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Modeling the search for the least costly opportunity

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ABSTRACT

With the continuing growth in the number of opportunities available at virtual stores over the Internet there is also a growing demand for the services of computer programs capable of scanning a large number of stores in a very short time. We assume that the cost associated with each scan is linear in the number of stores scanned, and that the resulting list of price quotes is not always satisfactory to the customer, in which case an additional scan is performed, and so on. In such a reality the customer, wishing to minimize her expected cost, must specify the requested sample size and a rule (control limit) to stop the search.

In the context of search theory, the above search model can be categorized as "fixed-sample-size, sequential, with infinite horizon". According to this model the expected search cost is a function of two decision variables: the sample size and the control limit. We prove that for arbitrary sample size the expected search cost is either quasi-convex or strictly decreasing in the control limit, and that the optimal expected search cost is quasi-convex in the sample size. These properties allow an efficient calculation of the optimal policy. We also develop analytic formulas to calculate the cost's variance, allowing customers to choose a slightly higher expected cost if there is a considerable decrease in the variance. Finally, we present detailed examples for price quotes that are distributed uniformly or exponentially.

1. Introduction and motivation

We envision a business environment where there is a large variety of products and services displayed at numerous virtual stores over the Internet, and assume that customers, while ignorant of individual stores' pricing policies, are acquainted with overall price distributions. We also foresee a growing demand for the services of "smart agents" (comparison-shopping agents), which, in contrast to the customers, are capable of randomly scanning many virtual stores in a very short time. We assume that for their services, these internet agents will charge a payment consisting of a fixed setup cost (for initiating the search) and a variable search cost (depending on the number of virtual stores that need to be scanned). As a search cycle ends the customer obtains a random sample of stores and price quotes. However, in our model the customer only focuses on the "best opportunity", i.e., the least price obtained, which may be disappointing, in which case she orders an additional price scan, again paying the fixed and variable search costs, and so on. The objective is to minimize the total product's cost and payment to the agent.

1.1. Comparison-shopping agents

According to Greenwald and Kephart (1999) comparison-shopping agents or Shopbots are Internet agents that automatically search for information that pertains to the price and quality of goods and services. These software agents can query multiple online vendors and then return sorted information to the customer within seconds. Some of the most popular services for price comparison are: PriceScan.com, Shopping.com (previously Dealtime.com) and MySimon.com.

Although, until recently, comparison-shopping agents, as well as many other information services over the Internet, were free of charge (based on the assumption that electronic advertisements would cover the costs of the site operators), today more and more of these services have begun charging for extracting information. Examples of this trend can be found in Yahoo's decision to charge for some of the services offered free until recently (music download, stock quotes, etc.) as well as the decision of the Wall Street Journal and the Economist to charge for viewing their online editions. The main drivers for this trend are two: reformation in the business models of Internet services ("no more free meals"); and the need for appropriate mechanisms to prevent massive abuse of these services. The latter reason is gaining importance as electronic merchants, wishing to adjust their prices according to the competition (dynamic prices) are seeking continuous real-time

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information about the market. Offering information free of charge may produce a frequency of queries that can escalate to service crash (big B2C players such as Amazon, might be interested in making comparative price inquiries for their numerous different products). Greenwald and Kephart (1999) extend this reasoning by introducing the notion of endless cyclic queries: a seller receiving an electronic request for a price quote may want first to check the price status of the competitors who may then, before replying, query him, and so on.

1.2. Search theory: Overview and related literature

Modeling the search for the best opportunity has attracted the attention of many researchers, prompting several literature reviews over the years, such as Lippman and McCall (1976), Burdett (1989) and McMillan and Rothschild (1994). A remarkably long list of articles has been dedicated to variations of the ''secretary problem'' (Ferguson, 1989).

Search models have developed to a point where their total contribution is referred to as ''search theory''. The two most elementary search models are: (a) The ''fixed-sample-size'' (or ''non-sequential'') model; and (b) The ''sequential'' model. Fixed-sample-size models are single-period models allowing for only a single sampling attempt. The problem is then to determine the optimal sample size. Sequential models are multi-period models allowing for only a single observation at each search period. The problem is then to determine an optimal control limit (stop the search if and only if the minimal price quote is smaller than the control limit; also referred to as reservation rate; or reservation wage in the labor market search literature). When sequential models are considered there is also a distinction between finite and infinite decision horizon. Thus, in the context of search theory, our model can be categorized as ''fixed-sample-size, sequential, with infinite horizon''. As this is a generalization of both the ''sequential'' and ''non-sequential'' scenarios, the problem is to determine, together, the optimal control limit and the sample size.

Search articles date back to the 1950s, e.g., Simon (1955). Stigler's (1961) introduced a fixed-sample-size model; McCall (1965, 1970) considered sequential models. Salop and Stiglitz (1977) point out that the fixed sample size is not necessarily dominated by the sequential search. Some of the sequential search literature, e.g., Rothschild (1974), and Lippman and McCall (1976) considered the presence of the recall option, the nature of the population and the decision time horizon. McMillan and Rothschild (1994) provide a long list of articles devoted to explaining the observed dispersion of prices in markets and others dealing with bargaining. Tesler (1973) studied sequential search empirically, focusing on the information the searcher has on price distribution and recall issues. Stahl (1989) studied a buyer–seller equilibrium.

Some researchers, e.g., Gal et al. (1981), Morgan (1983), Benhabib and Bull (1983), Morgan and Manning (1985) and McKenna (1986), suggested a combined (sequential and non-sequential) model in which a number of observations may be made in any period. This in fact is the model analyzed here. Gal et al. (1981) studied the case where the agent draws independently and identically distributed observations from a population with a known probability density function, though with uncertainty associated with the availability of the inspected offers. They establish some of the properties of a search rule, focusing mainly on the effects of decision time horizon. This approach has been recently applied to examining housing search behavior and the effects of brokers on the intensity and duration of search (Baryla et al., 2000).

