hep-th/9210137
IASSNS-HEP-92/76
October 1992

Actions for Integrable Systems and Deformed Conformal Theories*

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Abstract

I report on work on a Lagrangian formulation for the simplest 1+1 dimensional integrable hierarchies. This formulation makes the relationship between conformal field theories and (quantized) 1+1 dimensional integrable hierarchies very clear.

1. Introduction

It is a not widely appreciated fact that at least some (1+1) dimensional integrable hierarchies (of KdV type), in their "second" hamiltonian formulation, can be derived from an action principle[1]. Interestingly, as I will show later, *the same action, considered as a functional of different sets of fields, can give rise to different "gauge equivalent" integrable hierarchies*[2]. But the main merit of this Lagrangian approach to integrable systems is that when we quantize these theories in the obvious way, we see the relationship between (deformed) conformal field theories and quantized integrable systems emerge naturally. We need an action for integrable systems in their "second" hamiltonian formulation, because it is the "second" Poisson bracket algebras of integrable systems that are related to the operator algebras of conformal field theory[3].

I will focus here on the KdV action, summarizing the results of ref.4, but I will also give an action for NLS[5] hierarchy. In section 2, after presenting some

* Based on a talk given at the NSERC-CAP Workshop, *Quantum Groups, Integrable Models and Statistical Systems*, Kingston, Ontario, Canada, July 1992

results in classical KdV theory, including an explanation of the notion of the "gauge equivalence class" of the KdV equation, I give the KdV action. In section 3 I show how to quantize the theory defined by the KdV action, to obtain the usual notions of "quantum KdV" and "quantum MKdV"; we also obtain very naturally the result of Zamolodchikov[6], that the quantum KdV hamiltonians are conserved quantities in a certain deformation of the minimal conformal models. Similar treatment of the NLS action, given in section 4, reveals the result that the quantum NLS hamiltonians are conserved quantities in a certain deformation of the parafermion and $SL(2)/U(1)$ coset models.

2. Classical KdV Theory and the KdV Action

The meaning of the statement "the KdV equation is gauge equivalent to the MKdV equation" is that via the "Miura map" $u = j_x - \frac{1}{2}j^2$, a) a solution to the MKdV equation

$$j_t = j_{xxx} - \tfrac{3}{2}j^2 j_x \tag{1}$$

generates a solution of the KdV equation

$$u_t = u_{xxx} + 3uu_x \tag{2}$$

and b) the "second" Poisson bracket structure of the MKdV equation[7]

$$\{j(x), j(x')\} = \partial_x \delta(x - x') \tag{3}$$

induces the "second" Poisson bracket structure of the KdV equation

$$\{u(x), u(x')\} = (\partial_x^3 + u(x)\partial_x + \partial_x u(x))\delta(x - x') \tag{4}$$

Less well known, but of fundamental importance[8], is the fact that via the map $j = q_{xx}/q_x$, a) a solution of the Ur-KdV equation

$$q_t = q_{xxx} - \tfrac{3}{2}q_{xx}^2 q_x^{-1} \tag{5}$$

generates a solution of MKdV, and b) the Poisson bracket structure

$$\{q(x), q(x')\} = \partial_x^{-1} q_x \partial_x^{-1} q_x \partial_x^{-1} \delta(x - x') \tag{6}$$

induces the Poisson bracket structure (3). As recognized and explained by Wilson[8], Eqs.(5) and (6) are invariant under Möbius transformations

