Search Costs as a Means for Improving Market Performance

David Sarne and Yonatan Aumann

Computer Science Department
Bar-Ilan University
Ramat Gan, 52900 Israel

Abstract. In this paper we study the benefits of search costs in distributed multi-agent systems (MAS). These costs, often associated with obtaining, processing and evaluating information relating to other agents in the environment, can be either monetary or manifested in some tax on the agent's resources. Traditionally, such costs are considered as market inefficiency, and, as such, aimed to be reduced to the minimum. Here we show, in contrast, that in many MAS settings the introduction of search costs can actually improve market performance. This is demonstrated in three different settings. First we consider one-sided and two-sided (equilibrium-driven) search applications. In both settings we show that, while search costs may decrease the individual agents' outcomes, the overall *market throughput* may actually improve with the introduction of such costs. Next, we demonstrate a setting where, somewhat paradoxically, the introduction of search costs improves both the overall market throughput and the utility of each and every individual agent. We stress that we assume that the proceeds from the search costs are *wasted*, with no one directly benefiting from them. The importance of the results is for the design of MAS systems, where in many cases one should consider deliberately increasing (potentially artificially) the search friction to some desired level in order to improve the system's performance.

1 Introduction

In many cases, market friction and seemingly "inefficiency -based" mechanisms can be used for establishing desired market behaviors. For example, minimum wage has been shown as a mechanism that can, in some cases, improve employment rates [16] and Vickrey Clarke Groves (VCG) ensures assignment of items in a socially optimal manner and truth telling by requiring significant transfer of payments from agents to the center [8]. Similarly, particular taxes (known as marginal cost pricing) can eliminate inefficiencies of equilibria as demonstrated in transportation economics [5].

In this paper, we learn the positive role that search costs, incurred as part of a repeated search process, can play in enhancing performance of markets. In many multi-agent systems (MAS), agents may incur a cost when engaged in obtaining, processing or reasoning about information related to their environment and other agents that can be found in it [14, 9]. These costs can be either in the form of explicit monetary payments or resources that need to be consumed in order to carry out these activities (e.g., computational, communication). In economics and operations research such costs are often referred to as "search costs" or "environment friction" as they represent the inefficiency and lack of transparency of the environment the agents are operating in [3, 25]. Typical examples include distributed matching applications (where the interaction with other agents is costly) [1], shopbots (where price and product information must be actively gathered) [14], and reputation systems (where agents may need to pay a fee for querying the system) [13].

Traditionally, search costs are regarded as market *inefficiency*, and associated with reduced market performance. Indeed, in the presence of search costs, a rational player would not aim to find the optimal element in the environment, but rather settle for the "good enough", beyond which the marginal benefit of continuing the search is less than the search cost. Thus, search costs promote sub-optimal results (or so it would seem). As such, the traditional wisdom is that when designing a MAS environment, search costs should be avoided or reduced as much as possible. Taking eCommerce as an example, most researchers see a great benefit in the ability of electronic marketplaces to lower the buyers' cost to obtain information (e.g. about the price and product features) from multiple sellers, as well as the sellers' reduced costs to communicate their information [4]. The lowered buyer's search cost is associated in this case with an increased economic efficiency and enable new markets to emerge [4]. Similarly, many systems have been introduced in which central mechanisms or mediators are used in order to supply the agents full immediate information concerning market opportunities, eliminating the need to engage in costly search [10, 2].

In this paper we show that, not withstanding the above, search costs — "friction" — can also be beneficial, and may *improve* market performance, in some cases, when applied appropriately. We show this on two levels. First, we note that while each player's individual goal is to maximize its own utility, and as such may suffer from search costs, the market designer should consider the overall welfare produced by the system as a whole. This, we argue, is not the average utility of the individual players, but rather the total *utility throughput*, i.e. the aggregate utility *per time unit*. Note that in search settings the players' individual utilities and the utility throughput are not necessarily directly correlated, as players can stay in the system for longer or shorter time periods. With this understanding in mind, we show two examples where introducing search costs increases the market throughput, and hence also the social welfare. Specifically, we consider classical one and two-sided search settings, and show that in both, search costs can improve market throughput. It is important to stress that throughout the paper we assume that the proceeds from the search costs are discarded and do not benefit anyone in the system.

Next, we show that even when considering the benefit of each individual player, there are cases where search costs can increase the expected utility of *each and every* player in the system. This seemingly paradoxical phenomena is exhibited in a setting of two-sided search with multiple search rounds, each round consisting of several parallel searches. We show that if the cost for each round is given, then it may be beneficial to artificially increase (up to a certain level) the cost of each individual parallel search within the round, and that all players simultaneously benefit from this. The exact details are provided in Section 4.

The common rational to all these examples is that in some cases it may be beneficial (either to the system designer or to all) to reduce the amount of searches performed by the players, but when dealing with self-interested agents this cannot be directly dictated. Search costs provide a means to incite players to perform less searches. What is interesting in these examples is that even when the proceeds of the costs are wasted, the benefit of the reduced search outweighs the loss.

