The monotonically weak Lindelöf spaces

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joint work with M. Bonanzinga and F. Cammaroto

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Outline

Definitions

We assume all spaces to be *Tychonoff*.

Definition

The family of sets $\mathcal{A}$ *refines* a family of sets $\mathcal{B}$ we mean that every element of $\mathcal{A}$ is a subset of an element of $\mathcal{B}$.

Definition

A space X is *weakly Lindelöf* (wL) [Frolik, 1959] if for every open cover $\mathcal{U}$ of X there is a countable subfamily $\mathcal{U}_0 \subseteq \mathcal{U}$ with the *union dense* in X.

Definition

A space X is *monotonically Lindelöf* (mL) [Matveev, 1994] if there is a function r, henceforth called an mL-operator, that assigns to every open cover $\mathcal{U}$ of X a *countable open cover* $r(\mathcal{U})$ which refines $\mathcal{U}$ in such a way that $r(\mathcal{U})$ refines $r(\mathcal{V})$ whenever $\mathcal{U}$ refines $\mathcal{V}$.

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- $r(\mathcal{U})$ refines $\mathcal{U}$;
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Examples and counterexamples

The class of mwL spaces contains: all second countable spaces, the one-point Lindelöfication of discrete space of cardinality ω_1.

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Examples and counterexamples

Theorem 1

Let X be a space and D be a countable dense subspace of X consisting of isolated points. Then X is mwL.

Example

All Ψ-spaces are mwL.

$\Psi(\mathcal{A}) = \omega \cup \mathcal{A}$: Isbell and Mrowka's space, where $\mathcal{A}$ is an infinite maximal almost disjoint family of infinite subsets of ω.

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Recall that $L(\kappa)$, the one-point Lindelöfication of the discrete space of cardinality κ is the set $X = \kappa \cup \{p\}$ equipped with the topology in which the points of κ are isolated and every neighborhood of p has the form $\{p\} \cup (\kappa \setminus A)$ where $|A| \leq \omega$.
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Theorem (Levy-Matveev)

$L(\kappa)$ is mL iff $\kappa \leq \omega_1$.

Thus $L(\omega_1)$ is mwL.

Problem

For what $\kappa > \omega_1$ is $L(\kappa)$ mwL?

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Examples and counterexamples

Recall that the *Alexandroff duplicate AD(X)* of the topological space X is the set $X \times 2$ where the points of $X \times \{1\}$ are isolated while a basic neighborhood of a point $(x, 0) \in X \times \{0\}$ takes the form $(U \times 2) \backslash \{(x, 1)\}$ where U is a neighborhood of x in X.

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If X is a second countable space, then $AD(X)$ is mwL.

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Examples and counterexamples

Levy and Matveev proved that the one-point compactification of the discrete space of cardinality $\geq \omega_1$ is not mL.

Theorem

Let $\kappa \leq \mathfrak{c}$. Then the one-point compactification of the discrete space of cardinality κ is mwL.

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For what cardinals $\kappa > \mathfrak{c}$ is the one-point compactification of the discrete space of cardinality κ mwL?

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Subspaces

Recall that any hereditarily Lindelöf space having a base σ-(linearly ordered by $\supset$) is mL (see Levy and Matveev).

Theorem

Any space with σ-(linearly-ordered by $\supset$) π-base is mwL.

Corollary

Every space X with a dense countable set D of points of countable character is mwL.

Remark

In particular, every separable first countable space is mwL.

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Closed and regular closed subspaces

Recall that mL is preserved by closed subspaces. That mwL is not preserved by closed subspaces because a Ψ-space contains an uncountable closed discrete subspace which of course is not mwL.

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Example

There is a Hausdorff space X and a dense subspace $Y \subseteq X$ such that Y is mwL but X is not.

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Uncountable products

Levy and Matveev shown that if X is a dense subspace in the product of uncountably many nontrivial (that is T_1 and consisting of at least two points) factors, then X is not mL at any point.

Theorem

If X is a dense subspace in the product $Y = \prod_{a \in A} Y_a$, where $|A| > \omega$ and for each a, Y_a is a regular space and $|Y_a| \geq 2$, then X is not mwL at any point.

The following are immediate corollaries from previous Theorem:

- 2^κ is not mwL whenever $\kappa > \omega$
- A dense countable subspace in 2^κ (where $\omega_1 \leq \kappa \leq \mathfrak{c}$) is an example of a countable space which is not mwL (at any point).
- $C_p(X)$ is mwL iff X is countable.

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Cardinal Functions

Theorem

Every Tychonoff space of weight $\leq \mathfrak{c}$ can be embedded in an mwL Tychonoff space as a closed subspace.

So we see that an mwL space may have cardinality $\geq 2^{\mathfrak{c}}$, extent $\geq \mathfrak{c}$ and character $\geq \mathfrak{c}$.

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Cardinal Functions

Theorem

Every Tychonoff space of weight $\leq \mathfrak{c}$ can be embedded in an mwL Tychonoff space as a closed subspace.

So we see that an mwL space may have cardinality $\geq 2^{\mathfrak{c}}$, extent $\geq \mathfrak{c}$ and character $\geq \mathfrak{c}$.

Levy and Matveev asked if the weight of every mL space is not greater than $\mathfrak{c}$ and noted that a similar question makes sense apparently for any other "reasonable" cardinal function.

Further problems

Problem

If X is an mwL space, does it follow that $w(X) \leq \mathfrak{c}$? What can one say about other cardinal invariants of X?

Problem

Is $\omega^\star = \beta\omega \backslash \omega$ mwL?

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Is mwL preserved by closed irreducible maps?

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Suppose all continuous images of X are mwL. What can be concluded about X?

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Is every mwL LOTS or GO-space mL? In particular, is $[0, \omega_1]$ mwL?

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Final

Thank you!!!