Morgan (1983) suggested a model in which the searcher could choose his sample size and whether or not to stop the search at each of a sequence of decision points. This article focused on the necessary and sufficient conditions for the existence of optimal solutions of models with recall and fallback utilities. Nevertheless, calculating the optimal sample size and reservation price was recognized to be a complex task (Cosme, 1997). Both Benhabib and Bull (1983) and Morgan and Manning (1985) consider fixed-sample-size sequential models with finite horizon. Benhabib and Bull (1983) apply their model to job search. They prove that the reservation wage is non-decreasing in the number of remaining periods and show that the optimal sample size will always be less than that chosen in Stigler's (1961) model. Morgan and Manning (1985) allow the sample size to vary from period to period based upon the time remaining and the information obtained. They provide conditions that the optimal strategy must satisfy.

Based on the three above basic methods (sequential, non-sequential and the combined approach), many variants have been investigated. For example, the case where the consumer has a budget constraint (Manning and Morgan (1982) for Stigler's model, Veendorp (1984) for the sequential search) or when the searcher is mixing consumption with search (Benhabib and Bull (1983), where the job seeker is still working while searching) and accounting for inter-temporal budget allocation between consumption and search (Kohn and Shavell, 1974; Manning and Manning, 1997; Manning, 1989).

In parallel, search theory also evolved as an attempt of modeling Research and development (R&D) as a search process for a better technology, where applied research is a search in a given distribution, and basic research is a shift in the distribution searched (Evenson and Kislev (1975), see a review in Quyen and Vafa (2001)). Similar ideas associated with random search for lower cost methods from a fixed population of technological possibilities were proposed by Tesler (1982) and Muth (1986). Complementary models studying an endogenous growth in which technological improvements are generated by a costly and uncertain search process, with search efforts financed by capital whose alternative use is in the production of consumption goods were proposed by Jovanovic and Macdonald (1994) and Bental and Peled (1996, 2002).

Before continuing to the next section, it is important to note that a main goal of economic research in the area of search theory was developing theoretical models for analyzing the impact of search costs on the distribution of market prices (referred to as price dispersion). This notion was first introduced by Stigler (1961) and was reflected in many of the work cited above. As intuitively expected, any decrease in the search costs encourages consumers to increase search (i.e. sampling additional stores), resulting with an increased competition on the sellers' side, causing eventually a price reduction. With the recent growing interest in eCommerce we evidence a bloom in empirical research aiming to test the intuitive hypothesis that lower search costs increases search and result with lower prices (Brown and Goolsbee, 2002; Brynjolfsson and Smith, 2000; Clay et al., 2002; Lee, 1998; Cason and Friedman, 2003) yielding mixing results for different markets.

As stated above, the process of calculating the optimal sample size and the optimal reservation price was recognized to be a complex task. In this paper we address this issue for the ''fixed-sample-size, sequential, with infinite horizon'' search model.

1.3. A summary of results

We denote $V_N(x)$ the expected cost if the sample size is N and the control limit is x and refer to it as the ''cost function''. We provide a precise formulation of $V_N(x)$ in terms of the decision variables x and N. Mostly, the study of the fixed-sample-size, sequential, infinite-horizon model is through the analysis of $V_N(x)$. The main findings and contributions are:

- For arbitrary N, $V_N(x)$ is either quasi-convex or strictly decreasing in x. The situation is depicted in Figs. 1, 2a and 2b.
- For arbitrary N, let x_N^* and V_N^* be the optimal values of the control limit and cost function, respectively, then $x_N^* = V_N^*$. While this intuitive property has been observed and used in the past we are the first to prove it, formally, based on a cost function (also see Comment 2).
- V_N^* is quasi convex in N. Thus, the optimal sample size, N^*, can be found via a search over N, with a simple stopping rule.
- We develop analytic formulas to calculate the cost's variance. This has not been done before.

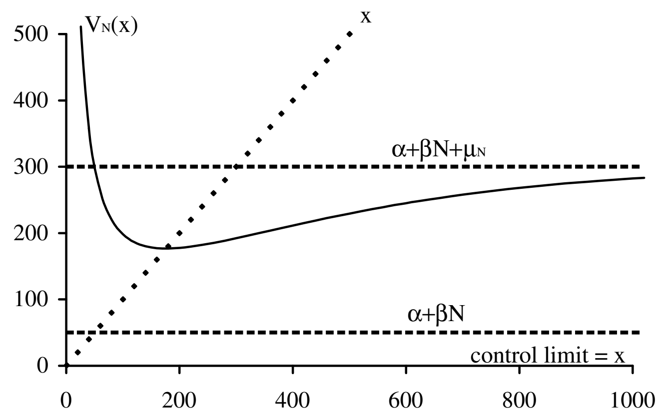

Fig. 1. The cost function for exponential observations. (The parameters used were: $\alpha = 40$, $\beta = 5$, $N = 2$ and $\lambda = 0.002$.)

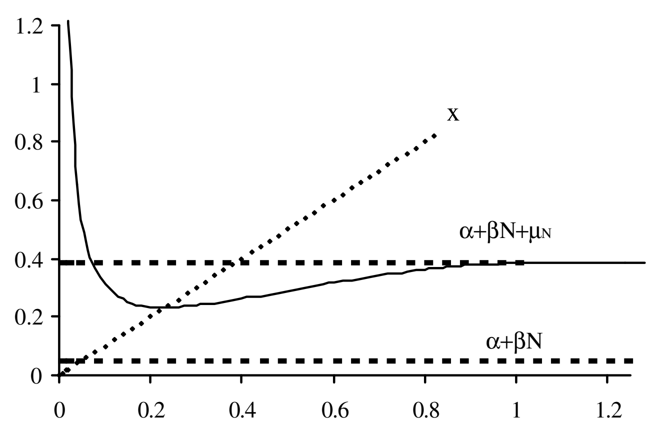

Fig. 2a. The cost function for uniform observations when $\alpha + \beta N + \mu_N < 1$. (The parameters used were: $\alpha = 0.040$, $\beta = 0.005$ and $N = 2$.)