The analysis given differs from classical examples of seemingly "inefficiency -based" mechanisms as it inherently derives from the modeling of the search strategy as a repeated process, and applies to the various models investigated as part of classical search theory [21, 15, 26, 6, 11, 18, 19]. The main contribution of this paper is thus in establishing the notion of market throughput within the context of search-based MAS applications and demonstrating that market friction in the form of search cost can often play a positive role market-wise, despite the reduction in individual performance experienced by the agents. In some cases, search costs can actually improve both individual and market performance. Therefore, when designing a new MAS, the system designer should carefully consider the option of deliberately generating some inefficiency in the system in the form of search costs.

2 One-Sided Search

Consider an environment with N homogeneous servers, and an infinite incoming flow of homogeneous agents requesting service from these servers.[1] In practice, the servers may represent medical specialists and agents - the patients, or servers may represent online retailers and the agents - shopbots. Each agent can send a query to any of the servers, where the utility of reply x received by agent j from a server i is randomly drawn from a distribution characterized by a probability distribution function (p.d.f.) $f(x)$, and cumulative distribution function (c.d.f.) $F(x)$, defined over the interval $(\underline{x}, \overline{x})$. For example, patients may seek for a second opinion from medical specialists, where the quality of the diagnosis received varies, and shopbots may query different retailers for price information. For simplicity we assume that all servers and all agents are homogeneous, and thus share the functions $f(x)$ and $F(x)$, and that an agent's utility from the returned value x equals x. We assume that the time it takes to execute a single query on any of the servers is fixed and WLOG will be considered a time unit. Based on the revealed utility of x, the agent can decide to send an identical query to an additional server, again obtaining a utility drawn from the same distribution function. This process continues until the agent decides that there is no point in sending any further queries, or until all N servers have been queried. In both cases, the resulting utility is the maximum among the utilities obtained from the queries sent. We assume that queries sent by agents that have already received service have priority over those of newly arriving agents.

Agents are assumed to be self-interested, and thus aiming to maximize their expected utility. The system designer, on the other hand, should be interested in the aggregate utility. Since the system is assumed to continue working indefinitely, the aggregate is not well defined, and what we are really interested in is the average *throughput*, i.e., the average aggregate utility *per time unit*. Formally, denote by $A(\underline{t}, \bar{t})$ the set of agents that have completed their querying process within the time interval $(\underline{t}, \bar{t})$, and by $U(A_i)$ the utility obtained by agent $A_i \in A(\underline{t}, \bar{t})$. The average throughput of the system during the interval $(\underline{t}, \bar{t})$, denoted $T(\underline{t}, \bar{t})$, is defined as:

$$T(\underline{t}, \bar{t}) = \frac{\sum_{A_i \in A(\underline{t}, \bar{t})} U(A_i)}{\bar{t} - \underline{t}} \quad (1)$$

[1] Since the purpose of this paper is to investigate the effect of search costs over throughput of MAS, we deliberately assume that the demand for the servers is infinite. Without this assumption, the analysis necessitates including queueing theory aspects, which add many complexities without adding to the understanding of the core phenomena illustrated.

For a system working indefinitely the throughput is defined as:

$$T_\infty = \lim_{\bar{t}\to\infty} \frac{\sum_{A_i \in A(0,\bar{t})} U(A_i)}{\bar{t}} \quad (2)$$

Since the agents are homogeneous and the setting is a symmetric game, they all use the same optimal querying strategy. Therefore, the following holds for large $(\underline{t}, \bar{t})$ intervals:

$$\sum_{A_i \in A(\underline{t},\bar{t})} U(A_i) = \frac{E[U(A_i)] \cdot N \cdot (\bar{t} - \underline{t})}{E[q]}$$

where $E[q]$ is the expected number of queries sent by each agent (identical to all agents as the agents are homogeneous). Therefore:

$$T(\underline{t}, \bar{t}) = \frac{N \cdot E[U(A_i)]}{E[q]} \quad (3)$$

Note that the right-hand side of Equation (3) is independent of $(\underline{t}, \bar{t})$. Hence, the same value also holds for the limit T_∞.

Suppose that each agent queries exactly $k \leq N$ servers, i.e., $E[q] = k$. The expected utility of each agent in this case is the expected maximum of a sample of size k taken from the distribution $f(x)$, which equals $\int_{y=\underline{x}}^{\overline{x}} y f_k(y)dy$, where $f_k(x)$ is the probability distribution function of the maximum of a sample of size k drawn from $f(x)$.[2] Therefore:

$$T_\infty = \frac{N \int_{y=\underline{x}}^{\overline{x}} y f_k(y)dy}{k} \quad (4)$$

Obviously, when there is no cost associated with querying a server, agents will query all servers (as the expected utility $\int_{y=\underline{x}}^{\overline{x}} y f_k(y)dy$, increases with k) . In this case, substituting $k = N$ in Equation 4, the average throughput reduces to the expected utility of a single agent, i.e., $T_\infty = \int_{y=\underline{x}}^{\overline{x}} y f_N(y)dy$. While this strategy maximizes individual agents' utility, it is certainly not so in terms of overall throughput. Given the limited number of servers and infinite demand, having each agent take advantage of all servers is clearly sub-optimal.

Therefore, for the sake of optimizing throughput, it would be best for a market designer to limit the number of queries sent by each agent (the throughput is inversely related to $E[q]$ - see by Equation (4)). While this is possible in some systems, in many systems, where agents are self-interested, the system designer cannot directly dictate such behavior. Introducing search costs is an implicit mechanism by which a system designer may incite agents to execute less queries. On the other hand, search costs themselves also reduce the throughput, as their proceeds are assumed to be wasted, and must thus be deducted from the overall utility. We show that nonetheless a certain level of search costs may still *increase* the overall throughput. The details follow.

