The Choice of Eliminating the Uncertainty Cloud in Auctions

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Abstract. This paper investigates auction settings where bidders private values depend on a common value element which encapsulates some inherent uncertainty concerning its value. In particular, we are interested in scenarios where the auctioneer may have access to accurate information that eliminates the uncertainty concerning the common value. The auctioneer in this case should reason about whether or not to disclose that information, or part of it, to the bidders. In particular, we distinguish between the case where the bidders are aware of the existence of such information on the auctioneer's side and when they are unaware of its existence. We show that the auctioneer's decision of whether to disclose information to the bidders is environment-dependent and affected by various factors, such as the number of bidders, the bidders' valuation functions and the a-priori level of uncertainty associated with the common value. Furthermore, we show that the awareness of the bidders to the option to obtain more accurate information plays a significant role in the decision of whether or not to disclose such information.

1 Introduction

The phenomenal success of e-commerce in general and online auctions in particular, gives rise to the role of agents as facilitators and mediators in electronic marketplaces. Moreover, the fact that some online auctions mechanisms require bidders to reason about various aspects of their strategies, and the resulting extended complexity of computing their strategies, further strengthen the need for the development of automated software agents [17].

Auctions are usually classified as either *"private value"* or *"common value"*, according to the way the bidders perceive their benefit from the auctioned item. In private value auctions, bidders know their own value for the auctioned item with certainty but are unsure about others' valuations (e.g., the sale of painting) [13, 12, 2]. In common value auctions bidders receive noisy signals about the auctioned item's true value, which is the same for all (e.g., firms competing for the rights to drill oil) [13, 12]. However, in most real world applications, the auctioned item is characterized by both private and common value elements [13, 6, 7, 14]. For example, consider the case of a PPC (pay-per-click) auction [11, 9]. Here, there is some uncertainty regarding the number of clicks the winner will likely experience if winning the auctioned advertisement area, which is the common value element in this case. At the same time, each bidder has a different valuation, i.e., private value, for her revenue per click. Another example is the classical oil drilling case [13]. Here, the amount of oil and its location deepness under the ground are the uncertain common values. However, each bidder's valuation of that oil depends on the stratum to which she needs to drill as each bidder can have different equipment and drilling technology. Similar arguments favoring this hybrid-value model can be suggested for other classical auction domains such as the U.S. Federal Communications Commission (FCC) [1] and landing slots at airports. Even the painting classic example for private value can be considered as an example for the hybrid model due to the resell factor [6]. Nonetheless, despite the better realism of the hybrid-value model, the majority of the work in these auction domains considers pure private-value or pure-common value models. Few works have considered models that combine both elements [6, 7], however the bidders values in these works are limited to an additive combination of the two or assume the same effect of the common value on all the bidders (see related work and discussion sections for more details).

In this paper, we analyze hybrid-value auction models in which the auctioned items' value for the bidders is a complex function of the common value element and differs among bidders. In these settings, we can envision scenarios where the auctioneer has access to information that can fully or partially eliminate the uncertainty associated with the common value. Whenever the information becomes available to the auctioneer, the question of how much of this information, if at all, should be revealed to the bidders arises.

We start our analysis by considering the case where no information that can eliminate the uncertainty regarding the common value is provided to the bidders. We then move to the next model where the auctioneer has the ability to extract the exact value of the auctioned item and chooses to fully disclose this information to the bidders. Many works claim that information disclosure is beneficial to the auctioneer in setting involving common value as it causes the bidders to bid more aggressively [6, 7, 15, 14]. Nevertheless, as we show in this paper, when a general valuation function is used, eliminating the uncertainty is not always the preferred strategy for the auctioneer. Furthermore, we show that various environmental settings can affect the choice of whether to fully disclose the information to the bidders or leave the uncertainty as is. In particular, we show that whenever the auctioneer's expected revenue is concerned a selective disclosure strategy is always preferred over a full disclosure and no disclosure pure strategies. This latter result depends on the fact that the bidders are unaware of the availability of accurate information to the auctioneer. Therefore, the next question we consider is the effect of having the bidders aware of the fact that the auctioneer is capable of eliminating the uncertainty associated with the common value has on the strategy she should use and her resulting expected revenue. Accordingly, we analyze the equilibrium strategy of the auctioneer and the bidders, which stems from the solution of the auction as a Stackelberg leadership game [5]. One interesting result of the analysis, is that in some settings, if the auctioneer mistakenly believes that the bidders are unaware of the fact that she has access to accurate information, then taking advantage of such information can result in expected revenue which is smaller than the expected revenue in the case where she has absolutely no access to such information. Several insights relating to implications of the results in the market-maker level are given.

