A TASTE OF SET THEORY: EXERCISE 4

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Question 2.12 (Page 26). Let F be a set of ordinals. Prove that $\cup F$ is an ordinal.

Question 2.15 (Page 27). Prove that the following properties are equivalent for an ordinal α:

(1) α is a limit ordinal.
(2) $\alpha = \sup\{\xi : \xi < \alpha\}$.
(3) For each cofinal $A \subseteq \alpha$, $\alpha = \sup A$.

Question 3.7 (Page 30). Let α, β be ordinals. Prove:

(1) $\alpha + 1 = S(\alpha)$.
(2) $\alpha + S(\beta) = S(\alpha + \beta)$.
(3) If $\alpha < \beta$, then for each ordinal γ, $\gamma + \alpha < \gamma + \beta$.
(4) If $\alpha < \beta$, then for each ordinal γ, $\alpha + \gamma \leq \beta + \gamma$.
(5) Give an example showing that $\leq$ cannot be improved to $<$ in (4).

Question 4.2 (Page 31). Prove that for each natural number n, $n \cdot \omega = \omega$.

Good luck!

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