^\omega$, we say that an infinite family $\mathcal{A}$ is *maximal* in $\mathcal{B}$ if $\mathcal{A} \subseteq \mathcal{B}$ and $\forall B \in \mathcal{B} \exists A \in \mathcal{A}(|A \cap B| = \omega)$.

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Ideals

Given an ideal I on ω, we set

$$I^{\perp} = \{B \in [\omega]^{\omega} : \forall A \in I(|B \cap A| < \omega)\}.$$

Notation

The *dual ideal* of a filter $\mathcal{F}$ is the ideal $I_{\mathcal{F}} = \{E \subseteq \omega : \omega \setminus E \in \mathcal{F}\}$. And the *dual filter* of an ideal I is the filter $\mathcal{F}_I = \{E \subseteq \omega : \omega \setminus E \in I\}$.

Given $S \in [\omega]^{\omega}$ and a filter $\mathcal{F}$, $S \to \mathcal{F}$ iff $S \in I_{\mathcal{F}}^{\perp}$. A filter $\mathcal{F}$ is a *FU*-filter iff $(I_{\mathcal{F}}^{\perp})^{\perp} = I_{\mathcal{F}}$.

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Example

If $\mathcal{A}$ is an AD-family, then

$$\mathcal{F}_{\mathcal{A}} = \{F \subseteq \omega : \forall A \in \mathcal{A}(A \subseteq^* F)\}$$

is a FU-filter.

Theorem[P. Simon, 1998]

A filter $\mathcal{F}$ is a FU-filter iff there is an AD-family $\mathcal{A}$ maximal in $I_{\mathcal{F}}^{\perp}$ such that $\mathcal{F} = \mathcal{F}_{\mathcal{A}}$.

The FAN-filter is precisely to $\mathcal{F}_{\mathcal{P}}$, where $\mathcal{P}$ is an infinite partition of ω in infinite subsets.

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For each *AD*-family $\mathcal{A}$, we let

$$I(\mathcal{A}) = \{E \subseteq \omega : \exists A_0, ..., A_k \in \mathcal{A}(E \subseteq^* A_0 \cup .... \cup A_k)\}.$$

If I is an ideal, then $I^+ = \mathcal{P}(\omega) \setminus I = \mathcal{F}^+$.

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For each *AD*-family $\mathcal{A}$, we let $S_{\mathcal{A}} = \mathcal{F}_{I(\mathcal{A})}$.

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Definition

An *AD*-family $\mathcal{A}$ is *nowhere maximal almost disjoint family* (*NMAD*-family) if for every $X \in I(\mathcal{A})^+$ there is $A \in \mathcal{A}^{\perp} \cap [X]^{\omega}$.

Proposition[P. Simon 2008]

$S_{\mathcal{A}}$ is a *FU*-filter iff $\mathcal{A}$ is a *NMAD*-family.

Example

If $\mathcal{P}$ is an infinite partition of ω in infinite subsets, then $S_{\mathcal{P}}$ is a *FU*-space with countable base.

If $\mathcal{A}$ is an *AD*-family, then $\chi(S_{\mathcal{A}}) = |\mathcal{A}|$.

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Theorem

For a *NMAD*-family $\mathcal{A}$ and an *AD*-family $\mathcal{B}$ with $\mathcal{A} \cap \mathcal{B} = \emptyset$, the following are equivalent:

- $S_{\mathcal{A}} = \mathcal{F}_{\mathcal{B}}$.
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Pre-orders

Given a filter $\mathcal{F}$ and a function $f : \omega \to \omega$, then

$$f[\mathcal{F}] = \{F \subseteq \omega : f^{-1}(F) \in \mathcal{F}\}$$

is a filter.

Definition

Let $\mathcal{F}$ and $\mathcal{G}$ be filters on ω.

- *(Katětov-order)* $\mathcal{F} \leq_K \mathcal{G}$ if there is a function $f : \omega \to \omega$ such that if $F \in \mathcal{F}$, then $f^{-1}(F) \in \mathcal{G}$ ($f[\mathcal{F}] \subseteq \mathcal{G}$).
- *(Rudin-Keisler order)* $\mathcal{F} \leq_{RK} \mathcal{G}$ if there is a function $f : \omega \to \omega$ such that $F \in \mathcal{F}$ iff $f^{-1}(F) \in \mathcal{G}$ ($f[\mathcal{F}] = \mathcal{G}$).
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- *(Katětov-order)* $\mathcal{F} \leq_K \mathcal{G}$ if there is a function $f : \omega \to \omega$ such that if $F \in \mathcal{F}$, then $f^{-1}(F) \in \mathcal{G}$ ($f[\mathcal{F}] \subseteq \mathcal{G}$).
- *(Rudin-Keisler order)* $\mathcal{F} \leq_{RK} \mathcal{G}$ if there is a function $f : \omega \to \omega$ such that $F \in \mathcal{F}$ iff $f^{-1}(F) \in \mathcal{G}$ ($f[\mathcal{F}] = \mathcal{G}$).
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Pre-orders

Given a filter $\mathcal{F}$ and a function $f : \omega \to \omega$, then

$$f[\mathcal{F}] = \{F \subseteq \omega : f^{-1}(F) \in \mathcal{F}\}$$

is a filter.

