FORCING THEORY: EXERCISE 5

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For all of this exercise, fix a countable transitive model of (enough of) ZFC, a poset $\langle P, \leq, 1\rangle \in M$, and an M-generic filter G (for P).

1. Prove:

(a) "τ is a P-name" is absolute for transitive models of ZFC.

(b) The interpretation function $\tau \mapsto \tau_G$ is absolute for transitive models of ZFC.

2. Prove:

(a) Assume that M' is a countable transitive model of (enough of) ZFC, such that $M \subseteq M'$ and $G \in M'$. Then $M[G] \subseteq M'$.

(b) $M[G]$ is a countable and transitive set.

3. $M[G]$ satisfies the axioms of Extensionality, Foundation, Pairing, and Union.

Hint. For Pairing, interpret the name $\{(\sigma, 1), (\tau, 1)\}$.
For Union, let $\mathrm{dom}(\tau) = \{\sigma : \exists p\ (\sigma, p) \in \tau\}$, and interpret $\bigcup \mathrm{dom}(\tau)$ to get a set containing τ_G.

Definition. $p \in P$ is called an *atom* if for all q, r stronger than p, we have that q, r are compatible.

Recall that if there are no atoms in P, then for each M-generic filter G, $G \notin M$.

4. If $p \in P$ is an atom, then there is a filter G intersecting *all* dense subsets of P, and such that $G \in M$.

Good luck!

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