In the presence of search cost, agents consider the tradeoff between the marginal utility of each additional query and its cost. This problem, of finding the agents' optimal querying strategy in the presence of a search cost, can be mapped to a variant of the

[2] The value of $f_k(x)$ is the derivative of the c.d.f. of the maximum of a sample of size k, $F_k(x)$. Since $F_k(x) = (F(x))^k$, we obtain: $f_k(x) = k(F(x))^{k-1} f(x)dx$.

known "Pandora's problem" [26].[3] Accordingly, in the optimal search strategy, each agent searches sequentially using a reservation value z, querying (randomly) additional servers as long as the maximum utility obtained so far is smaller than z or until all servers have been queried. Given a fixed search cost c (incurred when sending a query to a server) and assuming the cost is additive and expressed on the same scale as utilities, the reservation value z that maximizes the agent's expected utility can be extracted from:

$$c = \int_{y=z}^{\overline{x}} (y - z) f(y) dy \tag{5}$$

The expected number of queries each agent sends according to the above strategy is given by:

$$E[q] = \sum_{i=1}^{N} i P_q(i) \tag{6}$$

where $P_q(i)$ is the probability that exactly i queries will be sent ($P_q(i) = ((F(z))^{i-1}(1-F(z))$ for $i < N$ and $P_q(N) = (F(z))^{N-1}$ for $i = N$).

Hence, the expected utility of each agent using the optimal search strategy when incurring a search cost, denoted $V_{overall}$, is:

$$V_{overall} = E[x/x > z]\Big(\sum_{i=1}^{N-1} P_q(i) + P_q(N)(1-F(z))\Big) + P_q(N)F(z)E[Max(x_1, ..., x_N/x_i < z)] - cE[q] \tag{7}$$

where $E[x/x > z]$ is the expected utility of a single query if above z ($E[x/x > z] = \int_{y=z}^{\overline{x}} yf(y)/(1 - F(z)))$ and:[4]

$$E[Max(x_1, ..., x_N/x_i < z)] = \int_{\underline{x}}^{z} y f_N(y/y_i < z \forall i) dy = \int_{\underline{x}}^{z} \frac{yNf(y)}{F(z)} \Big(\frac{F(y)}{F(z)}\Big)^{N-1} dy \tag{8}$$

The expected throughput is thus $\frac{N(V_{overall} - cE[q])}{E[q]}$.

Figure 1 depicts the expected system throughput as a function of the search cost c, when using a uniform distribution function - $f(x) = 1, F(x) = x, 0 \leq x \leq 1$ - and ten servers ($N = 10$). The optimal market setting is obtained with a search cost of $c = 0.12$, in which case the market throughput is 2.5. Comparatively, if there is no search cost ($c = 0$) then each agent samples all servers, i.e., $E[q] = 10$. The expected throughput in this case is 0.91 (the maximum of a sample of size 10 drawn from a uniform distribution), significantly worse then when using $c = 0.12$. On the other hand, if market designer could *dictate* the number of queries per agent, then a throughput of 5 could have been obtained, by forcing each agent to query a single server (resulting with individual utility of 0.5, which is smaller than when using $c = 0.12$).

Figure 2 depicts the optimal search cost to be used and the resulting average throughput per server as a function of the the number of servers available, N, when using a uniform distribution function ($f(x) = 1, F(x) = x, 0 \leq x \leq 1$). The middle curve represents the marginal improvement to the overall throughput due to the addition of

[3] Alternatively, the problem can be mapped to a one-sided sequential search problem with a finite decision horizon and full recall [15, 21] and solved using backward induction, though with a greater complexity.

[4] Using $F_N(x/x < z) = (F(x/x < z))^N$, $f_N(x/x < z) = Nf(x/x < z)(F(x/x < z))^{N-1}dx$.

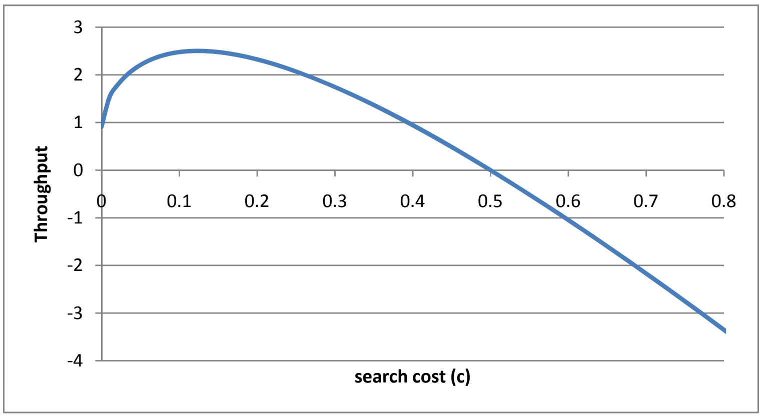

Fig. 1. Throughput as a function of search cost

each server (e.g., the transition from one server to two servers will be accompanied with a 0.2 addition to the overall system throughput). As can be observed from the figure, while adding more servers increases the overall throughput, it has no consistent marginal contribution to the overall throughput. For example, the transition from two to three servers is accompanied with the least marginal improvement in the system's throughput.