Definition

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$\mathcal{F} \leq_{RK} \mathcal{G}$ implies $\mathcal{F} \leq_{K} \mathcal{G}$ and $\mathcal{F} \leq_{RK^*} \mathcal{G}$.

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The following properties hold for the orders.

- $\mathcal{F}_r \leq_K \mathcal{F}$ for every free filter $\mathcal{F}$.
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- Let $\mathcal{F}$ be a filter on ω. Then, $\mathcal{F} \leq_K \mathcal{F}_r$ iff there is $S \in [\omega]^\omega$ such that $S \to \mathcal{F}$.
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- $\mathcal{F}_\mathcal{A} \approx_K \mathcal{F}_\mathcal{B}$ for all pair of *AD*-families $\mathcal{A}, \mathcal{B}$.
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FU-filters and *RK*-order

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Write $\mathcal{F} = \mathcal{F}_{\mathcal{A}}$ where $\mathcal{A}$ is an *AD*-family. Fix $A \in \mathcal{A}$ and define $f : \omega \to \omega$ so that $f : A \to \omega$ and $f : \omega \setminus A \to \omega$ are bijective. Clearly, f witnesses $\mathcal{F}_r \leq_{RK} \mathcal{F}_{\mathcal{A}}$.

(7)

Fix $B \in \mathcal{B}$. Define $f : \omega \to \omega$ so that $f : B \to \omega$ and $f : \omega \setminus B \to \omega$ are bijective. Clearly, f witnesses $\mathcal{F}_{\mathcal{A}} \leq_{RK^*} \mathcal{F}_{\mathcal{B}}$. In a similar way, we prove that $\mathcal{F}_{\mathcal{B}} \leq_{RK^*} \mathcal{F}_{\mathcal{A}}$.

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Definition

We say that a filter $\mathcal{F}$ on ω is *relatively equivalent* to the Fréchet filter if there is $S \in \mathcal{F}$ such that $S \to \mathcal{F}$.

A *FU*-filter $\mathcal{F}$ is relatively equivalent to the Fréchet filter iff $\mathcal{F} \approx_{RK} \mathcal{F}_r$.

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Let $\mathcal{F}$ and $\mathcal{G}$ filters such that $\mathcal{G} \neq \mathcal{F}_r$ and $\mathcal{F}$ is not relatively equivalent to the Fréchet filter. If $\mathcal{F} \leq_{RK} \mathcal{G}$, then there is a surjective function $g : \omega \to \omega$ such that $g[\mathcal{G}] = \mathcal{F}$.

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(b) (c) *FU*-filters and *RK*-order

Theorem

Let $\mathcal{A}$ and $\mathcal{B}$ be AD families on ω. The following conditions are equivalent.

(1) $\mathcal{F}_{\mathcal{B}} \leq_{RK} \mathcal{F}_{\mathcal{A}}$ via the surjective function $f : \omega \to \omega$.

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If $\mathcal{F}$ and $\mathcal{G}$ are filters on ω such that $\mathcal{F} \leq_{RK} \mathcal{G}$, then $\chi(\mathcal{F}) \leq \chi(\mathcal{G})$.

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(1) $\mathcal{F}_{\mathcal{B}} \leq_{RK} \mathcal{F}_{\mathcal{A}}$ via the surjective function $f : \omega \to \omega$.

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If $\mathcal{F}$ and $\mathcal{G}$ are filters on ω such that $\mathcal{F} \leq_{RK} \mathcal{G}$, then $\chi(\mathcal{F}) \leq \chi(\mathcal{G})$.

FU-filters and RK-order

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FU-filters and *RK*-order

Product

Let $\mathcal{A} = \{A_i : i \in I\}$ be an *AD*-family and for each $i \in I$ let $\mathcal{F}_i$ be a filter on ω with $A_i \in \mathcal{F}_i$. Then we define

$$\prod_{i \in I} \mathcal{F}_i = \{F \subseteq \omega : \forall i \in I(F \in \mathcal{F}_i)\}.$$

It is evident that $\prod_{i \in I} \mathcal{F}_i$ is a filter on ω and $\prod_{i \in I} \mathcal{F}_i$ is a *FU*-filter iff $\mathcal{F}_i$ is a *FU*-filter for all $i \in I$.

