

פתרון תרגיל 3 (חלק ב')

$$\begin{pmatrix} 1 & 2 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 3 & | & 0 & 1 & 0 \\ 3 & 8 & 7 & | & 0 & 0 & 1 \end{pmatrix} \begin{matrix} R_3 = R_3 - 3R_1 \\ \\ \end{matrix} \rightarrow \begin{pmatrix} 1 & 2 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 3 & | & 0 & 1 & 0 \\ 0 & 2 & 7 & | & -3 & 0 & 1 \end{pmatrix} \begin{matrix} R_1 = R_1 - 2R_2 \\ \\ R_3 = R_3 - 2R_2 \end{matrix} \quad \text{א. 1.}$$

$$\begin{pmatrix} 1 & 0 & -6 & | & 1 & -2 & 0 \\ 0 & 1 & 3 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & -3 & -2 & 1 \end{pmatrix} \begin{matrix} R_1 = R_1 + 6R_3 \\ \\ R_2 = R_2 - 3R_3 \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & | & -17 & -14 & 6 \\ 0 & 1 & 0 & | & 9 & 7 & -3 \\ 0 & 0 & 1 & | & -3 & -2 & 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -17 & -14 & 6 \\ 9 & 7 & -3 \\ -3 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 6 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & -6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

ב.

$$\begin{pmatrix} 1 & -2 & 0 & | & 1 & 0 & 0 \\ 2 & -3 & 1 & | & 0 & 1 & 0 \\ 1 & 1 & 5 & | & 0 & 0 & 1 \end{pmatrix} \begin{matrix} R_2 = R_2 - 2R_1 \\ \\ R_3 = R_3 - R_1 \end{matrix} \rightarrow \begin{pmatrix} 1 & -2 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & -2 & 1 & 0 \\ 0 & 3 & 5 & | & -1 & 0 & 1 \end{pmatrix} \begin{matrix} R_1 = R_1 + 2R_2 \\ \\ R_3 = R_3 - 3R_2 \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 2 & | & -3 & 2 & 0 \\ 0 & 1 & 1 & | & -2 & 1 & 0 \\ 0 & 0 & 2 & | & 5 & -3 & 1 \end{pmatrix} \begin{matrix} R_3 = R_3 / 2 \\ \\ \end{matrix} \rightarrow \begin{pmatrix} 1 & 0 & 2 & | & -3 & 2 & 0 \\ 0 & 1 & 1 & | & -2 & 1 & 0 \\ 0 & 0 & 1 & | & 2.5 & -1.5 & 0.5 \end{pmatrix} \begin{matrix} R_1 = R_1 - 2R_3 \\ \\ R_2 = R_2 - R_3 \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & | & -8 & 5 & -1 \\ 0 & 1 & 0 & | & -4.5 & 2.5 & -0.5 \\ 0 & 0 & 1 & | & 2.5 & -1.5 & 0.5 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} -8 & 5 & -1 \\ -4.5 & 2.5 & -0.5 \\ 2.5 & -1.5 & 0.5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

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$$\begin{aligned}
 & \left(\begin{array}{cc|cc} 1+\sqrt{3} & 5+\sqrt{3} & 1 & 0 \\ 2\sqrt{3} & 10 & 0 & 1 \end{array} \right) R_1 \leftrightarrow R_2 / 2\sqrt{3} \rightarrow \left(\begin{array}{cc|cc} 1 & \frac{5\sqrt{3}}{3} & 0 & \frac{\sqrt{3}}{6} \\ 1+\sqrt{3} & 5+\sqrt{3} & 1 & 0 \end{array} \right) R_2 = R_2 - (1+\sqrt{3})R_1 \rightarrow \\
 & \left(\begin{array}{cc|cc} 1 & \frac{5\sqrt{3}}{3} & 0 & \frac{\sqrt{3}}{6} \\ 0 & -\frac{2\sqrt{3}}{3} & 1 & -\frac{1}{2} - \frac{\sqrt{3}}{6} \end{array} \right) R_2 = -\frac{\sqrt{3}}{2} R_2 \rightarrow \left(\begin{array}{cc|cc} 1 & \frac{5\sqrt{3}}{3} & 0 & \frac{\sqrt{3}}{6} \\ 0 & 1 & -\frac{\sqrt{3}}{2} & \frac{1}{4} + \frac{\sqrt{3}}{4} \end{array} \right) \\
 & R_1 = R_1 - \frac{5\sqrt{3}}{3} R_2 \rightarrow \left(\begin{array}{cc|cc} 1 & 0 & \frac{5}{2} & -\frac{5}{4} - \frac{\sqrt{3}}{4} \\ 0 & 1 & -\frac{\sqrt{3}}{2} & \frac{1}{4} + \frac{\sqrt{3}}{4} \end{array} \right)
 \end{aligned}$$

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$$A^{-1}X^{-1}(A^T)^{-1} = B$$

$$\underbrace{\textcolor{blue}{A}A^{-1}}_I X^{-1} \underbrace{(A^T)^{-1}\textcolor{green}{A}^T}_I = \textcolor{blue}{A}B\textcolor{green}{A}^T$$

$$X^{-1} = ABA^T \Rightarrow X = (X^{-1})^{-1} = (ABA^T)^{-1}$$

$$ABA^T = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 11 & 6 & 3 \\ 2 & 1 & 0 \\ 5 & 2 & 1 \end{pmatrix}$$