FUNDAMENTAL TOPICS IN MATHEMATICS: EXERCISE 1

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Hints to some of the questions below are available in the material posted in the course's webpage.

1. Let $\langle A, <_A\rangle$ and $\langle B, <_B\rangle$ be well-ordered sets. Prove that the *lexicographic order* $<_{lex}$ on $A \times B$, defined by

$$(a_1, b_1) <_{lex} (a_2, b_2) \Leftrightarrow a_1 <_A a_2 \text{ or } (a_1 = a_2 \text{ and } b_1 <_B b_2)$$

is a well-ordering of $A \times B$.

2. Let $\langle A, <_A\rangle$ and $\langle B, <_B\rangle$ be well-ordered sets. Prove that one, and exactly one, of the following cases holds:

(1) $\langle A, <_A\rangle \cong \langle B, <_B\rangle$.

(2) There is $b \in B$ such that $\langle A, <_A\rangle \cong \langle \overset{b}{B}, <_B\rangle$.

(3) There is $a \in A$ such that $\langle \overset{a}{A}, <_A\rangle \cong \langle B, <_B\rangle$.

3. Prove that the two versions of the Axiom of Choice presented in class ("choice set" and "choice function") are equivalent.

4. Prove that the Axiom of Choice is equivalent to *partial invertibility of onto functions*, i.e. to the statement: For each onto function $g : A \to B$, there is a one-to-one function $f : B \to A$, such that $g(f(x)) = x$ for all $x \in B$.

5. Prove that the Well-Ordering Principle implies the Axiom of Choice.

Good luck!

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