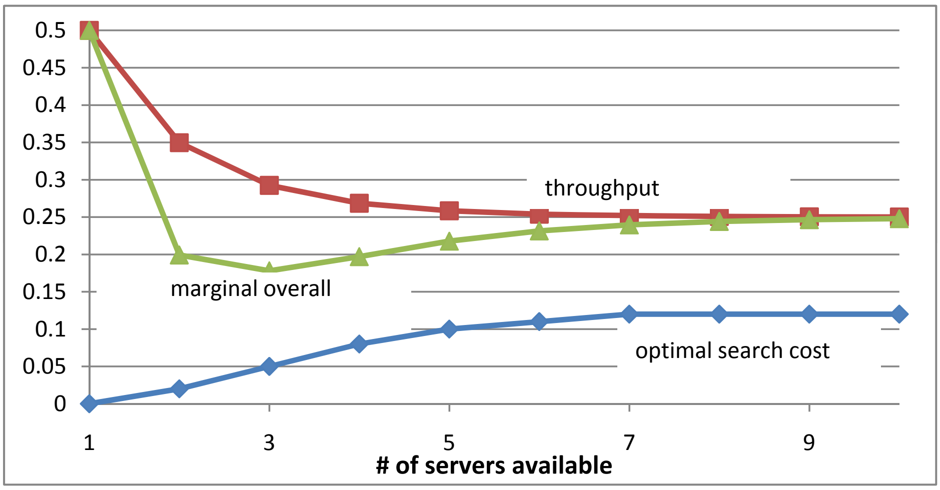

Fig. 2. Throughput per server, marginal improvement to throughput and optimal search cost as a function of the number of servers in the one-sided search model

As can be seen from Figures 1 and 2, the use of search cost in this model can improve the average throughput significantly, though the optimal magnitude of search cost to be used should be determined by analyzing the agents' resulting search strategies.

3 Two-Side Search

We next show that the benefits of search costs to market throughput are not limited only to settings where the agents are competing for limited resources, as in the previous section. In this section we demonstrate that search cost can also improve throughput in distributed matching environments, where the agents' value is generated from partnering with other agents, and where the search strategies are affected (in part) by the strategies of the other agents in the market. The model we use is a standard two-sided distributed search model, in which self-interested agents search for appropriate partners to form mutually acceptable pairwise partnerships [7]. The model postulates an environment populated with an infinite number of self-interested fully rational homogeneous agents[5]. Any agent A_i can form a partnership with any other agent A_j in the environment. A partnership between A_i and A_j results in utility $U(A_j \hookrightarrow A_i)$ for agent A_i and $U(A_i \hookrightarrow A_j)$ for agent A_j, where both $U(A_j \hookrightarrow A_i)$ and $U(A_i \hookrightarrow A_j)$ are drawn from a distribution characterized by a p.d.f. $f(x)$ and c.d.f. $F(x)$. The agents are assumed to know the utility distribution function $f(x)$. However, in the absence of central information source agents cannot tell a-priori what utility can be gained by a partnership with any specific agent. Therefore, the only way by which an agent A_i can learn the value it can obtain from partnering with a specific other agent A_j is by directly interacting with agent A_j. Since each agent in two-sided search models has no prior information concerning any of the other agents in its environment, it initiates interactions (i.e., search) with other agents randomly. The two-sided search model assumes that the agents are satisfied with having a single partner. Hence, once a partnership is formed the two partnering agents terminate their search process and leave the environment.

We define a search *round* as the interval in which the agent interacts with another agent and learns the utility it can obtain by partnership with it. Based on the learned values, the agents decide whether to commit or reject the partnership. If both agents mutually commit to the partnership, then the partnership is formed and both agents gain the corresponding utilities. If an agent does not form a partnership in a given round, it continues to the next search round and interacts with another agent in a similar manner.

We now define the market throughput in such a setting. Since the number of agents is infinite, it is meaningless to consider the total aggregate utility. Rather, we define the *market throughput* as the average expected utility per-agent, per time unit. If there would be no search costs, the agents' equilibrium strategy is to commit to partnerships only when the utility offered by that partnership is the maximum possible (i.e., $\overline{U}$). In such case, for any non-atomic p.d.f. the probability of actually attaining this maximum is zero, and partnerships will never be formed. Hence, in this case the expected throughput is zero. Adding a search cost to the model reduces each agent's expected utility, but can improve the overall throughput. We assume utilities and costs are additive and that the agents are trying to maximize their overall utility, defined as the utility from the partnership formed minus the aggregated search costs along the search process. Suppose that the agent's cost of interacting with another agent is c.

[5] The infinite number of agents assumption is common in two-sided search models (see [7, 23, 22]). In many domains (e.g., eCommerce) this derives from the high entrance and leave rates. In this case, the probability of running into the same agent twice is negligible.

The agents' optimal strategy in such model is reservation-based [22, 17, 26, 19].[6] The reservation value is used as a threshold for accepting/rejecting potential partnerships. The a reservation value is equal to the expected utility to be obtained from resuming the search; the agent will always prefer committing to an opportunity greater than the expected utility of resuming the search and will always prefer to resume the search otherwise.

