bilocal   M⁻¹NLS2 (2.23)

ham.struc. $U_t = V_x + 3UU_x$

$V_t = U_{xxx} + V_xU + 2U_xV$

$$\boxed{\begin{array}{l} U = j - \bar{j} \\ V = (j + \bar{j})_x - \frac{1}{2}(j + \bar{j})^2 \end{array}}$$

M⁻¹NLS (2.7)   bilocal

$A_t = A_{xx} + (2A^2 - B)_x$ ham.struc.

$B_t = -B_{xx} + (4AB)_x$

$$\boxed{\begin{array}{l} A = \psi_x/2\psi \\ B = \psi\bar{\psi} \end{array}}$$

struc.1 non-loc   NLS2 (2.15)

struc.2 local $j_t = (j_x + j^2 - 2j\bar{j})_x$

$\bar{j}_t = (-\bar{j}_x - \bar{j}^2 + 2j\bar{j})_x$

$$\boxed{\begin{array}{l} j = QP \\ \bar{j} = Q_x/Q - QP \end{array}}$$

NLS (2.1)   struc.1 local

$\psi_t = \psi_{xx} - 2\psi^2\bar{\psi}$ struc.2 non-loc

$\bar{\psi}_t = -\bar{\psi}_{xx} + 2\bar{\psi}^2\psi$

$$\boxed{\begin{array}{l} \psi = -\tilde{P} \\ \bar{\psi} = \tilde{Q}_x - \tilde{Q}^2\tilde{P} \end{array}}$$

DNLS

$P_t = P_{xx} + 2(P^2Q)_x$

$Q_t = -Q_{xx} + 2(Q^2P)_x$

$$\boxed{\begin{array}{l} P = p_x \\ Q = q \end{array}}$$

QNLS

$\tilde{P}_t = \tilde{P}_{xx} + 2\tilde{P}^2(\tilde{Q}_x - \tilde{P}\tilde{Q}^2)$

$\tilde{Q}_t = -\tilde{Q}_{xx} + 2\tilde{Q}^2(\tilde{P}_x + \tilde{Q}\tilde{P}^2)$

$$\boxed{\begin{array}{l} \tilde{P} = -e^{h-\bar{h}}h_x \\ \tilde{Q} = e^{\bar{h}-h} \end{array}}$$

AuxNLS (3.2)

$p_t = p_{xx} + 2p_x^2q$

$q_t = -q_{xx} + (2q^2p_x)_x$

$$\boxed{\begin{array}{l} p = T \\ q = S_x/T_x \end{array}}$$

PotNLS (2.18)

$h_t = h_{xx} + h_x^2 - 2h_x\bar{h}_x$

$\bar{h}_t = -\bar{h}_{xx} - \bar{h}_x^2 + 2h_x\bar{h}_x$

$$\boxed{\begin{array}{l} h = -\sigma \\ \bar{h} = \sigma + \ln \tau_x \end{array}}$$

bilocal inv.   Ur-NLS1 (2.28)

ham.struc. $T_t = T_{xx} + 2T_xS_x$

$S_t = 2S_xT_{xx}/T_x + 3S_x^2 - S_{xx}$

Ur-NLS2 (2.36)   bilocal inv.

$\tau_t = -\tau_{xx} - 2\tau_x\sigma_x$ ham.struc

$\sigma_t = \sigma_{xx} - 3\sigma_x^2 - 2\sigma_x\tau_{xx}/\tau_x$

Other Miura maps:	UrNLS1 to PotNLS:	$h = S$	$\bar{h} = -S + \ln(S_xT_x^{-1})$
	PotNLS to DNLS:	$P = e^{-(h+\bar{h})}h_x$	$Q = e^{h+\bar{h}}$
	QNLS to NLS2:	$j = -\tilde{P}\tilde{Q}$	$\bar{j} = \tilde{Q}_x/\tilde{Q} - \tilde{P}\tilde{Q}$
	NLS2 to M⁻¹NLS:	$A = \frac{1}{2}(j - \bar{j} + j_x/j)$	$B = j\bar{j}$
Useful composite maps:	PotNLS to NLS:	$\psi = e^{h-\bar{h}}h_x$	$\bar{\psi} = e^{\bar{h}-h}\bar{h}_x$
	PotNLS to NLS2:	$j = h_x$	$\bar{j} = \bar{h}_x$

Figure 1. The Equations of The NLS Tower.