

Around Malykhin's problem on metrizable of the separable Fréchet groups

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joint work with Dmitri Shakhmatov

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Dedicated to Ljubiša Kočinac in occasion of his 65th birthday

- Finding the right setting
 - the scale of convergence properties
 - Malykhin's problem
 - minimal groups and locally minimal groups
- The main results and their corollaries
- The non-abelian case
- How it works (a look into the kitchen)
- Open problems

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For Tychonov topological space X one has the following implications

$$w(X) \leq \omega \xrightarrow{(1)} \text{metrizable} \xrightarrow{(2)} \text{Fréchet} \xrightarrow{(3)} \text{sequential} \xrightarrow{(4)} t(X) \leq \omega$$

For compact spaces the implication (1) is invertible, while (2) and (3) are not invertible. Invertibility of the implication (4) (known as **Moore-Mrówka problem**) is independent on the axioms of ZFC ([\diamond] Ostaszewski, Fedorchuk 1976, Balough 1989, PFA).

For compact topological groups all implications are invertible, as $t(K) = w(K)$ holds for every compact group Ismail [1981] ($\{0, 1\}^{w(K)} \hookrightarrow K$ and $\{0, 1\}^{w(K)} = w(K)$).

Actually, he showed the more precise equality $t(K) = \chi(K)$ for locally compact groups, so in particular countable tightness implies metrizability for locally compact groups.

This motivates the question of whether countable tightness in the presence of a weaker compactness-like property can still imply some of the stronger properties like sequential, Fréchet-Urysohn or metrizable.

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Malykhin's problem

By the classical metrization theorem of Birkhoff and Kakutani every first countable group is metrizable. This justified:

Malykhin's problem [1978]

Are separable Fréchet groups metrizable ?

Here separable can be replaced by countable since a topological group with a dense metrizable subgroup is metrizable.

Malykhin was aware that $\{0, 1\}^{\omega_1}$ is a separable non-metrizable Fréchet group under the assumption $\mathfrak{p} > \omega_1$ (so this is consistency problem). The next example shows why "separable" is relevant:

Example

For every uncountable cardinal κ the Σ -product G of κ -many copies of the circle group \mathbb{T} is abelian, ω -bounded (closed separable subsets are compact) and Fréchet (hence $t(G) = \omega$), while G is non-metrizable (as $\chi(G) = w(G) = \kappa$). The group G is connected.

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Minimal topological groups

Definition (Choquet, Doitchinov, Stephenson 1970/71)

A Hausdorff group topology τ on a group G is **minimal** if every Hausdorff group topology τ' on G such that $\tau' \subseteq \tau$ satisfies $\tau' = \tau$.

Theorem [Arhangel'skiĭ 1981]

$nw(G) = w(G)$ for every minimal group G .

Since $t(X) \leq nw(X) \leq w(X)$ for every space X , this shows that minimality can be used as the desired compactness like property. Minimal abelian groups of countable pseudocharacter are metrizable. It was proved by Guran, by Pestov and by Shakhmatov that this fails to be true in the non-abelian case. This confirms

- the appropriateness of the minimal groups for this kind of problems;
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Locally minimal topological groups

Theorem [R.M. Stephenson, Jr. 1971]

Every locally compact minimal abelian group is compact.

So the class of the minimal abelian groups does not contain the locally compact abelian groups (where the equality $t(K) = \chi(K)$ holds). This suggests to replace the class of minimal groups by a larger class that contains also all locally compact groups.

Definition (Morris and Pestov 1998 and Banach 2000)

A Hausdorff topological group (G, τ) is **locally minimal** if there exists a τ -neighborhood U of e_G such that U fails to be a nbd of e_G in any Hausdorff group topology on G strictly coarser than τ .

Examples of locally minimal groups are the minimal groups, the locally compact groups and the subgroups of Banach-Lie groups (so abelian locally minimal groups need not be even locally precompact, i.e., subgroups of locally compact groups).