The *FAN*-filter is the filter $\prod_{n<\omega} \mathcal{F}_r(P_n)$ where $\{P_n : n < \omega\}$ is a partition of ω in infinite subsets. The product of finitely many filters $\mathcal{F}_0, \ldots.., \mathcal{F}_n$ will be denote by $\mathcal{F}_0 \oplus \mathcal{F}_1 \oplus \ldots.. \oplus \mathcal{F}_n$.

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Example

Let $\{M, N\}$ be a partition of ω in two infinite subsets. Fix two bijections $f : \omega \to M$ and $g : \omega \to N$. For each pair of *AD*-families $\mathcal{A}$ and $\mathcal{B}$, we consider the filter $\mathcal{F}_{\mathcal{A}} \oplus \mathcal{F}_{\mathcal{B}} := \mathcal{F}_{f[\mathcal{A}]} \oplus \mathcal{F}_{g[\mathcal{B}]}$.

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FU-filters and *RK*-order

Theorem

If $\mathcal{A}$ is an *AD*-family that is not *MAD*, then there is an *AD*-family $\mathcal{B}$ such that $\mathcal{F}_{\mathcal{B}} \not\leq_{RK} \mathcal{F}_{\mathcal{A}}$.

Question

Given an *AD*-family non-*MAD*-family $\mathcal{A}$, is there an *AD*-family $\mathcal{B}$ such that $\mathcal{F}_{\mathcal{A}}$ and $\mathcal{F}_{\mathcal{B}}$ are *RK*-incomparable ?

For any infinite partition $\mathcal{P}$ of ω in infinite subsets, the *FU*-filters $\mathcal{F}_{\mathcal{P}}$ and $S_{\mathcal{P}}$ are *RK*-incomparable.

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FU-filters and *RK*-order

Theorem

Let $\mathcal{A}$ be a *NMAD*-family of size $\mathfrak{c}$. If $\kappa < \mathfrak{c}$ satisfies that $2^\kappa > \mathfrak{c}$, then exist 2^κ-many *FU*-filters non-*RK*-successors of $S_{\mathcal{A}}$. In particular, there is a *FU*-filter that is not *RK*-incomparable with $S_{\mathcal{A}}$.

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Contenido

FAN-filter

Theorem

Let $\mathcal{P}$ be a partition of ω. If $\mathcal{C}$ is an *AD*-family such that $|\mathcal{C}| < \mathfrak{a}$, then there is an *AD*-family $\mathcal{B}$ such that $\mathcal{B} \subseteq \mathcal{C}^{\perp}$ and $\mathcal{F}_{\mathcal{P}} \not\leq_{RK} \mathcal{F}_{\mathcal{B}}$.

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$S_{\mathcal{Q}}$

Lemma

Let $\mathcal{Q}$ be a partition of ω. For an AD-family $\mathcal{B}$ the following conditions are equivalent.

- $S_{\mathcal{Q}}$ is equivalent to $\mathcal{F}_{\mathcal{B}}$.
- $S_{\mathcal{Q}} \approx_{RK} \mathcal{F}_{\mathcal{B}}$.
- There is an partition $\mathcal{P}$ of ω such that $\mathcal{B}$ is maximal in $\mathcal{P}^{\perp}$.

Notation

Given an infinite partition $\mathcal{P} = \{P_n : n < \omega\}$ of ω,

$$Sel(\mathcal{P}) = \{A \in [\omega]^{\omega} : \forall n < \omega(|A \cap P_n| \leq 1)\}.$$

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Proposition

Let $\mathcal{Q}$ be a partition of ω. If $\mathcal{A}$ is an *AD*-family of size $< \mathfrak{b}$, then $S_{\mathcal{Q}} \not\leq_{RK} \mathcal{F}_{\mathcal{A}}$.

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Let $\mathcal{Q}$ be a partition of ω. If $\mathcal{A}$ is an *AD*-family of size $< \mathfrak{b}$, then $S_{\mathcal{Q}} \not\leq_{RK} \mathcal{F}_{\mathcal{A}}$.

$S_{\mathcal{Q}}$

Theorem

Let $\mathcal{Q} = \{Q_n : n < \omega\}$ be a partition of ω. For an *AD*-family $\mathcal{A}$ the following conditions are equivalent.

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- There is a surjection $f : \omega \to \omega$ such that if $P_n = f^{-1}(Q_n)$ for each $n < \omega$, and $\mathcal{P} = \{P_n : n < \omega\}$, then
  - $\mathcal{A} \subseteq \mathcal{P}^{\perp}$, and
  - for every $S \in Sel(\mathcal{P})$ there is $A \in \mathcal{A}$ such that $|A \cap f^{-1}(f[S])| = \omega$.
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For any partition $\mathcal{P}$, we have that $S_{\mathcal{P}} <_{RK} S_{\mathcal{P}} \oplus \mathcal{F}_{\mathcal{P}}$ and $\mathcal{F}_{\mathcal{P}} <_{RK} S_{\mathcal{P}} \oplus \mathcal{F}_{\mathcal{P}}$.

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