The paper is organized as follows. In the coming section we introduce and analyze the model and its different variants, and supply examples to the strategies to be used in different settings. In section 3 we give a discussion of the results obtain. Related work review is given in section 4. Finally, we conclude and give directions for future research in Section 5.

2 Model and Analysis

We consider an environment with an auctioneer, offering a single item for sale in a second-price sealed-bid auction, and n bidders that are interested in that item. Both the auctioneer and the bidders are assumed to be risk-neutral and fully rational. The auctioned item is assumed to have a characteristic X which value, denoted x, is associated with some uncertainty. The a-priori estimate of the value of X is captured by a probability density function $f(x)$ and a cumulative distribution function $F(x)$, defined over a finite continuous interval.[3] The probability density function $f(x)$ is assumed to be known to all the bidders.

Each bidder i is assumed to be of an independent private type, t_i. A bidder's type defines the way she values the proposed item, given the true value of the characteristic X. We use the function $V_{t_i}(x)$ to define the value for bidders of type t_i from the auctioned item if its characteristic X has a value x. The value of X thus can be seen as a common value in this context, and the function $V_{t_i}(x)$ defines the way each bidder of type t_i's valuation is affected by this common value. It is notable that the value function $V_{t_i}(x)$ in our model is assumed to be general, unlike former works that assumed such function to be linear in the common value [7] or that all of the bidders' valuations depend on the common value in the same manner and each bidder's valuation is a symmetric function of the other bidders' signals [14]. While the auctioneer is not familiar with the specific type of each bidder, she is assumed to be acquainted with the distribution of bidder types, defined by the probability density function $h(t_i)$.

We assume that in some settings, information regarding the true type of X becomes available to the auctioneer and it is up to the auctioneer to decide whether she wants to disclose this information to the bidders. The receipt of such information from the auctioneer is the only way the bidders can obtain the true value x and the model assumes symmetry in a sense that if the information is disclosed then it becomes available to all bidders. Furthermore, we assume that if the auctioneer decides to disclose information she supplies reliable information and commits to the disclosed value. In the following subsections we analyze the bidding strategies of bidders of the different types, and the expected revenue for the auctioneer

[3] For exposition purposes we consider the value of X to be drawn from a continuous distribution function. An analysis similar to the one introduced for the continuous case can be produced for the discrete case. This applies also to the use of a continuous distribution function of the types as follows.

in settings differing in: (a) the availability of the true value of x to the auctioneer; (b) the auctioneer's choice of disclosing this information; and (c) the bidders' awareness of the existence of such information.

2.1 "No Information" model

In many settings, both the auctioneer and the bidders cannot obtain the true value of the common value. For example, in oil drilling example where the geological maps are common values all parties are symmetric in their knowledge concerning the common value, which is a probabilistic function. Having no other information concerning the value of the characteristic x of the proposed item, the bidders may use only the probability density function $f(x)$ (which the models assumes to be common knowledge) for setting their bids. As in any second-price sealed-bid auction in the private value model, bidders bid truthfully (according to the auction theory), i.e., a bidder of type t_i will set her bid, denoted by $B_I(t_i)$, as the expected benefit from the item given the different values that the characteristic X may obtain. Formally, a bidder of type t_i will bid:

$$B_I(t_i) = \int_y V_{t_i}(y) f(y) dy \tag{1}$$

The expected revenue for the auctioneer in this case, denoted ER_I^{auc}, is thus the expected second best bid. In order to formally express ER_I^{auc}, the types' distribution $h(t_i)$ needs to be transformed into a bids' distribution. For this purpose, we use $G_I(w)$ to denote the probability that a random bidder is of a type that bids no more than some value w.[4] The function $G_I(w)$ can be calculated as:

$$G_I(w) = \int_{B_I(t_i) \leq w} h(t_i) dt_i \tag{2}$$

and hence the corresponding probability density function of a bid received from a random bidder, denoted $g_I(w)$, is:

$$g_I(w) = \frac{dG_I(w)}{dw} \tag{3}$$

Using the functions $G_I(y)$ and $g_I(y)$, the expected second best bid given n bidders, denoted ER_I^{auc}, can be calculated as:

$$ER_I^{auc} = \int_y n(n-1)(G_I(y))^{n-2}(1 - G_I(y)) g_I(y) y dy \tag{4}$$

2.2 "Full Information" Model

If the value x of the characteristic X becomes available to the auctioneer and the auctioneer decides to disclose it to the bidders (as in the case where the regulator forces the disclosure of such information, e.g., when ecological aspects are