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Theorem [R.M. Stephenson, Jr. 1971]

Every locally compact minimal abelian group is compact.

So the class of the minimal abelian groups does not contain the locally compact abelian groups (where the equality $t(K) = \chi(K)$ holds). This suggests to replace the class of minimal groups by a larger class that contains also all locally compact groups.

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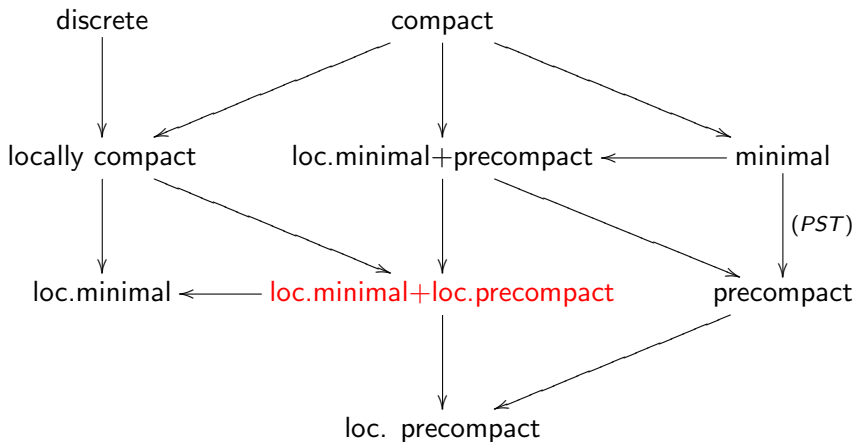
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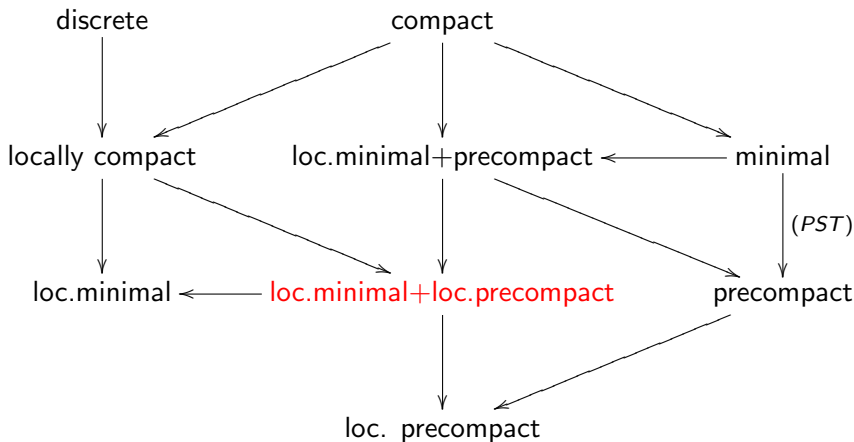
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Theorem (D. Shakhmatov, DD)

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For a topological group G let

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Then from $w(G) = \chi(G)b(G)$ and the above corollary we get:

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A locally minimal and locally precompact abelian group G satisfies $w(G) = t(G)b(G)$.

Since $t(X) \leq nw(X)$ for every space X , our next corollary strengthens the above mentioned result of Arhangel'skiĭ for abelian groups.

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$t(G) = w(G)$ for every locally minimal precompact abelian group G .

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The next corollary answers a question of Oleg Okunev [2007]:

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Minimal groups of countable tightness in the **non-abelian case**

The hypothesis "abelian" cannot be removed in the above results.

Example (Shakhmatov, DD - 1992)

Let κ be an uncountable cardinal.

- (a) Let G be the Σ -product of κ -many copies of a finite simple non-abelian group F . Then G is both ω -bounded and **totally minimal** (every Hausdorff quotient is minimal), since every closed normal subgroup of the completion F^κ of G has the form F^A for some subset $A \subseteq \kappa$.

On the other hand, G is Fréchet, hence of countable tightness, while G is non-metrizable. The group G is zero-dimensional.

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Do resolvable groups with similar properties exist ?