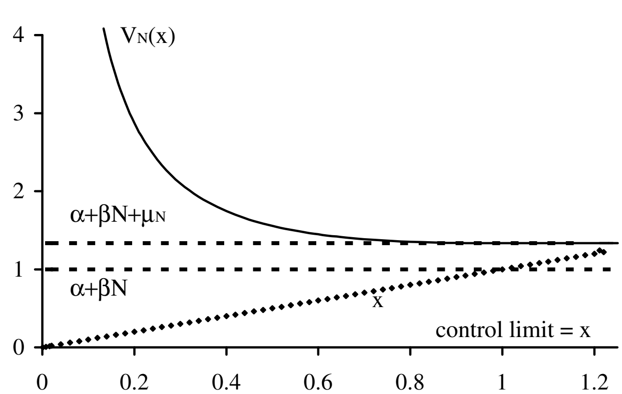

Fig. 2b. The cost function for uniform observations when $\alpha + \beta N + \mu_N > 1$. (The parameters used were: $\alpha = 0.6$, $\beta = 0.2$ and $N = 2$.)

- As an illustration and application we examine prices that are either uniform or exponential.

2. The model: Formulation and analysis

We consider a business environment where there is a considerable demand for the services of Internet search agents and assume that for a random sample of N observations agents charge $\alpha + \beta N$, where α and β are the fixed and variable search costs, respectively.

We denote $X_1, \ldots, X_N$ a random sample (of price quotes at different stores) drawn from a population with p.d.f. $g(x)$ and c.d.f. $G(x)$, $0 \leqslant x < \infty$, and define $Y_N = \min\{X_1, \ldots, X_N\}$.

We denote $f_N(y)$ and $F_N(y)$ the p.d.f. and c.d.f. of Y_N, respectively, and $\mu_N = E(Y_N)$. Clearly (using the notation $\overline{G}(y) = 1 - G(y)$)

$$F_N(y) = 1 - \overline{G}^N(y) \quad \text{and} \quad f_N(y) = N * g(y) * \overline{G}^{N-1}(y). \tag{1}$$

After obtaining a random sample the customer has the option to end the search, paying Y_N for the commodity; or, if she is discontented with the result (because she thinks the least costly opportunity in the sample is too high) orders an additional search iteration, paying, again, $\alpha + \beta N$ to the agent. Thus, in total, the customer's optimal policy is characterized by two numbers: N^*, the optimal sample size to be ordered; and x^*, the optimal control limit which is the highest price she is willing to accept.

Note that in our model the time horizon is infinite. Also, as we assume that search iterations are very fast, typically lasting several seconds, we will not use a discounting factor.

Why base the stopping rule on the lowest price quote, disregarding the number of search iterations? Or, in other words, why not use a finite horizon model? In the Appendix we present a finite horizon search model, proving that it leads to higher expected search costs. In addition, a finite horizon model may be preferable when iterations are lengthy; however, over the Internet search is fast.

For fixed (arbitrary) N, we denote $V_N(x)$ the expected search cost (until the search is terminated) if the search ends only as $Y_N \leqslant x$. If $Y_N > x$ the search proceeds (a new sample of size N is taken). Thus

$$V_N(x) = E[\alpha + \beta N + Y_N \cdot 1(Y_N \leqslant x) + V_N(x) \cdot 1(Y_N > x)].$$

Here $1(Y_N \leqslant x)$ represents the indicator of the event $\{Y_N \leqslant x\}$. Note that $E[1(Y_N > x)] = 1 - F_N(x)$ and that $E[Y_N \cdot 1(Y_N \leqslant x)] = \int_{y=0}^{x} y f_N(y)\,dy$. Thus

$$V_N(x) = \frac{\alpha + \beta N + \int_{y=0}^{x} y f_N(y)\,dy}{F_N(x)}. \tag{2}$$

We will refer to $V_N(x)$ as the "cost function". Probabilistically, Eq. (2) resembles Example 7.11 from Sheldon (2002, Chapter 7). Note that $V_N(x) \geqslant \alpha + \beta N$ and that

$$\text{(a)} \quad \lim_{x \to 0^+} V_N(x) = \infty \quad \text{and} \quad \text{(b)} \quad \lim_{x \to \infty} V_N(x) = \alpha + \beta N + \mu_N. \tag{3}$$

The content of (3) is intuitive: if the control limit, x, is very small, the chances of obtaining $Y_N \leqslant x$ are small. Thus, repeated samples must be taken, leading to high overall costs. If, on the other hand, the control limit, x, is very large, almost surely $Y_N \leqslant x$ will be obtained in the first sample.

Comment 1

(a) The cost function, $V_N(x)$, can be decomposed into two parts: $(\alpha + \beta N)/F_N(x)$, and $(\int_{y=0}^{x} y f_N(y)\,dy)/F_N(x)$, representing, respectively, the expected payment to the agent and the expected cost of purchasing the commodity. (The expected number of search cycles is $1/F_N(x)$; the expected purchasing cost is $E(Y_N | Y_N \leqslant x) = \int_{y=0}^{x} y f_N(y)\,dy / F_N(x)$.)

(b) Next we will perform various mathematical operations concerning $V_N(x)$. As we envision the search to last a short time we assume that the number of stores and distribution of prices remain unchanged during the search, i.e., the system is in steady-state. This assumption is supported by recent empirical research in online retail markets, e.g., Baye et al., 2005 and Clay et al. (2002). Also, we assume sampling is with replacement, i.e., performing additional iterations the comparison-shopping may return to some of the same stores. In any case, even if these agents will learn to do sampling without replacement we assume the sampled population is large, thus statistical correction terms are negligible.

(c) To ease technicalities we assume that either $g(x) > 0$ over $[0, \infty)$, or that $g(x) > 0$ only over $[0, A]$, for some A, i.e., $g(x) = 0$ over (A, ∞). Note that if $g(x) > 0$ only over $[0, A]$, then $V_N(x) = \alpha + \beta N + \mu_N$, $x \geqslant A$. Next, through Theorems 1–4, we explore several important properties of $V_N(x)$.

Theorem 1

(a) *Suppose that $g(x) > 0$ over $[0, \infty)$; then, for arbitrary N*
 (a1) *$V_N(x)$ is quasi convex in x with a unique minima satisfying $V_N^* = x_N^*$, and*
 (a2) *$V_N(x)$ is asymptotic from below to $\alpha + \beta N + \mu_N$.*
 The situation is depicted in Fig. 1.

