A TASTE OF SET THEORY: EXERCISE NUMBER 4

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Remark. In all of our exercises, you may use the Axiom of Choice when it seems needed.

1. Recall that a (binary) relation E on a set P is *well-founded* if in each nonempty $A \subseteq P$ there is an element a which is minimal with respect to E (i.e., there is no $b \in A$ such that $b\,E\,a$). Prove the equivalence of the following assertions:

(1) E is well-founded.
(2) There is no sequence of elements $a_0, a_1, a_2, \ldots \in P$ such that for each n, $a_{n+1}\,E\,a_n$.

2. Recall that for a set A,

$$|A| = \min\{\alpha : \exists\text{bijection } f : \alpha \to A\}.$$

For sets A, B, write $A \preceq B$ if there exists a one-to-one function $f : A \to B$. Prove that the following are equivalent:

(1) $A \preceq B$.
(2) There is a function $g : B \to A$ which is onto.
(3) $|A| \leq |B|$.

3. Prove that the cardinal arithmetic operations satisfy:

$$\kappa^{\lambda+\mu} = \kappa^\lambda \cdot \kappa^\mu$$

for all cardinals κ, λ, μ.

Good luck!

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