$$q \to \frac{aq + b}{cq + d} \qquad\qquad ad - bc = 1 \tag{7}$$

It follows that if f is some functional of q invariant under some subgroup of the Möbius transformations, then Eq.(5) will imply some KdV-type equation for f and the bracket (6) will imply some bracket for f. Examples of such f's are j, which is invariant under the $c = 0$ subgroup of Möbius transformations, and $u = q_{xxx}q_x^{-1} - \frac{3}{2}q_{xx}^2 q_x^{-2}$, which is invariant under the full group of Möbius transformations. Another such f is $\tilde{j} \equiv q_{xx}q_x^{-1} - 2q_x q^{-1}$, which is invariant under the $b = 0$ subgroup; $\tilde{j}$ satisfies MKdV*, satisfies the same brackets as j, and $u = \tilde{j}_x - \frac{1}{2}\tilde{j}^2$. We can also take f's that are invariant under one parameter subgroups of the Möbius transformations, such as $h \equiv \ln q_x$ (invariant under $a = d = 1$, $c = 0$), $\tilde{h} \equiv \ln(q_x/q^2)$ (invariant under $a = d = 1$, $b = 0$), and $\eta \equiv q_x/q$ (invariant under $b = c = 0$). All this is explained in ref.8. Note that we could have written any function of q_x instead of h above; we have chosen h and $\tilde{h}$ so that $j = h_x$ and $\tilde{j} = \tilde{h}_x$. The complete set of equations obtained from Ur-KdV in this way is what I call the gauge equivalence class of KdV; I should point out that this notion is usually introduced via a zero-curvature formulation, but I will have no need for this here.

Consider now the following action:

$$\begin{aligned} S &= S_0 + H \\ S_0 &= -\frac{c}{48\pi}\int dxdt\ q_{xt}q_{xx}q_x^{-2} \\ H &= \int dxdt\ p[u] \end{aligned} \tag{8}$$

Here c is a constant and $p[u]$ is some function of u and its x-derivatives. S_0 is the "geometric Virasoro action" of Polyakov, Bershadsky and Ooguri and others[9], which is invariant under Möbius transformations (7), as is H. H has no time derivatives in it, so the Poisson brackets are determined purely by S_0; on the other hand since S_0 is first order in time derivatives it will give no contribution to the hamiltonian, which is therefore H. The Poisson brackets determined by S_0 are exactly those in Eq.(6) multiplied by $24\pi/c$. We can write

$$S_0 = -\frac{c}{48\pi}\int dxdt\ h_x h_t \tag{9}$$

so S can also be considered as an action for h. Treating S as an action for q the equation of motion is found to be

$$u_t = -\frac{24\pi}{c}(\partial_x^3 + u(x)\partial_x + \partial_x u(x))\frac{\delta p}{\delta u} \tag{10}$$

* So there are two distinct maps from Ur-KdV to MKdV.

Here $\delta p/\delta u$ is defined by $\delta H = \int dxdt\ (\delta p/\delta u)\delta u$. Treating S as an action for h the equation of motion is

$$j_t = -\frac{24\pi}{c}\partial_x(\partial_x + j)\frac{\delta p}{\delta u} \tag{11}$$

where here we understand that we should write $\delta p/\delta u$ in terms of j. From (10) and (11) it is clear that if we choose

$$p[u] = \sum_{n=1}^{\infty} \lambda_n p_n[u] \tag{12}$$

where the λ_n's are constants and the p_n's are the densities of the conserved quantities of the KdV equation,

$$\begin{aligned} p_1[u] &= u \\ p_2[u] &= \tfrac{1}{2}u^2 \\ p_3[u] &= \tfrac{1}{2}(u^3 - u_x^2) \\ &\vdots \end{aligned} \tag{13}$$

then (10) will give an arbitrary equation in the KdV hierarchy and (11) an arbitrary equation in the MKdV hierarchy. Note that by treating S as a non-local functional of u we can also obtain an arbitrary equation in the Ur-KdV hierarchy from S.[4]