(b) *Suppose that $g(x) > 0$ only over $[0, A]$; then, for arbitrary N, either*
 (b1) *$V_N(x)$ is quasi convex in x with a unique minima satisfying: $V_N^* = x_N^*$, or*
 (b2) *$V_N(x)$ strictly decreases over $[0, A]$.*
 The situation is depicted in Figs. 2a and 2b.

Proof

$$V'_N(x) \equiv \frac{\mathrm{d}V_N(x)}{\mathrm{d}x} = \frac{xf_N(x)F_N(x) - f_N(x)[\alpha + \beta N + \int_{y=0}^{x} yf_N(y)\,\mathrm{d}y]}{F_N^2(x)}$$
$$= \frac{f_N(x)}{F_N(x)}[x - V_N(x)] \equiv r_N(x)[x - V_N(x)]. \quad (4)$$

(a1) Let x_N^* be a value satisfying $V'_N(x_N^*) = 0$, then $V_N^* \equiv V_N(x_N^*) = x_N^*$. Note that $r_N(x) > 0$, hence

$$V''_N(x_N^*) = r'_N(x_N^*)(x_N^* - V_N^*) + r_N(x_N^*)[1 - V'_N(x_N^*)] = r_N(x_N^*) > 0.$$

Thus $V_N(x)$ is quasi convex.

(a2) This is a consequence of (a1) and (3-b).

(b1) Observe Eq. (4). If a value x, satisfying $V'_N(x) = 0$, exists over $[0, A]$, the situation is similar to (a1).

(b2) If a value x, satisfying $V'_N(x) = 0$, does not exist over $[0, A]$, then $V_N(x)$ either strictly increases or strictly decreases over $[0, A]$. Also, from (4) the sign of $V'_N(x)$ is determined by $[x - V_N(x)]$. However, from (3-a) it follows that for small values of x, $[x - V_N(x)] < 0$. □

Comment 2. The notion of using a reservation rate that satisfies $V_N^* = x_N^*$ is intuitive: at the end of each search cycle the customer may stop the search, paying Y_N, or proceed with the search at a total future expected cost equal to V_N^*. Thus, it is optimal to proceed with the search as long as $Y_N > V_N^*$. We should add that several authors assumed that $V_N^* = x_N^*$ in analyzing their model; however, we are first to prove it, formally, in conjunction with a cost function.

Theorem 2. *Suppose that $g(x) > 0$ only over $[0, A]$. Then, the condition $\alpha + \beta N + \mu_N > A$ is necessary and sufficient for $V_N(x)$ to be strictly decreasing in x.*

Proof. Note that, as a consequence of Theorem 1b, $V_N(x)$ strictly decreases in x if and only if it strictly decreases at $x = A$. Thus, using (4), $V_N(x)$ strictly decreases in x if and only if $A - V_N(A) < 0$ which is equivalent to $\alpha + \beta N + \mu_N > A$.

Theorems 1 and 2 reveal the shape of $V_N(x)$, $x \geqslant 0$. Also, for arbitrary N, Theorem 1 provides a practical tool to calculate the control limit, x_N^*; namely to solve $V_N(x) = x$. The next theorem reveals another useful property of the control limit. This property can also be used to calculate x_N^* and helps in further analyzing the model. □

Theorem 3. *For arbitrary N, the optimal control limit satisfies:*

$$\alpha + \beta N = \int_{y=0}^{x_N^*} F_N(y)\,\mathrm{d}y, \quad (5)$$

and, therefore, $V_N^* = x_N^*$ strictly increases in the parameters α and β.

Proof. First note that for $x = x_N^*$ the relationship $V_N(x) = x$ is equivalent to $xF_N(x) = \alpha + \beta N + \int_{y=0}^{x} yf_N(y)\,\mathrm{d}y$. Next observe that (using integration by parts) $\int_{y=0}^{x} yf_N(y)\,\mathrm{d}y = xF_N(x) - \int_{y=0}^{x} F_N(y)\,\mathrm{d}y$. □

Comment 3. Theorems 1–3 reveal the shape of the cost function and provide an effective methodology to calculate the optimal policy: set $N = 1$ and calculate $V_1^* = x_1^*$, then set $N = 2$ and calculate $V_2^* = x_2^*$ and so on. Then choose a sample size N^* for which the value of the cost function is minimal. Note that, by Eq. (2), $\alpha + \beta N^* \leqslant V^*$ and $V_1^* \leqslant \alpha + \beta + \mu_1$. Also, clearly, $V^* \leqslant V_1^*$; therefore, $N^* \leqslant 1 + \mu_1/\beta$, which provides an upper bound to stop the search for N^*. While such a bound is useful we will provide a stronger result, namely, that $V_N^* = x_N^*$ is quasi convex in N.

Theorem 4

(a) *The optimal expected search cost, V_N^*, is quasi convex in N.*

For the proof see the Appendix.

Proposition 1. *The optimal policy for some special cases, namely, $\alpha = 0$ or $\beta = 0$, seems trivial. Yet the proofs require some work.*

(a) *If $\alpha = 0$ it is optimal to use a sample of size one, i.e., $V_1^* \leqslant V_N^*$. (For this case our model coincides with Lippman and McCall (1976).)*

(b) *If $\beta = 0$ it is optimal to use the largest possible sample, i.e., $V_{N+1}^* \leqslant V_N^*$. That is, the optimal sample size is infinite.*

For the proof see the Appendix.

3. Examples: Specific distribution functions

In this section we apply the theory developed above to observations that are drawn from either the exponential or the uniform distribution.

3.1. Observations drawn from the exponential distribution

Assume that prices are exponential, i.e., for $x \geqslant 0$

$$g(x) = \lambda e^{-\lambda x} \quad \text{and} \quad f_N(x) = \lambda N e^{-\lambda N x}; \quad F_N(x) = 1 - e^{-\lambda N x}.$$

Also, $\mu_N = 1/\lambda N$. Therefore, using (2)

$$V_N(x) = \frac{\alpha + \beta N - xe^{-\lambda Nx} - \frac{e^{-\lambda Nx}}{\lambda N} + \frac{1}{\lambda N}}{1 - e^{-\lambda Nx}}.$$

Next, to calculate x_N^*, we use (5):

$$\alpha + \beta N = \int_{y=0}^{x_N^*} F_N(y)\,dy = \int_{y=0}^{x_N^*} (1 - e^{-\lambda Ny})\,dy = x_N^* + \frac{1}{\lambda N}(e^{-\lambda N x_N^*} - 1).$$

This leads to (note that solving $V_N(x) = x$ leads to the same result):

$$\lambda N(\alpha + \beta N - x_N^*) + 1 = e^{-\lambda N x_N^*},$$

from which x_N^* can be easily obtained.

3.2. Observations drawn from the uniform distribution

Assume that prices are uniform, i.e., for $0 \leqslant x \leqslant 1$

$$g(x) = 1 \quad \text{and} \quad f_N(x) = N(1-x)^{N-1}; \quad F_N(x) = 1 - (1-x)^N.$$

Also, $\mu_N = 1/(N+1)$. Therefore (using (3)),

$$V_N(x) = \begin{cases} \frac{\alpha+\beta N-((1+Nx)(1-x)^N-1)\mu_N}{1-(1-x)^N}, & 0 \leqslant x \leqslant 1, \\ \alpha + \beta N + \mu_N, & 1 \leqslant x \leqslant \infty. \end{cases}$$

Clearly, if $\alpha + \beta N + \mu_N > 1$, $x_N^* = 1$. Otherwise, to calculate x_N^*, we use (5):

$$\alpha + \beta N = \int_{y=0}^{x_N^*} F_N(y)\,dy = \int_{y=0}^{x_N^*} [1 - (1-y)^N]\,dy = x_N^* + \frac{1}{N+1}[(1 - x_N^*)^{N+1} - 1].$$

This leads to:

$$(N+1)(x_N^* - \alpha - \beta N) = 1 - (1 - x_N^*)^{N+1},$$

from which x_N^* can be easily obtained.

Table 1 summarizes a numerical comparison between exponential and uniform observations, with equal means, for $\beta = 0.001$. The column "cycles" is the expected number of sample draws, i.e., $1/F_N(x)$ as $N = N^*$ and $x = x^*$. Note that, for uniform observations, if the solution of $V_N(x) = x$ yields a value greater than one, then any $x \geqslant 1$ may serve as a control limit. For example, for $\alpha = 1.024$, we have $V^* = x^* = 1.086$. Thus effectively, $x^* = 1$ and only one sample needs to be taken. However, to achieve the optimal expected search cost of $V^* = 1.086$ one must use a sample of $N^* = 31$ observation.

To demonstrate the results of Table 1 in terms of dollars, assume that the uniform and exponential quotes are in terms of thousands of dollars, i.e., the mean is \$500. Thus the variable cost is β = \$1. If the setup cost is small, for example α = \$4, and prices are uniformly distributed, the expected number of search cycles is 1.87 and the expected total cost (agent + commodity) is \$57. If, on the other hand, α = \$64, the expected number of search cycles is 1.02 and the expected total cost (agent + commodity) is \$126. Note that unless α is very large, the customer pays considerably less than the mean. Also note that both N^* and V^* increase in α, which is intuitive.

To further explore the impact of α and β on V^* and N^* we performed the following experiment. For each of the values α in Table 1 we varied β to assume the values:

$$\beta = 0.001 * \{1, 2, 4, 8, 16, 32, 64, 128, 256, 512\},$$

i.e., a total of $10 * 11 = 110$ experiments. (Actually, 220 experiments, as each was performed for both uniform and exponential prices). Results are given in Tables 3a and 3b in the Appendix.

These experiments affirm that any increase in α and β increases the optimal reservation value x^* (and hence, the expected cost). Furthermore, any change in β has a greater effect on x^* than a similar change in α. Also, as expected, N^* is non-decreasing in α and non-increasing in β.

4. Calculating the cost's variance

Customers may want to consider the tradeoff between using a strategy with low expected cost versus a strategy with a slightly higher expected cost, but with a considerable lower variance.

To calculate the variance we need to represent the search cost as a random variable. We denote $V_N(x, Y_N)$ the random cost if the reservation rate is x and the random outcome is Y_N. The following is the most elementary mathematical statement of the search problem:

$$V_N(x, Y_N) = \alpha + \beta N + Y_N \cdot 1(Y_N \leqslant x) + V_N(x, \widehat{Y}_N) \cdot 1(Y_N > x)]; \qquad (6)$$

here Y_N and $\widehat{Y}_N$ are independent and identically distributed random variables. Therefore

$$V_N(x) = E[V_N(x, Y_N)] = \alpha + \beta N + E[Y_N * 1(Y_N \leqslant x)] + V_N(x) * \overline{F}_N(x),$$

which is consistent with (2). Now

$$[V_N(x, Y_N)]^2 = (\alpha + \beta N)^2 + 2(\alpha + \beta N)[Y_N \cdot 1(Y_N \leqslant x) + V_N(x, \widehat{Y}_N) \cdot 1(Y_N > x)] + (Y_N)^2 \cdot 1(Y_N \leqslant x) + (V_N(x, \widehat{Y}_N))^2 \cdot 1(Y_N > x)].$$

Thus

$$[V_N(x, Y_N)]^2 = (\alpha + \beta N)^2 + 2(\alpha + \beta N)[V_N(x, Y_N) - (\alpha + \beta N)] + Y_N^2 \cdot 1(Y_N \leqslant x) + [V_N(x, \widehat{Y}_N)]^2 \cdot 1(Y_N > x).$$

Let $L_N(x) \equiv E\,[V_N(x, Y_N)]^2$, then

$$L_N(x) = \frac{(\alpha + \beta N)([2V_N(x) - (\alpha + \beta N)]) + \int_0^x y^2 f_N(y)\,dy}{F_N(x)}.$$

Table 1
Numerical values for exponential and uniform observations (β = 0.001)

