A TASTE OF SET THEORY: HINT FOR EXERCISE NUMBER 2

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Here is a hint for:

4. Recall that a set X is *inductive* if: $\emptyset \in X$, and for each $x \in X$, $x \cup \{x\} \in X$ too. Define

$$\omega = \bigcap \{X : X \text{ is inductive}\}.$$

Prove the following facts:

(1) If a set X is inductive, then $X \cap \mathit{Ord}$ is inductive.
(2) ω is an ordinal.
(3) ω is a *limit* ordinal.
(4) ω is the *least* nonzero limit ordinal.

Fill in the details in the following claims:

ω is a set (of ordinals).
Since ω is a set, it cannot be that $\mathit{Ord} \subseteq \omega$.
So let γ be the first ordinal such that $\gamma \notin \omega$.
$\gamma \subseteq \omega$.
Prove that γ is inductive, and therefore $\omega \subseteq \gamma$.
γ is an ordinal. It is not a successor ordinal.
$\gamma \neq 0$.
If $\alpha < \gamma$ and $\alpha \neq 0$ and α is a limit ordinal, then α is inductive, so $\alpha \in \omega \subseteq \alpha$. Contradiction.

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