A selective version of c.c.c.

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Definitions

An R-separable space is a space where, for every sequence $(D_n)_{n\in\omega}$ of dense subsets, one can find $(d_n)_{n\in\omega}$ such that $\{d_n : n \in \omega\}$ is dense and each $d_n \in D_n$.

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Definition

We say that a topological space X is a **selectively c.c.c.** space if, for every sequence $(\mathcal{A}_n)_{n\in\omega}$ of maximal pairwise disjoint open families, one can find a sequence $(A_n)_{n\in\omega}$ such that $\bigcup_{n\in\omega} A_n$ is dense in X and each $A_n \in \mathcal{A}_n$.

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Proof.

In [1], Bella, Bonanzinga and Matveev shown that there is a countable dense subspace of $2^{cov(\mathcal{M})}$ that is not R-separable. By the previous result such a space is selectively c.c.c.. □

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For every $s \in \omega^{<\omega}$, let $V_s = \{t \in \omega^{<\omega} : s \subset t\}$. Then, consider the topology over $\omega^{<\omega}$ where, for each $s \in \omega^{<\omega}$, an basic open neighborhood for it is of the form

$$V_s \setminus \bigcup_{t \in \mathcal{F}} V_t$$

where $\mathcal{F} \subset V_s$ and for every $n \in \omega$, $\{t \in \mathcal{F} : dom t = n\}$ is finite.

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where $\mathcal{F} \subset V_s$ and for every $n \in \omega$, $\{t \in \mathcal{F} : dom t = n\}$ is finite.
For each $n \in \omega$, let $\mathcal{C}_n = \{V_t : t \in \omega^{<\omega}, dom t = n\}$. Then $(\mathcal{C}_n)_{n \in \omega}$ witnesses that such a space is not selectively c.c.c.. $\square$

Products

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This question is more interesting when one see the results of Bonanzinga, Cammaroto, Pansera and Tsaban in [2].

Bibliography

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Announcement

August, 2013, São Paulo - Brazil.
In honor of the (20th + something) anniversary of Ofelia Alas.
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