In the last paragraph we pulled the KdV conserved quantities out of a hat. In fact we could have chosen $p[u]$ as in Eq.(12) with the p_n's *any* set of densities such that the quantities $I_n = \int dx\ p_n[u]$ mutually commute under the bracket (4). This would have given a different integrable hierarchy. I am not aware of a classification of all possible sets of p_n's. But when we write the p_n's of the KdV hierarchy in terms of h we find that the I_n's commute with both $I_+ = \int dx\ e^h$ and $I_- = \int dx\ e^{-h}$ (note that I_+ and I_- do not commute though); in fact it is known[10] that requiring the p_n's to be functions of $j = h_x$ and its derivatives such that the I_n's commute with I_+ and I_- *uniquely* determines the p_n's of the KdV hierarchy. Note that in our formalism $e^h = q_x$ so (assuming periodic boundary conditions on q) I_+ is zero. I strongly suspect (from conformal field theoretic considerations) that a general set of p_n's can be obtained by requiring commutation of the I_n's with I_+ and $I(\lambda) = \int dx\ e^{-\lambda h}$ for some λ (not all λ's will be allowed); but I am unaware of a proof of this statement. In quantization we will for one purpose use "commutation with I_+ and I_-" as the definition of the KdV hamiltonians, and for another purpose use "commutation with I_2" as the definition (this is also sufficient to define the other I_n's at the classical level[10]).

3. Quantization

When we quantize a theory we choose a set of Poisson brackets and elevate them to the level of operator commutation relations. In quantizing the theory based on the action S_0 we have a choice; either we can treat the field q as fundamental, in which case we should use the u bracket (4), as u is the dynamical field, or we can treat the field h as fundamental, in which case we should use the j bracket (2). But in the latter approach we should not completely ignore the fact that S_0 can be treated as an action for q; this reflects the fact that we can impose a consistent constraint on the theory defined by $S_0[h]$, namely the constraint $I_+ = \int dx\ e^h = 0$ (by "consistent" in this context I mean that this constraint is preserved under the dynamics). We will do this.

But first a few words on the standard notions of quantum integrable systems. A common feature of classical integrable systems is the existence of at least one Poisson bracket structure and an infinite number of quantities in involution with respect to this bracket. Given this situation, we can investigate whether upon elevating the brackets to operator commutation relations there is still an infinite number of quantities in involution, with "classical limit" (suitably defined) the hamiltonians of the classical integrable system. If the answer is positive then we can regard the quantities in involution as conserved quantities of some operator evolution equation, which we dub the "quantum" version of the original classical equation. Remarkably it seems that there are infinite numbers of conserved quantities for the quantum KdV equation (quantized using its first[10,11,12] and second[10,11,13] brackets), the quantum MKdV equation (quantized using its second bracket[10,13]), the quantum NLS equation (quantized using its first[14,12] and second[5] brackets), and the quantum $SL(N)$ KdV equations (quantized using its second bracket[11,13]). These are remarkable results because the "bihamiltonian" structure of integrable systems, often regarded as responsible for the existence of the infinite number of conserved quantities, is *lost* on quantization[11].

Returning now to the quantization of our action, the first quantization of S_0 proposed above consists of making the u Poisson bracket (4) into an operator commutation relation. Looking at ref.3 we see that if we write

$$u = -\frac{12}{c} \sum_{n=-\infty}^{\infty} L_n e^{inx} + \tfrac{1}{2} \tag{14}$$

then the modes L_n satisfy a Virasoro algebra with central charge c. The natural choice of Hilbert space is the "Verma module of the identity", i.e. the states

$$L_{-n_1} L_{-n_2} .... L_{-n_r} |0\rangle, \qquad n_1 \geq n_2 \geq .... \geq n_r \geq 2 \tag{15}$$

where $|0\rangle$ is a vacuum state satisfying

$$L_n|0\rangle = 0, \qquad n \geq -1 \tag{16}$$

Since this quantum theory knows nothing of the classical fields h and j, we proceed by defining a quantum analog of I_2, namely

$$\mathcal{I}_2 = \frac{1}{2}\int dx\ (uu) \tag{17}$$

where the parentheses denote normal ordering. We seek quantum KdV hamiltonians as operators that commute with $\mathcal{I}_2$; this is just the quantum KdV theory of Kupershmidt and Mathieu[11]. It has been proven that an infinite number of quantum hamiltonians exist[13].