Since the agents are not limited by a decision horizon, and their search process does not imply any new information about the market structure (e.g., about the utility distribution of future partnership opportunities), their strategy is stationary - an agent will not accept an opportunity it has rejected beforehand and will use the same reservation value along its search.

We now derive the general formula for the optimal reservation value. The expected utility of an agent when using a reservation value x, assuming all other agents are using reservation value x', denoted $V(x, x')$, is given by:

$$V(x, x') = -c + (1 - F(x')) \int_{y=x}^{\overline{x}} y f(y) dy + (1 - (1 - F(x'))(1 - F(x)))V(x, x') \quad (9)$$

Here, $(1 - F(x'))$ is the probability that the agent is found adequate by the other agent, in which case the partnership will form only if the value obtained from the partnership is greater than x. Otherwise, if the utility obtained from partnering with the other agent is below x or the other agent obtains a utility lesser than x' (i.e., with probability $(1-(1-F(x'))(1-F(x)))$), the search is resumed and the expected cost is $V(x, x')$. Using some simple mathematical manipulations, Equation 9 can be expressed as:

$$V(x, x') = \frac{-c + (1 - F(x')) \int_{y=x}^{\overline{x}} y f(y) dy}{(1 - F(x'))(1 - F(x))} \quad (10)$$

Differentiating the last equation according to x and setting it to zero, we obtain (using integration by parts) that the optimal reservation value to be used when all other agents are using reservation value x' can be derived from:

$$c = (1 - F(x')) \int_{y=x}^{\overline{x}} (1 - F(y)) dy \quad (11)$$

The equilibrium reservation value, x^*, is obtained by setting $x' = x = x^*$ in Equation 11. The equilibrium expected utility of each agent is thus given by:

$$V(x^*) = \frac{-c + (1 - F(x^*)) \int_{y=x}^{\overline{x}} y f(y) dy}{(1 - F(x^*))^2} \quad (12)$$

Once search cost is introduced, the expected time it takes until a partnership is formed becomes finite, and its value is: $\frac{1}{(1-F(x^*))^2}$. The throughput is thus $\frac{V(x^*)}{(1-F(x^*))^2}$.

Figure 3 depicts the expected average throughput and individual agents' utility in the two-sided search model as a function of the search cost c, when using a uniform distribution function ($f(x) = 1, F(x) = x, 0 \le x \le 1$). As can be observed from the

[6] Notice the reservation value used here is different from a reservation price concept (that is usually used as buyers' private evaluation). While the reservation price represents an agent's valuation of its own utility from a given opportunity, the reservation value is a threshold defined over the objective (or common) value of the opportunity.

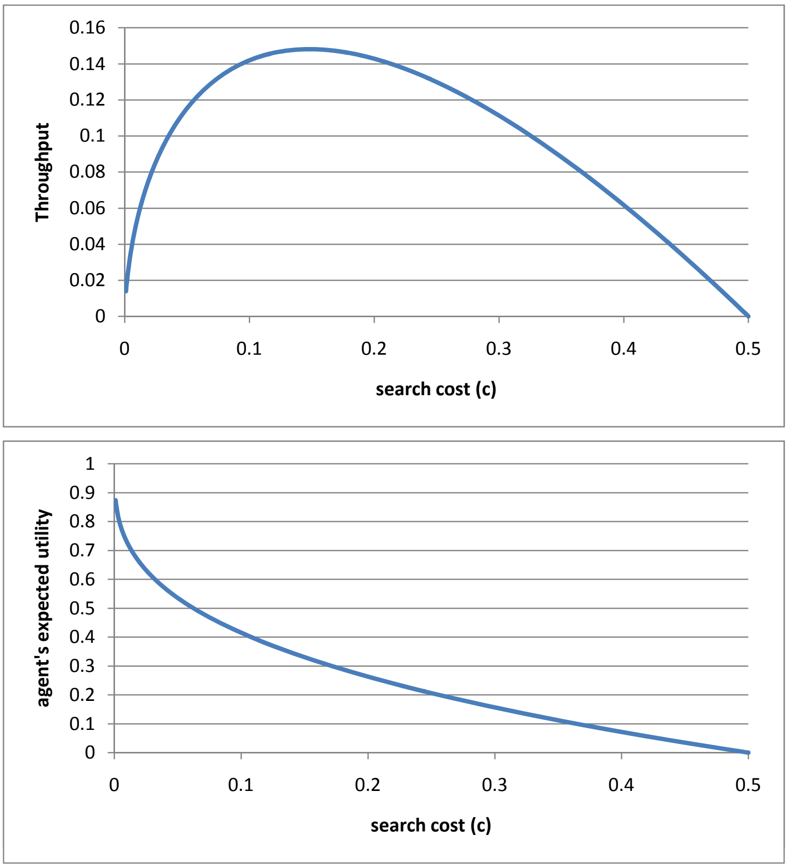

Fig. 3. Throughput and agents' utility as a function of search cost in two-sided search

example, the introduction of search cost up to some extent (0.5 in this case) improves system throughput. The optimal market setting is the one where the search cost is $c = 0.148$, in which case the expected throughput is 0.148. The agents' expected utility in this case drops to 0.333 and their expected search length (i.e., the number of search rounds) is in this case 2.25. Comparatively, when there is no search cost ($c = 0$) the expected throughput is zero, but each agent's (theoretical) expected utility is 1. This is yet another example for a case where an increase in the search cost can improve overall throughput however with the price of harming individual utilities. If the agents were fully cooperative and obey the market designer's instructions, a throughput of 0.5 could be attained, by having each agent search for a single search stage and then commit to whatever partnership it is offered. However, this behavior is not individually rational.

