A Simple Problem of Error Analysis

Suppose I want to compute $f'(x)$ where $f(x) = x^3$, but for some reason I insist to do it making the 3 following errors:

1. Instead of putting in the right value of x, I put in $x + \Delta x$, where Δx is some small error. (Thus we have *data error.*)
2. Instead of computing the derivative directly I use the approximation

$$\frac{f(x+h) - f(x)}{h}$$

   where h is small. (Thus we have *algorithmic error.*)
3. I do all calculations in double precision on a computer. (Thus we have *rounding error.*)

The magnitudes of the errors associated with these three effects are roughly $6x\Delta x$, $3xh$ and $2\epsilon_{\text{machine}} x^3/h$ respectively. Depending on what h I take, the error will behave very differently.

The a graph of the total error in the above procedure, with $x = 10$, $\Delta x = 0.00001$ and h running from 10^{-9} to 10^{-5} was found by running the following lines of code in matlab or scilab:

```
x=10.00001;
h=10.^[-9:0.02:-5];
f=((h+x).^3-x^3)./h;
err=abs(f-300);
plot(log10(h),err)
```

Here's the result: