Cryptography or Ramsey theory? The cool or the beautiful

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Topic I: Public Key Crypto (The cool)

Key Agreement Protocols

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WIN/WIN: Better KAP / Efficient algorithms.

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There are similar algorithms in the noncommutative case.

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Learn and use cutting-edge software and C/C++ libraries to attack the underlying mathematical problems.

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Requirements:

1. Top student.
2. Loves programming and optimizing ("hacker").

Topic II: Ramsey theory (The beautiful)

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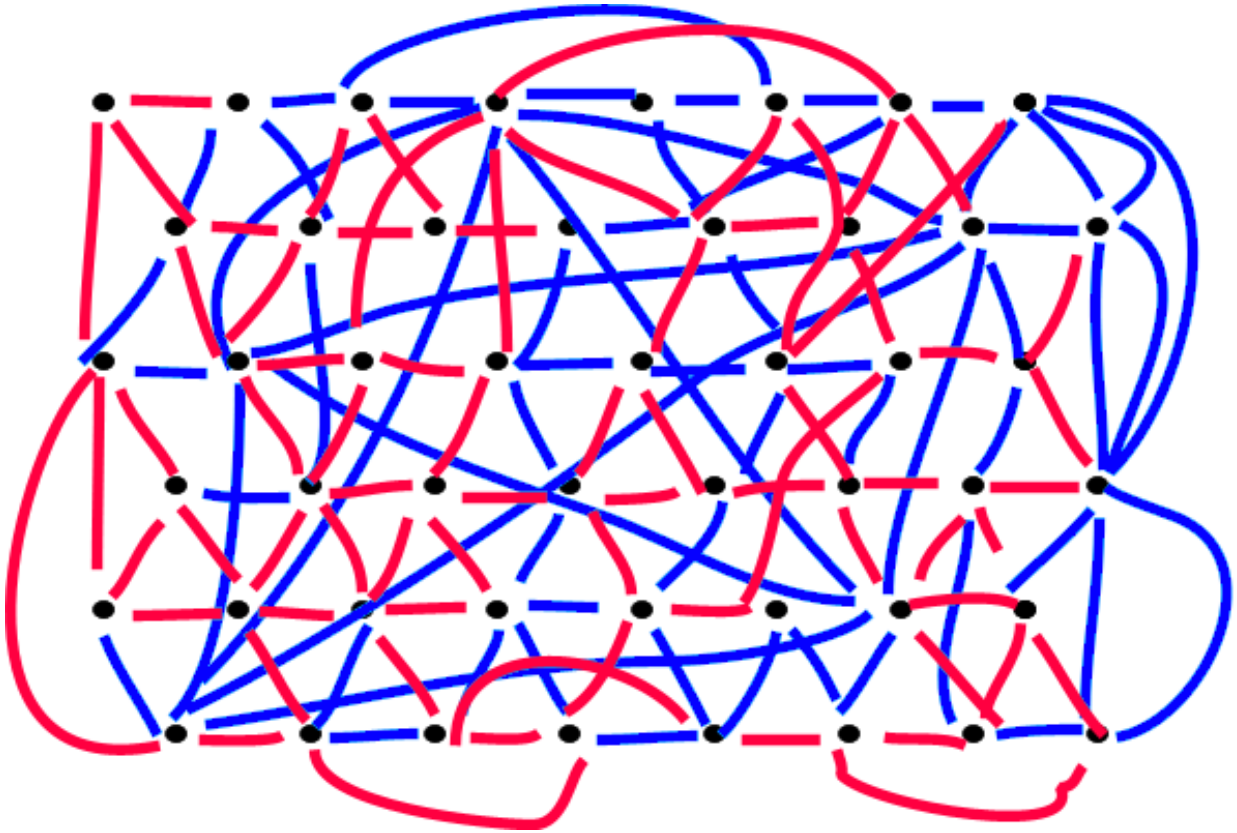

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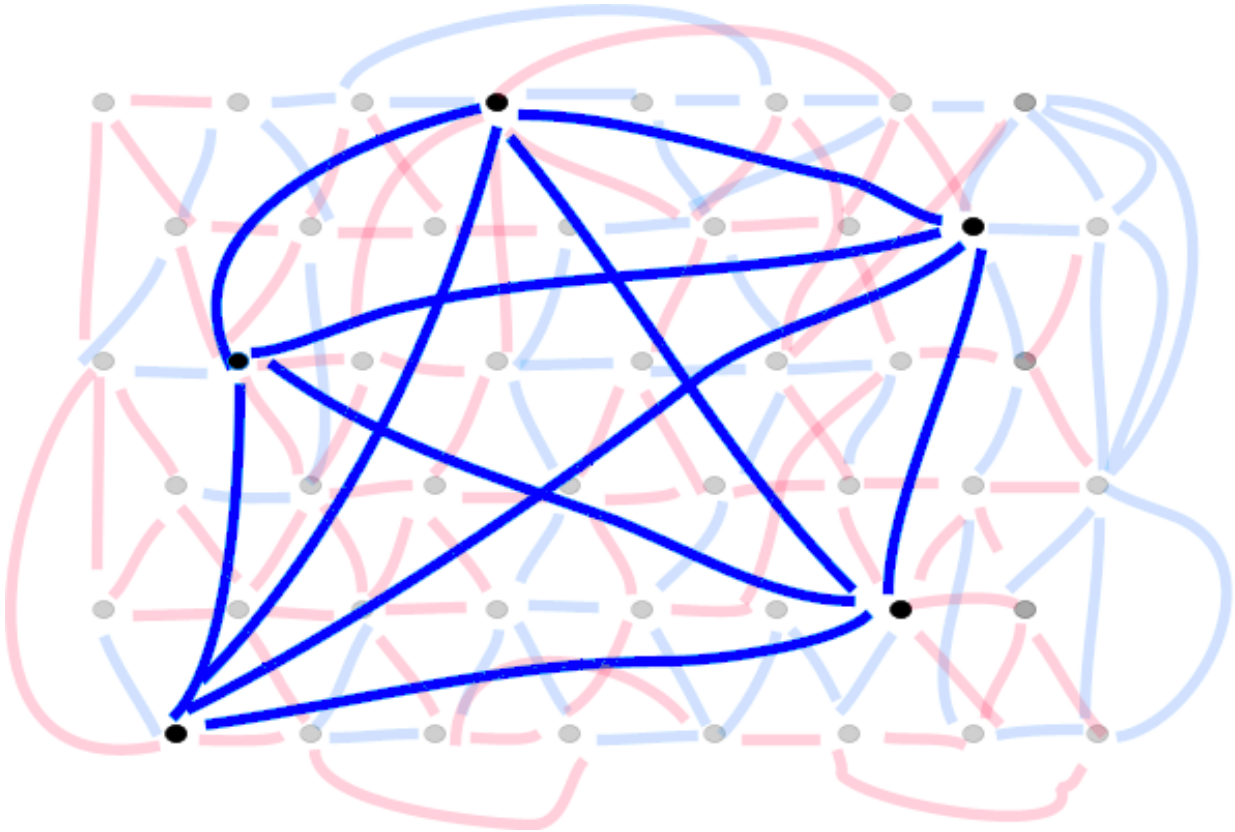

Then $\exists$ infinite complete monochromatic subgraph.

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