[4] The function $G_I(w)$ is thus the cumulative probability function of the bid placed by a random bidder.

considered), then each bidder bids (according to second-price auction theory) her private value, calculated according to the value x. Formally, the bid of type t_i when realizing value x for the characteristic X, denoted $B_{II}(t_i, x)$, is given by:

$$B_{II}(t_i, x) = V_{t_i}(x) \tag{5}$$

The expected revenue of the auctioneer in this case, denoted ER_{II}^{auc}, is the expected second best bid given the item characteristics' exact value. In order to formally express ER_{II}^{auc}, we use $G_{II}(w, x)$ to denote the probability that a random bidder is of type that bids no more than some value w, given the true value x. The probability $G_{II}(w, x)$ can be calculated as:

$$G_{II}(w, x) = \int_{B_{II}(t_i, x)) \leq w} h(t_i) dt_i \tag{6}$$

and hence the corresponding probability distribution function of a bid received from a random bidder if value x is known a-priori, denoted $g_{II}(w, x)$, is:

$$g_{II}(w, x) = \frac{dG_{II}(w, x)}{dw} \tag{7}$$

Using the functions $G_{II}(w, x)$ and $g_{II}(w, x)$, the auctioneer's expected revenue given that the true value of X is x, denoted $ER_{II}^{auc}(x)$, can be calculated as:

$$ER_{II}^{auc}(x) = \int_{y} n(n-1)(G_{II}(y, x))^{n-2}(1 - G_{II}(y, x))g_{II}(y, x)ydy \tag{8}$$

Consequently, the expected revenue for the auctioneer from using the strategy of always disclosing the value of X to the bidders, denoted ER_{II}^{auc}, is:

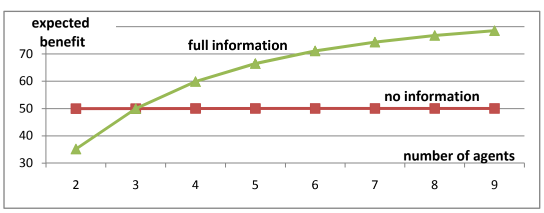
$$ER_{II}^{auc} = \int_{x} ER_{II}^{auc}(x)f(x)dx = \int_{x} f(x) \int_{y} n(n-1)(G_{II}(y, x))^{n-2}(1-G_{II}(y, x))g_{II}(y, x)ydydx \tag{9}$$

While we do not associate in this paper the act of obtaining the true value x with a cost, such a cost if incurred by the auctioneer should be subtracted from ER_{II}^{auc} calculated in Equation 9 in order to obtain the net revenue when the strategy is to obtain and always disclose the information. Therefore, given a cost c_e for obtaining the true value of the characteristic X, the auctioneer will prefer to obtain and distribute the information only if $ER_{II}^{auc} - ER_{I}^{auc} > c_e$.

2.3 The Choice of Revealing the Information

Assuming there is an option for the auctioneer to obtain the true value of X, none of the two above models generally dominates the other in terms of the expected revenue for the auctioneer. The decision of whether to run the auction as is, with the a-priori uncertainty, or to obtain and disclose the information relating to the characteristic X depends on: (1) the number of bidders; (2) the valuation functions as defined by the types; (3) the distribution of types; and (4) the distribution of the characteristic X's potential values. This result is different from former results that were obtained when using models with more restricted valuation functions [7, 14] as discussed in Section 3. The following figures illustrate how changes in each of these parameters can alter the decision of whether to leave the uncertainty in place or attempt to eliminate it, when all the other three parameters are fixed. For simplicity, the examples are based on discrete distribution functions and thus modifications of Equations 1-9 are used, where the integrals are replaced with weighted sums according to the discrete probabilities.