α	Uniform			Exponential (λ = 2)		
	N^*	$V^* = x^*$	Cycles	N^*	$V^* = x^*$	Cycles
0.001	7	0.050	3.30	6	0.037	2.81
0.002	10	0.053	2.38	8	0.039	2.15
0.004	13	0.057	1.87	11	0.043	1.64
0.008	18	0.064	1.44	14	0.049	1.35
0.016	22	0.074	1.23	17	0.058	1.16
0.032	26	0.092	1.09	20	0.076	1.05
0.064	29	0.126	1.02	22	0.109	1.01
0.128	30	0.190	1.00	22	0.173	1.00
0.256	31	0.318	1.00	22	0.301	1.00
0.512	31	0.574	1.00	22	0.557	1.00
1.024	31	1.086	1.00	22	1.069	1.00

Table 2
The optimal cost and standard deviation as a function of N, for exponential observations (λ = 0.1, α = 1, β = 0.1)

N	x_N^*	$\sigma_N(x_N^*)$	N	x_N^*	$\sigma_N(x_N^*)$	N	x_N^*	$\sigma_N(x_N^*)$
1	5.088	6.705	11	2.975	0.645	21	3.576	0.225
2	3.915	3.472	12	3.011	0.574	22	3.654	0.206
3	3.449	2.385	13	3.055	0.513	23	3.735	0.188
4	3.207	1.831	14	3.105	0.459	24	3.817	0.173
5	3.069	1.489	15	3.161	0.411	25	3.900	0.160
6	2.989	1.251	16	3.221	0.369	26	3.985	0.148
7	2.947	1.074	17	3.286	0.333	27	4.070	0.137
8	2.930	0.935	18	3.354	0.300	28	4.157	0.128
9	2.932	0.821	19	3.426	0.272	29	4.245	0.119
10	2.948	0.726	20	3.500	0.247	30	4.333	0.111

Finally, let $\sigma_N^2(x) \equiv \mathrm{Var}(V_N(x, Y_N))$, hence

$$\sigma_N^2(x) = L_N(x) - V_N^2(x).$$

The expressions of $L_N(x)$ for the exponential and uniform distributions can be found in the Appendix.

As a numerical example we consider the search for a commodity whose prices are exponential with mean of 10 dollars. We assume the fixed cost for initiating the search is one dollar and that the variable cost is 0.1 dollar. That is: $\lambda = 0.1$, $\alpha = 1$, $\beta = 0.1$.

The results are depicted in Table 2. Note that the optimal policy is to use $N = 8$; the resulting expected cost \$2.93 with a standard deviation of \$0.935. Note that in spite of the relative high values of α and β, the customer benefits substantially – relative to the mean. Also, the customer may prefer to use, for example, $N = 14$ with an expected cost of \$3.105 (an increase of about 6%) with a substantial reduction in the standard deviation (about 50%).

5. Conclusions

While our study was motivated by an application of search over the Internet; the academic and practical problem of searching for the best opportunity is shared by all disciplines. In our paper we have presented a precise formulation of the "fixed-sample-size, sequential, infinite horizon" search model. This model is a natural extension of the classical "fixed-sample-size" and "sequential" models. Our analysis characterizes the cost function, $V_N(x)$ as either quasi-convex or convex-strictly-decreasing in x, and reveals that x_N^* is quasi-convex in N. These results make it easy to calculate the optimal policy and generalize earlier results obtained for the classical more elementary models.

How practical is the use of an infinite-horizon search model? Our numerical examples (Table 1) reveal that even for small values of the fixed cost (α) the expected number of search cycles is quite moderate. Such calculation can also help the operators of comparison-shopping agents to determine reasonable pricing and offer packages which are guaranteed to have a small expected number of search cycles.

Some of the analytical computations required to solve the search problem may be quite demanding. The main difficulty is in the need to calculate the integral $\int_0^x y f_N(y)\,\mathrm{d}y$, which involve the distribution of a minimum of random variables. One course of action is to evaluate the integral, numerically, for each N. What we should report is that we have developed an algorithm that allows for a very quick approximation of the optimal policy. In brief, our method approximates any given p.d.f. with a histogram. The number of rectangles used determines the precision. The histogram is linked to a computer program that takes advantage of the theoretical properties proved in Theorems 1–4. More details are available from the authors.

An important contribution of our paper is in the derivation of formulas to calculate the cost's variance. This has been achieved via presenting the cost as a random variable. While a clear application is in considering the tradeoff between expected cost and variance, we believe that other researchers will come up with additional applications. One course of action could be in considering a cost function equal to a weighted total of the expected search cost and the standard deviation.

It is interesting to observe that the content of Theorems 1–3 remains unchanged if the direct search cost, i.e., $(\alpha + \beta N)$, is replaced by $c(N)$, a general function of N. However, this is not necessarily true for Theorem 4, i.e., for general $c(N)$, V_N^* is not necessarily quasi-convex in N.

We like to conclude our article offering several possible extensions and directions for future research. (a) In some applications the customer does not have complete knowledge of the distribution of the product's prices. In such a case, after each search iteration, the customer may want to revise her prior over this distribution. (b) A customer may want to acquire several products. This may influences the size of the control limit and the number of stores that need to be scanned. (c) Similar to (b) several customers may want to collaborate by ordering several products. While some of the benefits of such collaboration are obvious there is also the problem of how they should split the surplus among themselves.

