FORCING THEORY: EXERCISE 4

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For all of this exercise, fix a countable transitive model of (enough of) ZFC, a poset $\langle P, \leq, 1\rangle \in M$, and an M-generic filter G (for P).

1. Prove that $M[G]$ is a transitive set.

2. $M[G]$ satisfies the axioms of Extensionality, Foundation, Pairing, and Union.
Hint. For Pairing, interpret the name $\{(\sigma, 1), (\tau, 1)\}$.
For Union, let $\mathrm{dom}(\tau) = \{\sigma : \exists p\ (\sigma, p) \in \tau\}$, and interpret $\bigcup \mathrm{dom}(\tau)$ to get a set containing τ_G.

Definition. $p \in P$ is called an *atom* if for all q, r stronger than p, we have that q, r are compatible.

Recall that if there are no atoms in P, then for each M-generic filter G, $G \notin M$.

3. If $p \in P$ is an atom, then there is a filter G intersecting *all* dense subsets of P, and such that $G \in M$.

4. Let $\mathbb{C}$ be Cohen's forcing, G be $\mathbb{C}$-generic over M, and $F = \bigcup G$. Let $A = \{n \in \omega : F(n) = 1\}$. Prove that for each k, A contains an arithmetic progression of length k, and is disjoint from (another) arithmetic progression of length k. [1]

Good luck!

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[1]In fact, your proof—which should use a density argument—shows things are even more flexible.