A TASTE OF SET THEORY: EXERCISE NUMBER 6

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1. Prove the following assertions:

(a) Assume that $f : \alpha \to ON$ is an increasing function (that is, for all $\beta < \gamma < \alpha$, $f(\beta) < f(\gamma)$). Then for each $\beta < \alpha$, $f(\beta) \geq \beta$.

(b) Let α be an ordinal and $F \subseteq \alpha$. Then the order type of F is $\leq \alpha$.

Hint for (b): Let β be the order type of F, and $f : \beta \to F$ be an order isomorphism. Then f is increasing.

2. Prove that the following are equivalent:

(1) $|A| \leq |B|$.

(2) There is a function $g : B \to A$ which is onto.

3. Prove, by defining a bijection between the relevant sets, that the cardinal arithmetic operations satisfy:

$$\kappa^{\lambda+\mu} = \kappa^{\lambda} \cdot \kappa^{\mu}$$

for all cardinals κ, λ, μ.

Good luck!

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