For the second quantization the fundamental field is j; writing

$$j = \sqrt{\frac{6}{c}}\sum_{n=-\infty}^{\infty} j_n e^{-inx} \tag{18}$$

we find the modes j_n satisfy the Heisenberg algebra

$$[j_n, j_m] = 2n\delta_{n,-m} \tag{19}$$

Without imposing the constraint the natural Hilbert space is the set of states

$$j_{-n_1}j_{-n_2}....j_{-n_r}|0\rangle, \qquad n_1 \geq n_2 \geq .... \geq n_r \geq 1 \tag{20}$$

where here $|0\rangle$ is a vacuum state satisfying $j_n|0\rangle = 0$, $n \geq 0$. To impose the constraint, the quantum analog of $I_+ = 0$, we restrict to states $|\psi\rangle$ satisfying

$$\mathcal{I}_+|\psi\rangle = 0 \tag{21}$$

where

$$\mathcal{I}_+ = \int dx\ :e^h: \tag{22}$$

In Eq.(22) the colons denote normal ordering and $h = \partial_x^{-1}j$. Operators in the constrained theory should commute with $\mathcal{I}_+$ so that they map physical states to physical states. But before we work out the simplest such operator, let us first do some rescalings to make our formulae appear more like the conformal field theory literature; writing

$$\begin{aligned} \phi &= i\sqrt{\frac{c}{6}}h \\ J &= \phi_x \end{aligned} \tag{23}$$

we have

$$S_0 = \frac{1}{8\pi}\int dxdt\ \phi_x\phi_t$$
$$\mathcal{I}_+ = \int dx\ :e^{-i\beta\phi}:, \qquad \beta = \sqrt{\frac{6}{c}} \tag{24}$$

Following ref.10, we can use conformal field theoretic techniques to evaluate commutators. We find that the operator $T = -\frac{1}{4} : J^2 : +i\alpha J_x$ commutes with $\mathcal{I}_+$ if $\alpha = \frac{1}{2}(\beta - \beta^{-1})$. T is the analog of u in this quantization of the theory, and the modes of T satisfy a Virasoro algebra, but with central charge $\tilde{c} = 1 - 24\alpha^2 = 13 - 6(\beta^2 + \beta^{-2}) = 13 - c - 36c^{-1}$.

For $\beta = \sqrt{m/(m+1)}$, $m = 3, 4, ...$, we obtain in this way the central charges of the minimal conformal models. In fact what we have seen here is that quantizing S_0, treated as an action for the constrained field h, leads us naturally to certain features of the Dotsenko-Fateev-Feigin-Fuchs construction for the minimal models. As explained by Felder[15], this construction works because the Hilbert spaces of the minimal models (which are representation spaces of the Virasoro algebra) can be realised as the cohomology of a certain operator acting between certain representation spaces of the Heisenberg algebra ("Fock spaces"). We have obtained a Lagrangian prescription of a part of this; the states in our theory are restricted to lie in the kernel of $\mathcal{I}_+$, which on the single charge-zero Fock space we have been considering, is Felder's BRST operator. It might be hoped that a more careful analysis of S_0 might lead to a more complete Lagrangian prescription of Felder's work; in particular in ref.4 I explained why the field h should be regarded as compactified, and this would motivate us to enlarge the Hilbert space of the unconstrained theory to include Fock spaces of exactly the charges required. But it is at the moment not clear to me how the constraint operator becomes Felder's operator on these spaces.

Returning to the main subject, we have seen that operators in the quantum theory of S_0 we are now considering must commute with $\mathcal{I}_+$, and it is therefore natural to seek quantum KdV hamiltonians in this context by seeking operators that commute both with $\mathcal{I}_+$ and with $\mathcal{I}_- \equiv \int dx\ :e^{-h}:$. It is not clear that the quantum KdV hamiltonians defined this way will coincide with those defined previously. But it turns out that the second hamiltonian constructed this way can be written in the form $\int dx\ (TT)$ (cf. Eq.(17)), so the set of quantum KdV hamiltonians defined here does coincide with those defined above (up to a replacement of c with $\tilde{c}$). This is a non-trivial result, that the two definitions of the classical KdV hamiltonians given at the end of section 2, namely "commutation with I_+ and I_-" and "commutation with I_2", give rise to the same set of quantum hamiltonians (up

to $c \to \tilde{c}$)*; we will appreciate this result more in the next section.