Figure 1 relates to the effect of changes in the number of bidders on the expected revenue. It uses three possible values for the the characteristic X (x_1, x_2, x_3) and three different types of bidders. The probability associated with each value x_i, the probability of the different types and the private value of each type, given the true value of X, are given in the accompanied table. The two curves depict the expected revenue for the auctioneer when supplying no information at all to the bidders (marked as "no information") and when fully disclosing the value of X (marked as "full information"). Here, when having only two bidders it is more beneficial for the auctioneer to leave the uncertainty as is, whereas for more than three bidders it is better to reveal the true value of X. As expected, as the number of bidders increases the expected revenue from revealing the true information increases. This has a simple intuitive explanation - since the bidders are bidding according to their valuation given the value x, the expected second best bid's value increases as the number of bidders increases. Notice that in the example given in Figure 1, the expected revenue for the auctioneer when not disclosing information does not depend on the number of bidders. This is because in this example the mean private value is equal for all three types. Nevertheless, this is not the general case.



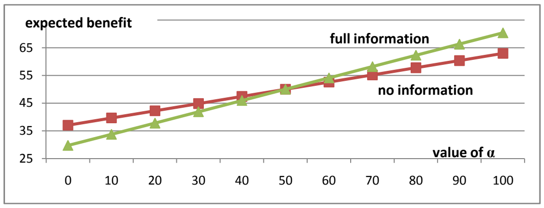
		private value		
		x1	x2	x3
type	P(type) \ P(value)	0.333	0.333	0.334
1	0.333	0	50	100
2	0.333	50	50	50
3	0.334	100	50	0

Fig. 1. Different number of bidders values may affect the choice of auctioneer's strategy.

Figure 2 relates to changes in the bidders' valuation assigned to each value of the characteristic X. It uses the same environment as in Figure 1, however with three bidders and varies the valuation of bidders of type 2 (the parameter α). As can be observed from the figure, the "full information" strategy dominates the "no information" strategy for some α values and vice versa.

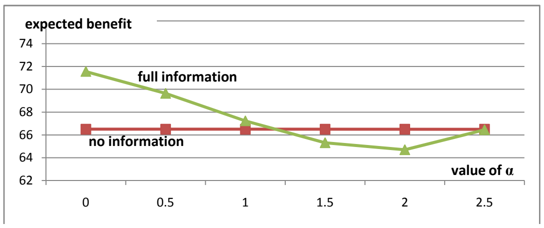
Figure 3 uses an environment with three bidders, and four possible bidder types, varying the distribution of the bidders' type. The parameter α is used to control the skewness of the distribution $h(t)$. Here, again, we can see that the dominance of one strategy over the other changes as a function of the tested parameter with no observed consistency.

Finally, Figure 4 uses an environment with four possible values for the the characteristic X (i.e., the common value), three bidders and three bidder types,



N=3		private value		
		x1	x2	x3
type	P(type) \ P(value)	0.333	0.333	0.334
1	0.333	0	50	100
2	0.333	α	α	α
3	0.334	100	50	0

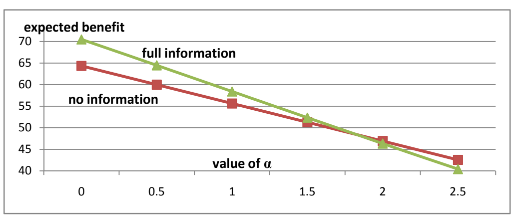
Fig. 2. Different private value functions of bidders may affect the choice of auctioneer's strategy.



N=3		private value		
		x1	x2	x3
type	P(type) \ P(value)	0.333	0.333	0.334
1	0.5-2α	100	100	0
2	0.35-α	0	100	100
3	0.15+α	100	0	100
4	2α	200	0	0

Fig. 3. Different types distributions may affect the choice of auctioneer's strategy.

varying the probability function associated with the value of the characteristic X. As in the other three examples, the preferred strategy for the auctioneer changes as the tested parameter changes.



N=3		private value			
		x1	x2	x3	x4
type	P(type) \ P(value)	0.5-2α	0.35-α	0.15+α	2α
1	0.333	100	100	0	0
2	0.333	0	100	50	100
3	0.334	100	0	100	0

Fig. 4. Different common value distribution may affect the choice of auctioneer's strategy.