4 Search Costs Benefiting Individual Performance

In the two previous sections we exhibited settings where the market throughput is improved by introducing search costs. This is of interest to a market designer who's goal is to maximize the overall social welfare. However, in both previous examples, the search costs did reduce the agents' individual utilities. In this section we give a setting where, somewhat paradoxically, the introduction of search costs improves both the overall throughput and each and every player's individual expected utility. We exhibit this in the model introduced in [22], which is an extension of the standard two-sided search model. We now briefly review this model.

As in the standard two sided search, the model considers an environment populated by an infinite number of agents, each seeking a single partner, and utilities drawn from a distribution function (as in the previous section). The difference in the [22] model is in the search process. The model postulates a two-leveled search process, as follows. The search is conducted in discrete *rounds*. Within each round, each agent can choose to meet *in parallel* any number of other agents, and learn the utility associated with partnering with any of them. Given this information, each agent chooses if and with whom to partner. If two agents both choose to pair with one another then they obtain the said utilities and leave the system. Otherwise, they continue to the next search round. It is assumed that each agent make its decision independently of the decisions of all other agents, including potential partners. Furthermore, the agent can choose to pair with at most one of the agents it meets in a search round, and due to the synchronous nature of the mechanism has to reject all the rest of the agents met in that search round. There are two potential costs associated with each round: α - a fixed per-round cost, and β - an additional cost associated with each parallel probe in the round. Thus, if an agent chooses to meet with N potential partners then its total cost for the round will be $\alpha + N\beta$. The values of α and β are assumed to be the same for all agents.

Since the agents are not limited by a decision horizon and can control the intensity of their search, and the interaction with other agents does not imply any new information about the market structure, their strategy is stationary - an agent will not accept an opportunity it has rejected beforehand. Thus agents will use a reservation value strategy.

For analysis purposes, we'll use several notations. A strategy of sampling N other agents in each search round, and acting according to a reservation value x_N will be denoted (N, x_N). The expected utility of an agent when using strategy (N, x_N), will be denoted $V(N, x_N)$. Clearly, since all utilities for all agents are randomly selected from an identical distribution, there is no sense for any agent to consider partnering with anyone but the maximum of any given round. Thus, if an agent chooses to meet with N other agents in a given round, then the possible utility it can obtain during this round is distributed as the maximum of N random variables from the distribution with p.d.f. $f(x)$. As in Section 2 we use $f_N(x)$ and $F_N(x)$ to denote the p.d.f. and c.d.f. of the maximum distribution, respectively.

We start by formulating the expected utility for the agent when using a strategy (N, x_N), given that the strategy (k, x_k) is being used by all other agents in the environment. The expected future utility $V(N, x_N)$ is:

$$V(N,x_N) = -\alpha-\beta N+\frac{1}{k}(1-F_k(x_k))\int_{y=x_N}^{\overline{x}} y f_N(y)dy+(1-\frac{1}{k}(1-F_k(x_k))(1-F_N(x_N)))V(N,x_N) \tag{13}$$

Here, $\frac{1}{k}(1-F_k(x_k))$ is the probability that the utility obtained by the agent associated with the best value in the N-agents sample of agent A from partnering with agent A is greater than the reservation value x_k that is used by that agent and the maximum

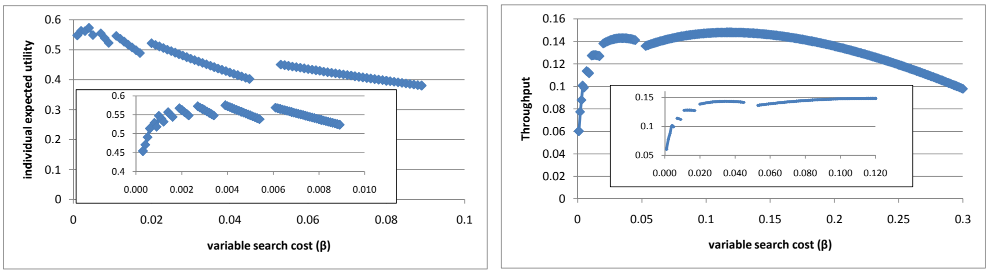

Fig. 4. (a) The expected utility of each agent as a function of the search cost used; (b) The throughput as a function of the search cost used.

in its k-agents sample. Similarly, the probability $(1-F_k(x_k))(1-F_N(x_N)))V(N,x_N)$ applies to any scenario other than the one where both agents choose to pair with each other, in which case the agent resumes its search with an expected utility $V(N,x_N)$. Equation 13 can also be formulated as:

$$V(N,x_N)=\frac{(1-F_k(x_k))\int_{y=x_N}^{\overline{x}} yf_N(y)dy-\alpha k-\beta Nk}{(1-F_k(x_k))(1-F_N(x_N))} \tag{14}$$

Deriving Equation 14 according to x_N and applying several mathematical manipulations, we obtain that the agent's optimal reservation value x_N, satisfies:

$$\alpha k+\beta Nk=(1-F_k(x_k))\int_{y=x_N}^{\overline{x}}(1-F_N(y))dy \tag{15}$$

Since all agents are homogeneous, the equilibrium strategy is the pair (N,x_N) that satisfies Equation 15 when substituting $k=N$ and $x_k=x_N$. While for each N value there is potentially a corresponding x_N value that satisfies the latter, the equilibrium strategy is the one yielding the maximum expected utility according to Equation 14 (for more details of the analysis of the model the reader is referred to [22]).