Finally in this section we note the significance of the quantum KdV hamiltonians in the minimal models. Defining the quantum KdV hamiltonians via commutation with $\mathcal{I}_+$ and $\mathcal{I}_-$, we see that (for appropriate values of β) they are operators in the minimal model which commute with $\int dx \ : e^{-h} :$; but $: e^{-h} :$ is exactly the (1,3) primary field, so *the quantum KdV hamiltonians are conserved quantities in the* $\Phi_{(1,3)}$ *deformations of the minimal models*[6].

4. An action for the Nonlinear Schrödinger Hierarchy

I will now give the NLS action. In the KdV case, while the action gave us an interesting perspective, we did not really learn anything new. In writing the NLS action a) we gain insight into the gauge equivalence class of the NLS equation (it only takes fragmented knowledge of the class to write the action, and then it can be used to deduce more), b) on quantization we see how just as quantum KdV is related to (a deformation of) the minimal models, similarly quantum NLS is related to (a deformation of) the parafermion and $SL(2)/U(1)$ coset models, and c) the most cryptic element in the bosonization of these models, the form of (one of the) screening operators is obtained naturally from the classical theory. I will just give a few details here; for a more complete discussion see ref.5.

The gauge equivalence class of NLS is specified by giving the Ur-NLS equations, their second Poisson bracket structure and the group action that leaves the equations and brackets invariant. Calling the Ur-NLS fields S, T, the equations, brackets and group action are

$$\begin{aligned} T_t &= T_{xx} + 2T_x S_x \\ S_t &= \frac{2S_x T_{xx}}{T_x} + 3S_x^2 - S_{xx} \end{aligned} \tag{25}$$

$$\begin{pmatrix} \{S(x),S(y)\} & \{S(x),T(y)\} \\ \{T(x),S(y)\} & \{T(x),T(y)\} \end{pmatrix} = \begin{pmatrix} 0 & -\partial_x^{-1}T_x\partial_x^{-1} \\ \partial_x^{-1}T_x\partial_x^{-1} & 2\partial_x^{-1}T_x\partial_x^{-1}T_x\partial_x^{-1} \end{pmatrix} \delta(x-y) \tag{26}$$

$$\begin{aligned} e^S &\to \lambda(cT+d)e^S \\ T &\to \frac{aT+b}{cT+d} \end{aligned} \tag{27}$$

* There are several other results we are taking for granted; for example, on the classical level it is straightforward to show that the set of objects that commute with I_2 generate an abelian algebra, but this is not so clear on the quantum level. Such issues are discussed in ref.10.

where in Eq.(27) $ad - bc = 1$. The group is $SL(2) \times \mathbf{R}$. In the KdV case we had four different variables that were useful, q, h, j, u. S, T are the analogs of q, and the analogs of h, j, u are, respectively the three sets of variables

$$\begin{aligned} h &= S \\ \bar{h} &= -S + \ln(S_x T_x^{-1}) \end{aligned} \tag{28}$$

$$\begin{aligned} j &= h_x \\ \bar{j} &= \bar{h}_x \end{aligned} \tag{29}$$

$$\begin{aligned} A &= \tfrac{1}{2}(j - \bar{j} + j_x/j) \\ B &= j\bar{j} \end{aligned} \tag{30}$$