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- (c) Replace the finite simple group F by any center-free resolvable finite group (e.g., $F = S_3$), then a similar argument shows that the (resolvable) group G will still be minimal. Hence a resolvable minimal ω -bounded (hence, precompact) group of countable tightness (actually, Fréchet) need not be metrizable.

The group G from (c) is not totally minimal due to the following

Fact.

If G is a totally minimal countably compact group, then $G \cdot \overline{K'} = K$, where K is the completion of G .

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- (c) Replace the finite simple group F by any center-free resolvable finite group (e.g., $F = S_3$), then a similar argument shows that the (resolvable) group G will still be minimal. Hence **a resolvable minimal ω -bounded (hence, precompact) group of countable tightness (actually, Fréchet) need not be metrizable.**

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Example (Megrelishvili, DD 2012)

For every integer $m > 1$ there exists locally countably compact (so, locally precompact) **non-metrizable Fréchet minimal groups** G of arbitrarily large character with some additional options:

- (a) the center $Z(G)$ is either the circle group, or a finite cyclic group of size m ;
- (b) the quotient $G/Z(G)$ is abelian;
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Theorem (D. Shakhmatov, DD)

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Proof by induction, using as parameter the nilpotency class of the group and the fact that the center of a minimal group is still a minimal group.

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Recognizing dense locally minimal subgroups

Definition

A subgroup G of a topological group H is called:

- (a) *essential* if $G \cap N = 0$ for a closed normal subgroup N of H implies $N = 0$.
- (b) [ACDD] *locally essential* if there exists a neighborhood V of 0 in G such that $H \cap N = \{0\}$ implies $N = \{0\}$ for all closed normal subgroups N of G contained in V .

The next criterion for local minimality was inspired by Banaschewski-Prodanov-Stephenon's minimality criterion for dense subgroups, using (in a similar way) essential subgroups:

Theorem [Aussenhofer, Chasco, Dominguez, DD]

A dense subgroup G of a topological group H is locally minimal iff H is locally minimal and G is locally essential in H .

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Corollary

A locally precompact group G is locally minimal iff G is a locally essential subgroup of its completion.

Main Theorem [D. Shakhmatov – DD]

If K is a locally compact abelian group, then $\chi(K) \leq t(G)$ for every locally essential subgroup G of K .

The next lemma is relevant for the proof (\mathbb{Z}_p - the p -adic integers).

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If κ is an infinite cardinal, $p \in \mathbb{P}$ and G is a locally essential subgroup of \mathbb{Z}_p^κ , then $t(G) \geq \kappa$.

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If K is a compact abelian group, then $w(K) \leq t(G)$ for every locally essential subgroup G of K .

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Theorem

If K is a compact abelian group containing a locally essential radial subgroup, then K is metrizable.

Lemma

Let p be a prime and let N be a non-trivial abelian pro- p -group. Then no locally essential subgroup of N^{ω_1} can be radial.

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Does the tightness of an arbitrary locally minimal abelian group coincide with its character?

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Is every locally minimal abelian group of countable tightness necessarily metrizable? Is it necessarily Fréchet or sequential ?

Question

- (a) Is every totally minimal (precompact) resolvable group of countable tightness metrizable.
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Is every pseudoradial minimal abelian group necessarily metrizable?

(a space X is **pseudoradial** if, for any subset A of X , A is closed if along with any transfinite sequence it contains all its limits).

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Theorem [Alan Dow and Doyel Barman 2012]

In the Cohen model, every countable Fréchet space has π -weight at most \aleph_1 .

Here comes one of the main recent results of M. Hrusak and his student U. Ramos Garcia

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It is consistent with continuum arbitrarily large that every countable Fréchet space of weight $< \mathfrak{c}$ has countable π -weight.

Moreover, in the forcing model they build

- there is a countable Fréchet space of weight and π -weight equal to \mathfrak{c} (noticed by Alan Dow);
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In other words, Malykhin's problem is resolved for **precompact groups**.

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