2.4 Selective Information Disclosure Model

If the bidders are unaware of the auctioneer's access to the true value of X, then based on the results obtained in the former section, the auctioneer can improve its expected revenue by selectively disclosing information. An example for such scenario is the case where the auctioneer is a dealer offering a used car for sale, and the bidders are unaware of the fact that it has access to the car history. A dominating strategy for the auctioneer in this case is to obtain the information, however reveal it selectively, i.e., based on the value x, reveal x only if the expected second best bid, when the value x is known to the bidders ($ER_{II}^{auc}(x)$ as calculated in Equation 8) is greater than the expected second best bid when the bidders have no a-priori information (ER_I^{auc} as calculated in Equation 4). Formally, the auctioneer will choose to reveal the true value x only if the following holds:

$$n(n-1)\int_y y\Big((G_{II}(y,x))^{n-2}(1-G_{II}(y,x))g_{II}(y,x) - (G_I(y))^{n-2}(1-G_I(y))g_I(y)\Big)dy > 0 \quad (10)$$

The expected revenue for the auctioneer in this case, denoted ER_{III}^{auc}, equals the expected second best bid when selecting the maximum among two strategies, weighted according to the distribution $f(x)$ and thus can be calculated as:

$$ER_{III}^{auc} = n(n-1)\int_x f(x)max\Big(\int_y((G_{II}(y,x))^{n-2}(1-G_{II}(y,x))g_{II}(y,x)ydy, \int_y(G_I(y))^{n-2}(1-G_I(y))g_I(y)ydy\Big)dz \quad (11)$$

Proposition 1. *The selective information disclosure strategy, when the bidders are assumed to be unaware of the fact that the auctioneer knows the true value of X, is always preferred by the auctioneer.*

The proof for the proposition derives directly from Equations 11, 9 and 4.

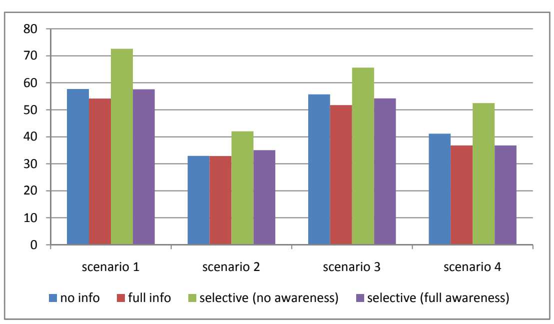
2.5 Equilibrium with Selective Disclosure

The selective disclosure is indeed the preferred method for the auctioneer (as given in proposition 1) as long as the bidders are unaware of the option to obtain such information. Nevertheless, if the bidders are aware of the existence of such information, they will integrate this knowledge as part of their bidding strategy, even if the auctioneer does not disclose the information concerning the value x. An example for such scenario is the case where the auctioneer is an individual offering a car for sale. Unlike in the dealer's case, here the bidders can assume that the auctioneer (as the car owner) has access to the car history. In this case, the bidders update their estimate of the distribution of the item characteristic's value, $f(x)$, eliminating all the values that the auctioneer prefers to disclose according to Equation 10. We use T to denote the set of values that the auctioneer discloses if revealed to be the true value of X, i.e., if the true value of X is $x \in T$ then the auctioneer discloses this value to the bidders. Given T, the bidders now update the probability distribution function of X, if no value is revealed from the auctioneer. The updated probability distribution function, denoted $f^*(x)$, is given by:

$$f^*(x) = \begin{cases} \frac{f(x)}{1-\int_{y \notin T} f(y)d(y)} & \text{if } x \notin T \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

Consequently, the bids received for $x \notin T$ can be calculated by substituting $f(x) = f^*(x)$ in Equation 1. The bids for $x \in T$ remain the same as in the full information model, i.e., can be calculated using Equation 5. The expected value for the auctioneer in this case can thus be obtained using Equation 11 where the values of $g_I(y)$ and $G_I(y)$ are calculated using the bids obtained from having the bidders update their estimate of the distribution of the value of the characteristic X.

The new bidding strategy used by the bidders in this case necessarily leads to worsen expected revenue for the auctioneer, in comparison to the case where the bidders are unaware of the availability of the information to the auctioneer. Yet, this can yield better expected revenue for the auctioneer in comparison to the full information and no information strategy variants in some settings. Figure 5 compares, for four illustrative settings (denoted "scenarios") the expected revenue for the auctioneer when she selectively discloses the information relating to X, according to the strategy described in Section 2.4, with and without the bidder's knowledge of her access to such information, and the expected revenue achieved in the two other approaches. The description of the scenarios is given to the right of the figure. The set of values of X that the auctioneer chooses not to disclose in each scenario (according to Equation 10) is shaded in the table. The goal of Figure 5 is to show that once the bidders become aware of the fact that the auctioneer can have access to the information about the value of X, the relation between the resulting expected revenue for the auctioneer when the latter is using selective disclosure and the expected revenue when using "no information" and "full information" is environment-dependent. The expected revenue in this case can either be as low as the minimum among the two (scenario 4), higher than the two (scenario 2), better than one but worse than the other (scenario 3) or equal to the maximum among the two (scenario 1). In any case, it is always lower than the expected revenue when using selective disclosure while the bidders are unaware of the availability of the information to the auctioneer.