The variables A, B are invariant under the full transformation group. Eq.(25) induces the NLS equation for the quantities $\psi, \bar{\psi}$, where

$$\begin{aligned} \psi &= e^{h-\bar{h}} h_x \\ \bar{\psi} &= e^{-(h-\bar{h})} \bar{h}_x \end{aligned} \tag{31}$$

these are invariant under the $SL(2)$ subgroup of the transformation group; indeed the NLS equations

$$\begin{aligned} \psi_t &= \psi_{xx} - 2\psi^2\bar{\psi} \\ \bar{\psi}_t &= -\bar{\psi}_{xx} + 2\bar{\psi}^2\psi \end{aligned} \tag{32}$$

display an obvious invariance under $\psi \to \alpha\psi$ $\bar{\psi} \to \alpha^{-1}\bar{\psi}$, which is the residual $\mathbf{R}$. The NLS action is

$$S_{NLS} \;=\; \tilde{k} \int dxdt \; h_x \bar{h}_t \;+\; \sum_{n=1}^{\infty} \lambda_n \int dxdt \; p_n[A, B] \tag{33}$$

where the p_n's are certain functionals of A, B and their x-derivatives; one way to define them is to require commutation with $\mathcal{H}_1 \equiv \int dx \, e^{h+\bar{h}}$ and $\mathcal{H}_2 \equiv \int dx \, h_x e^{-(h+\bar{h})}$, which play the role of I_- and I_+ in the KdV theory. In fact, when written in terms of S, T, and assuming periodic boundary conditions, $\mathcal{H}_2$ vanishes, just as I_+ vanishes in terms of q in KdV theory; similarly A, B commute with $\mathcal{H}_2$, just as u commutes with I_+ in KdV theory. S_{NLS} gives the A, B hierarchy when varied with respect to S, T, and vice-versa; it gives the $j, \bar{j}$ hierarchy when varied with respect to $h, \bar{h}$, and vice-versa; it gives the usual NLS hierarchy when varied with respect to variables T and S_x/T_x.

Quantizing S_{NLS} along the lines of the second quantization method in section 3, we are naturally led to consider states in a Fock space annihilated by a normal-ordered version of $\mathcal{H}_2$, and we have to seek operators that commute with this constraint operator. It is easy to find quantized analogs of $\psi, \bar{\psi}, B$ (written in terms of $h, \bar{h}$), and these take exactly the forms of the fundamental parafermion, its conjugate, and the stress-energy tensor in the bosonized version of the parafermion and $SL(2)/U(1)$ coset models. And as I have already said, the normal-ordered version of $\mathcal{H}_2$ *is* the mysterious screening operator in these theories, which we have now obtained from a simple classical argument. The quantum NLS hamiltonians, defined by "commutation with $\mathcal{H}_1$ and $\mathcal{H}_2$" (both normal ordered) can thus be identified as conserved quantities in a deformation of the parafermion and $SL(2)/U(1)$ coset models by an operator $e^{h+\bar{h}}$, which is just the first "thermal operator". Finally I should mention that searching for the conserved quantities in the deformed theories by looking for operators (written in terms of $j, \bar{j}$) that commute with $\mathcal{H}_1$ and $\mathcal{H}_2$ is quite a bit easier than trying to build such operators out of the quantized $\psi, \bar{\psi}$ fields.

Acknowledgements

Useful discussions with Didier Depireux are acknowledged. This work was supported by the U.S.Department of Energy under grant #DE-FG02-90ER40542.

References

1. From an action one derives both equations of motion *and* Poisson brackets for the fields involved (see ref.4). It is fairly easy to write an action which gives the KdV equation and its "first" hamiltonian structure, and this has been done explicitly in (for instance) L.A.Dickey, *Ann.N.Y.Acad.Sci.* **410** (1983) 301 and A.Das, *Integrable Models*, World Scientific (1989). I am also informed, by Boris Kupershmidt, that it is known that one can write an action for the MKdV equation with its "second" hamiltonian structure, but the novel point that I will be stressing here is that this same action can be viewed as an action for the KdV equation in its "second" hamiltonian formulation as well. Note also that from the work of Dickey just mentioned it is clear such an action exists, but it is not clear how to write it in a simple form.
2. For the action mentioned in note 1. for the KdV in its "first" hamiltonian formulation this does not seem to be the case.
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