FORCING THEORY: EXERCISE 5

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For all of this exercise, fix a countable transitive model M of ZFC, a poset $\langle \mathbb{P}, \leq, 1\rangle \in M$, and an $\mathbb{P}$-generic filter G over M.

1. Prove the Extension Lemma for $\Vdash^*$: If $p \Vdash^* \varphi$ and $q \leq p$, then $q \Vdash^* \varphi$, either.

2. Complete the remaining cases in the inductive proof of the Truth Lemma for $\Vdash^*$:

$$M[G] \models \varphi \Leftrightarrow \exists p \in G, p \Vdash^* \varphi.$$

(a) Case 1(b): $p \Vdash^* \tau \neq \sigma$.
(b) Case 2(a): $p \Vdash^* \varphi \vee \psi$.
(c) Case 2(b): $p \Vdash^* \exists v\ \varphi(v)$.

3. Kunen, Chapter VII, Exercise (A9).

Definitions. $A \subseteq \mathbb{P}$ is an *antichain* if for each distinct $a, b \in A$, a, b are incompatible. A is a *maximal antichain* if it is an antichain and no antichain properly contains it.

4. Kunen, Chapter VII, Exercise (A12), not including the "Furthermore" part.

Good luck!

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