As in the former section, since the number of agents is infinite, the *market throughput* is defined as the average expected utility per-agent, per time unit. Figure 4 depicts the expected individual utility of each agent and the throughput as a function of β - the cost for each parallel search within a round, when using a uniform distribution function $(f(x)=1, F(x)=x, 0\leq x\leq 1)$ and $\alpha=0.03$. The small graphs inside each of the graphs are enlargements of the originals over the more interesting parts of the graphs. As can be observed from the graphs, with α fixed, each and every individual agent actually benefits from the introduction of search costs into the market. The optimal individual utility of 0.575 is obtained for $\beta=0.004$, in which case each agent meets with 5 possible partners in each round and the throughput is 0.1. The maximum throughput (0.148) is obtained when using a search cost of 0.118, yielding a utility of 0.333 to the agents. It is notable that the change in system's throughput as well as in individual expected utility is not consistent over large portions of the interval. The market designer should thus be careful when considering any change (either an increase or a decrease in search cost) in the setting represented by this example, as there is no way to predict the usefulness of such a change. In order to understand the implications of a suggested deviation from one value to another over the horizontal axis a direct calculation is required.

5 Related Work

Search is an inherent process in MAS, in particular when there is no central source that can supply full immediate reliable information on the environment and the state of the other agents that can be found. The introduction of search costs into MAS models leads to a more realistic description of MAS environments. In particular, search cost is highly recognized in eCommerce environments where agents need to invest/consume some of their resources in order to obtain information concerning the good or the transaction offered by other prospective agents [3, 9]. The overall agreement is that despite the significant reduction in search costs in MAS, due to recent advances in communication technologies, these cannot be ignored completely [3, 14].

Optimal search strategies for settings where individuals aim to locate an opportunity that will maximize the expected utility, while minimizing the associated search costs have been widely studied ([17, 15], and references therein). Within the framework of search theory, three main clusters of search models can be found. These are (a) the fixed sample size model; (b) the sequential search model; and (c) the variable sample size model. In the fixed sample size model, the searcher executes a single search round in which it obtains a large set of opportunities simultaneously [24] and chooses the one associated with the highest utility. In the sequential search strategy [21, 15], which for the general finite decision horizon case is also known as "Pandora's Problem" [26], the searcher obtains a single opportunity at a time, allowing multiple search stages. Several attempts were made to adopt the fixed sample size search [14] and the sequential search [9] models in agent-based electronic trading environments associated with search costs. In these cases the main focus was on establishing the appropriate characteristics of the environment and search strategy rather than the computational aspects of extracting it. Last, the variable sample size search method [6, 11, 18, 19] suggests a combined approach in which several opportunities are obtained during each search period.

In an effort to understand the effect of dual search activities in such models, the "two-sided" search research followed. This notion was explored within the equilibrium search framework [1, 7, 23]. While the literature in the area of one-sided and two-sided is rich and thorough, its focus is individual performance and search cost (often modeled as the discounting of gains) is considered as a non-favorable factor.

Finally, it is notable that the role of friction in distributed environments has been studied in several contexts before. For example, many authors rationalize the price dispersion (i.e., variation in prices across sellers of the same item, holding fixed the item's characteristics) observed in both offline and online markets by the cost for consumers to gather information about prices [24, 20, 12]. Others, have shown that an increase in the minimum wage, which is often considered by economics to be market inefficiency, can have positive employment effects [16]. In the auctions domain, the Vickrey Clarke Groves mechanism ensures several desired bidding characteristics by requiring significant transfer of payments from agents to the center [8]. Nevertheless, none of the above work has considered the positive effect search costs may have on system throughput.

6 Discussion and Conclusions

The main focus of the paper is in exhibiting that search costs are not necessarily and universally harmful to the system's performance. The illustrations in Sections 2-4 exhibit the fact that search costs can also play a positive role in improving market performance. Sections 2 and 3 exhibit the benefits of search costs for improving throughput.

The last example demonstrates that the improvement in system's throughput does not necessarily have to be at the expense of individual utilities. These results are non-intuitive, as traditionally search costs are considered to be market friction and as such their reduction is intuitively favorable. The examples given in the paper suggest that market designers should not take this latter claim as a general truth.

Notwithstanding, it is notable that the introduction of the measure of market *throughput* as a key measure for evaluating the performance and effectiveness of MAS systems is by itself an important contribution to the research of MAS. Unlike individual or collective agents' utilities, the throughput focuses in the value generated by the system as a whole over time. As such, we believe that this measure should be central for MAS designers, and understanding its behavior is crucial for the success of future mechanisms. As evident from Sections 2-4, the throughput is not necessarily correlated with individual utilities as these lack the aspect of time.

The paper relies on three established models from the "search theory" research area to support its main claims. Justification and legitimacy considerations for the applicability of these models to day-to-day settings were widely discussed in the literature we referred to throughout the paper.