Appendix A

Proof of Theorem 4. First note that $F_{K+1}(y) - F_K(y) = (1 - \overline{G}(y))[\overline{G}(y)]^K$ is non-negative, strictly decreasing in K. Hence

Table 3a
Values of V^* and N^* as a function of α and β for uniform observations

Uniform distribution			β									
			0.001	0.002	0.004	0.008	0.016	0.032	0.064	0.128	0.256	0.512
α	0.001	N^*	7	4	3	2	1	1	1	1	1	1
		x^*	0.05	0.07	0.10	0.13	0.18	0.26	0.36	0.51	0.72	1.01
	0.002	N^*	10	6	4	2	1	1	1	1	1	1
		x^*	0.05	0.07	0.10	0.14	0.19	0.26	0.36	0.51	0.72	1.01
	0.004	N^*	13	8	5	3	2	1	1	1	1	1
		x^*	0.06	0.08	0.11	0.14	0.20	0.27	0.37	0.51	0.72	1.02
	0.008	N^*	18	11	7	4	3	2	1	1	1	1
		x^*	0.06	0.08	0.11	0.15	0.21	0.28	0.38	0.52	0.73	1.02
	0.016	N^*	22	14	9	5	3	2	1	1	1	1
		x^*	0.07	0.10	0.13	0.17	0.22	0.30	0.40	0.54	0.74	1.03
	0.032	N^*	26	17	11	7	4	3	2	1	1	1
		x^*	0.09	0.12	0.15	0.19	0.25	0.33	0.43	0.57	0.76	1.04
	0.064	N^*	29	20	13	8	5	3	2	1	1	1
		x^*	0.13	0.15	0.18	0.23	0.29	0.37	0.48	0.62	0.80	1.08
	0.128	N^*	30	21	14	10	6	4	3	2	1	1
		x^*	0.19	0.22	0.25	0.30	0.36	0.45	0.56	0.71	0.88	1.14
	0.256	N^*	31	21	15	10	7	4	3	2	1	1
		x^*	0.32	0.34	0.38	0.43	0.49	0.58	0.70	0.84	1.01	1.27
	0.512	N^*	31	21	15	10	7	5	3	2	1	1
		x^*	0.57	0.60	0.63	0.68	0.75	0.84	0.95	1.10	1.27	1.52
	1.024	N^*	31	21	15	10	7	5	3	2	1	1
		x^*	1.09	1.11	1.15	1.19	1.26	1.35	1.47	1.61	1.78	2.04

Table 3b
Values of V^* and N^* as a function of α and β for exponential observations

Exponential distribution			β									
			0.001	0.002	0.004	0.008	0.016	0.032	0.064	0.128	0.256	0.512
α	0.001	N^*	6	4	2	1	1	1	1	1	1	1
		x^*	0.04	0.05	0.07	0.10	0.14	0.19	0.28	0.41	0.61	0.94
	0.002	N^*	8	5	3	2	1	1	1	1	1	1
		x^*	0.04	0.05	0.07	0.10	0.14	0.20	0.28	0.41	0.61	0.94
	0.004	N^*	11	7	4	3	2	1	1	1	1	1
		x^*	0.04	0.06	0.08	0.11	0.15	0.20	0.29	0.41	0.61	0.94
	0.008	N^*	14	9	6	3	2	1	1	1	1	1
		x^*	0.05	0.06	0.09	0.12	0.16	0.21	0.29	0.42	0.62	0.94
	0.016	N^*	17	11	7	4	3	2	1	1	1	1
		x^*	0.06	0.07	0.10	0.13	0.17	0.23	0.31	0.43	0.63	0.95
	0.032	N^*	20	13	9	6	4	2	1	1	1	1
		x^*	0.08	0.09	0.12	0.15	0.19	0.26	0.35	0.46	0.65	0.97
	0.064	N^*	22	15	10	7	4	3	2	1	1	1
		x^*	0.11	0.13	0.15	0.19	0.23	0.30	0.39	0.51	0.70	1.01
	0.128	N^*	22	16	11	7	5	3	2	1	1	1
		x^*	0.17	0.19	0.22	0.25	0.30	0.37	0.47	0.61	0.78	1.08
	0.256	N^*	22	16	11	8	5	4	3	2	1	1
		x^*	0.30	0.32	0.35	0.38	0.43	0.51	0.61	0.75	0.93	1.22
	0.512	N^*	22	16	11	8	6	4	3	2	1	1
		x^*	0.56	0.58	0.60	0.64	0.69	0.76	0.87	1.01	1.22	1.50
	1.024	N^*	22	16	11	8	6	4	3	2	1	1
		x^*	1.07	1.09	1.11	1.15	1.20	1.28	1.38	1.53	1.77	2.03

$$\int_{y=0}^{x}(F_{N+2}(y)-F_{N+1}(y))\,\mathrm{d}y<\int_{y=0}^{x}(F_{N+1}(y)-F_{N}(y))\,\mathrm{d}y. \qquad \text{(A1)}$$

Recall that $V_N^*=x_N^*$. We will show that $x_N^*\leqslant x_{N+1}^*$ implies $x_{N+1}^*\leqslant x_{N+2}^*$. From Theorem 3 we have $\alpha+\beta N=\int_{y=0}^{x_N^*}F_N(y)\,\mathrm{d}y$. Also, $\alpha+\beta(N+1)=\int_{y=0}^{x_{N+1}^*}F_{N+1}(y)\,\mathrm{d}y$. Therefore

$$\beta=\int_{y=0}^{x_{N+1}^*}F_{N+1}(y)\,\mathrm{d}y-\int_{y=0}^{x_N^*}F_N(y)\,\mathrm{d}y. \qquad \text{(A2)}$$

Similarly,

$$\beta=\int_{y=0}^{x_{N+2}^*}F_{N+2(y)\,\mathrm{d}y}-\int_{y=0}^{x_{N+1}^*}F_{N+1}(y)\,\mathrm{d}y. \qquad \text{(A3)}$$

By hypothesis, $x_N^*\leqslant x_{N+1}^*$. Hence, from (A2)

$$\beta\geqslant\int_{y=0}^{x_{N+1}^*}F_{N+1}(y)\,\mathrm{d}y-\int_{y=0}^{x_{N+1}^*}F_N(y)\,\mathrm{d}y$$
$$=\int_{y=0}^{x_{N+1}^*}(F_{N+1}(y)-F_N(y))\,\mathrm{d}y. \qquad \text{(A4)}$$

Assume now that $x_{N+1}^*\geqslant x_{N+2}^*$. Hence, from (A3)

$$\beta\leqslant\int_{y=0}^{x_{N+1}^*}F_{N+2}(y)\,\mathrm{d}y-\int_{y=0}^{x_{N+1}^*}F_{N+1}(y)\,\mathrm{d}y$$
$$=\int_{y=0}^{x_{N+1}^*}(F_{N+2}(y)-F_{N+1}(y))\,\mathrm{d}y. \qquad \text{(A5)}$$

This, together with (A4), contradicts (A1). □

Proof of Proposition 1

(a) From (5) it follows that $\beta N=\int_{y=0}^{x_N^*}F_N(y)\,\mathrm{d}y=\int_{y=0}^{x_N^*}(1-\overline{G}^N(y))\,\mathrm{d}y$; and, in particular, $\beta=\int_{y=0}^{x_1^*}F_1(y)\,\mathrm{d}y=\int_{y=0}^{x_1^*}(1-\overline{G}(y))\,\mathrm{d}y$. Therefore,

$$N\int_{y=0}^{x_1^*}(1-\overline{G}(y))\,\mathrm{d}y=\int_{y=0}^{x_N^*}(1-\overline{G}^N(y))\,\mathrm{d}y.$$

Note that

$$1-\overline{G}^N(y)=[1+\overline{G}(y)+\cdots+\overline{G}^{N-1}(y)](1-\overline{G}(y))\leqslant N(1-\overline{G}(y)),$$

to obtain

$$\int_{y=0}^{x_1^*}(1-\overline{G}^N(y))\,\mathrm{d}y\leqslant\int_{y=0}^{x_N^*}(1-\overline{G}^N(y))\,\mathrm{d}y.$$

Thus, $x_1^*\leqslant x_N^*$.

(b) From (5) it follows that $\alpha=\int_{y=0}^{x_N^*}F_N(y)\,\mathrm{d}y=\int_{y=0}^{x_N^*}(1-\overline{G}^N(y))\,\mathrm{d}y$; and, similarly, $\alpha=\int_{y=0}^{x_{N+1}^*}F_{N+1}(y)\,\mathrm{d}y=\int_{y=0}^{x_{N+1}^*}(1-\overline{G}^{N+1}(y))\,\mathrm{d}y$. Therefore,

$$\int_{y=0}^{x_N^*}(1-\overline{G}^N(y))\,\mathrm{d}y=\int_{y=0}^{x_{N+1}^*}(1-\overline{G}^{N+1}(y))\,\mathrm{d}y\geqslant\int_{y=0}^{x_{N+1}^*}(1-\overline{G}^N(y))\,\mathrm{d}y.$$

Thus, $x_{N+1}^*\leqslant x_N^*$.

A.1. The expressions of $L_N(x)$ for the exponential and uniform distributions

For exponential observation we have:

$$\int_0^x y^2 f_N(y)\,\mathrm{d}y=\frac{2}{(\lambda N)^2}-\mathrm{e}^{-\lambda Nx}\left(x^2+\frac{2x}{\lambda N}+\frac{2}{(\lambda N)^2}\right).$$

Thus

$$L_N(x)=\frac{(\alpha+\beta N)[2V_N(x)-(\alpha+\beta N)]-x(x+2/\lambda N)\mathrm{e}^{-\lambda Nx}}{1-\mathrm{e}^{-\lambda Nx}}+\frac{2}{(\lambda N)^2}.$$

For uniform observation we have:

$$E[(Y_N)^2\cdot 1(Y_N\leqslant x)]=\int_0^x y^2 f_N(y)\,\mathrm{d}y=\begin{cases}\int_0^x y^2N(1-y)^{N-1}\mathrm{d}y, & 0\leqslant x<1,\\ \int_0^1 y^2N(1-y)^{N-1}\mathrm{d}y, & 1\leqslant x<\infty.\end{cases}$$

Thus, after integration:

$$L_N(x)=\begin{cases}\frac{(\alpha+\beta N)(2V_N(x)-(\alpha+\beta N))}{F_N(x)}+\frac{2-(1-x)^N(N^2x^2+N(x+2)x+2)}{(N+1)(N+2)F_N(x)}; & 0\leqslant x<1\\ (\alpha+\beta N)(2V_N(x)-(\alpha+\beta N))+\frac{2}{(N+1)(N+2)}; & 1\leqslant x<\infty\end{cases}$$

A.2. A finite horizon model

Notation is similar to that in the paper with the exception that N_k and V_k are the sample size and expected cost, respectively, as there are k periods (iterations) remaining.

One period remaining (k = 1). Conducting only one search cycle, the customer will pay $\alpha + \beta N_1$ for the search and Y_{N1} (the minimum of N_1 quotes) for the commodity. The only decision variable is the sample size. The expected cost is: $V_1(N_1) = \alpha + \beta N_1 + E(Y_{N1})$.

Note that $E(Y_{N1})$ decreases in N_1. A search over N_1 (or analytical method, at times) leads to optimal values: N_1^* and V_1^*.

Two periods remaining (k = 2). Note that after obtaining N_2 quotes (a decision variable) the customer follows the rule: If $Y_{N2} \leqslant V_1^*$ she stops. Otherwise, i.e., if $Y_{N2} > V_1^*$ she will take an additional sample of N_1^* quotes. Thus, again, the only decision variable is N_2, and the expected cost is: $V_2(N_2) = \alpha + \beta N_2 + E[1(Y_{N2} \leqslant V_1^*)Y_{N2} + 1(Y_{N2} > V_1^*)V_1^*]$. Now minimize over N_2 to obtain V_2^* and N_2^*. Observe that $V_2^* \leqslant V_1^*$, because

$$V\,{}_2^* = \min_{N2}\{V_2(N_2)\} = \min_{N2}\{\alpha + \beta N_2 + E[1(Y_{N2} \leqslant V_1^*)Y_{N2} + 1(Y_{N2} > V_1^*)V_1^*]\} \leqslant \min_{N2}\{\alpha + \beta N_2 + E[1(Y_{N2} \leqslant V_1^*)Y_{N2} + 1(Y_{N2} > V_1^*)Y_{N2}]\} = \min_{N2}\{\alpha + \beta N_2 + E[Y_{N2}]\} = V_1^*.$$

Similarly, using induction, V_k decreases in k, which is intuitive. In particular, the infinite horizon problem gives a better result than the finite horizon problem. Also note that the finite-horizon problem is considerably simpler in that there is only one decision variable.

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