It is notable that search costs can have many forms, and there are various methods for the market designer to control them. For example, search costs can be introduced as a payment an agent needs to pay in order to meet other agents or obtain a service, additional communication and computational overhead that result from the interaction protocol and even a payment per time unit for operating in the system. In this paper we adopted a pessimistic approach that assumes that the proceeds from any search costs are wasted and do not benefit anyone. In many cases, However, the proceeds of these costs can also be somehow redistributed back to the agents (e.g., equally split the proceeds among all agents when leaving the system - leaving their searching strategy unaffected). This could further improve individual utilities and the system's throughput.

Generally, as can be seen from the analysis given, the introduction of search costs should be carefully considered and their optimal magnitude should be calculated taking into consideration the resulting changes in the agents' strategies and equilibrium considerations whenever applicable. When search costs are already an inherent part of the system, there is no general answer for whether or not a decrease in these costs will improve system performance. In some settings, an increase rather than a decrease can actually contribute to improving throughput. In other cases, a decrease in search costs can contribute to improving system throughput, however decreasing these costs beyond to a certain point can result with the opposite effect. The analysis methodology given in this paper can facilitate the calculation of the right search cost to which the market designer should strive.

Acknowledgment

This work was supported in part by the Israeli Science Foundation grant no. 1401/09 and BSF grant 2008-404.

References

1. A. Atakan. Assortative matching with explicit search costs. *Econometrica*, 74(3):667–680, 2006.

2. M. Attane, I. Garcia, J. Martinez, and J. Papi. Tecapsy trans european car-pooling and parking systems. In C. A. Brebbia, editor, *The Sustainable City II: Urban Regeneration and Sustainability.* Spring Verlag, 2002.
3. Y. Bakos. Reducing buyer search costs: Implications for electronic marketplaces. *Mgmt. Science*, 42:1676–92, 1997.
4. Y. Bakos. The emerging role of electronic marketplaces on the internet. *Commun. ACM*, 41(8):35–42, 1998.
5. W. C. Beckmann M., McGuire C.B. *Studies in the economics of transportation.* Yale University Press, 1959.
6. J. Benhabib and C. Bull. Job search: The choice of intensity. *Journal of Political Economy*, 91:747–764, 1983.
7. K. Burdett and R. Wright. Two-sided search with nontransferable utility. *Review of Economic Dynamics*, 1:220–245, 1998.
8. R. Cavallo. Optimal decision-making with minimal waste: strategyproof redistribution of vcg payments. In *AAMAS '06: Proceedings of the fifth international joint conference on Autonomous agents and multiagent systems*, pages 882–889, New York, NY, USA, 2006. ACM.
9. S. Choi and J. Liu. Optimal time-constrained trading strategies for autonomous agents. In *Proc. of MAMA'2000*, 2000.
10. K. Decker, K. P. Sycara, and M. Williamson. Middle-agents for the internet. In *IJCAI*, pages 578–583, 1997.
11. S. Gal, M. Landsberger, and B. Levykson. A compound strategy for search in the labor market. *International Economic Review*, 22(3):597–608, 1981.
12. M. C. W. Janssen and J. L. Moraga-Gonzalez. Strategic pricing, consumer search and the number of firms. *Review of Economic Studies*, 71(4):1089–1118, October 2004.
13. A. Josang, R. Ismail, and C. Boyd. A survey of trust and reputation systems for online service provision. *Decision Support Systems*, 43(2):618–644, March 2007.
14. J. Kephart and A. Greenwald. Shopbot economics. *JAAMAS*, 5(3):255–287, 2002.
15. S. Lippman and J. McCall. The economics of job search: A survey. *Economic Inquiry*, 14:155–189, 1976.
16. A. M. Masters. Wage posting in two-sided search and the minimum wage. *International Economic Review*, 40(4):809–826, 1999.
17. J. McMillan and M. Rothschild. Search. In R. Aumann and S. Hart, editors, *Handbook of Game Theory with Economic Applications*, pages 905–927. 1994.
18. P. Morgan. Search and optimal sample size. *Review of Economic Studies*, 50(4):659–675, 1983.
19. P. Morgan and R. Manning. Optimal search. *Econometrica*, 53(4):923–944, 1985.
20. M. R. Preston. Multiproduct equilibrium price dispersion. *Journal of Economic Theory*, 67(1):83–105, October 1995.
21. M. Rothschild. Searching for the lowest price when the distribution of prices is unknown. *Journal of Political Economy*, 82(4):689–711, 1974.
22. D. Sarne and S. Kraus. Agents strategies for the dual parallel search in partnership formation applications. In *Proc. of AMEC2004, LNCS 3435*, pages 158 – 172, 2004.
23. R. Shimer and L. Smith. Assortative matching and search. *Econometrica*, 68(2):343–370, 2000.
24. G. Stigler. The economics of information. *Journal of Political Economy*, 69(3):213–225, 1961.
25. J. Stiglitz. Imperfect information in the product market. In R. Schmalensee and R. Willig, editors, *Handbook of Industrial Organization*, volume 1, chapter 13, pages 769–847. Elsevier, 1 edition, 1989.
26. M. L. Weitzman. Optimal search for the best alternative. *Econometrica*, 47(3):641–54, May 1979.