N=3	private value			
type	x1	x2	x3	x4
1	200	0	0	0
2	0	170	0	30
3	100	0	50	50

	type			private value			
	1	2	3	x1	x2	x3	x4
scenario 1	0.330	0.339	0.330	0.363	0.281	0.288	0.068
scenario 2	0.378	0.228	0.394	0.106	0.065	0.378	0.451
scenario 3	0.310	0.366	0.324	0.270	0.327	0.088	0.315
scenario 4	0.359	0.411	0.230	0.078	0.343	0.334	0.246

Fig. 5. The expected revenue (vertical axis) using different strategy disclosure strategies in different environment settings.

If the auctioneer knows that the bidders are aware of her ability to obtain the accurate value of X, she should take this into consideration when reasoning about what information to disclose to the bidders. The auctioneer in this case should model the reaction of the different bidders to her decision to partially disclose information. The auctioneer's problem is thus to find a set T (of values to be disclosed if revealed to be the true value of X) that is in equilibrium. That is the bidders and the auctioneer are aware of this set T and the auctioneer has no incentive to deviate from this set given that the bidders are aware of it.

We denote the expected revenue for the auctioneer when using strategy T by $ER_{III}^{auc*}(T)$. The expected revenue $ER_{III}^{auc*}(T)$ in this case can be calculated as:

$$ER_{III}^{auc*}(T) = \int_{x\in T} f(x) \int_y (n(n-1)(G_{II}(y,x))^{n-2}(1-G_{II}(y,x))g_{II}(y,x)ydydx + \int_{x\notin T} f(x) \int_y n(n-1)(G_I^*(y))^{n-2}(1-G_I^*(y))g_I^*(y)ydydx \tag{13}$$

where $G_I^*(y) = \int_{\int_{z\in T}\frac{V_{t_i}(z)f(z)}{1-\int_{w\in T}f(w)dw}dy\leq y} h(t_i)dt_i$ and $g_I^*(y) = \frac{dG_I^*(y)}{dy}$.

3 Discussion

Based on the analysis and examples given in the former section, it is important for the auctioneer to be able to calculate her expected revenue under different information disclosure settings and levels of bidders' awareness of the availability of such information. In particular, the auctioneer's decision to use the selective disclosure strategy should be carefully made, as this strategy generally dominates the other strategies only if the bidders are completely unaware of the fact that the auctioneer has accurate information that can eliminate the uncertainty associated with the common value of the auctioned item. Failing to recognize whether the bidders are aware of the existence of such information, may result in expected revenue lower than the one that could have obtained if the auctioneer had not have access to such information (or would have obtained the information but decided not to disclose it) as illustrated in scenario 3 in Figure 5.

It is notable that the results given in Subsection 2.3 differ from findings given in former literature in the area of hybrid-value auctions [6, 7, 15, 14] due to the limitations of the value functions used in these works as discussed in the introduction. For example, Goeree and Offerman, 2003, [7] show that a decrease in the uncertainty associated with the common value necessarily increases auctioneer's revenues. This is due to their assumption that the private and common values are independent in their contribution to the total value for the bidder, i.e., using an additive valuation function, rather than a general one. Milgrom and Weber, 1982, show that in settings where all of the bidders' valuations depend on the common value in the same manner "honesty" (always report all information completely) maximizes the expected price and thus is the best strategy for the auctioneer [14]. The model given in this paper, on the other hand, uses a general valuation function, thus the dominance of disclosing information (i.e., eliminating the uncertainty) is not straightforward and depends, as evidenced in the examples given, on the environmental settings. For the general case, eliminating the uncertainty is not necessarily the best strategy from the auctioneer's perspective.

While we do not consider the auctioneer's access to the information (if available) to be costly, we could also think of scenarios where the auctioneer will need to invest some resources (that could be translated to monetary scale) in order to obtain this information (e.g., buy this information or hire an expert that will supply this information). This was briefly discussed in Subsection 2.2 but has further implications. For example, in case the auctioneer needs to decide whether she wants to invest the required resources in order to obtain the information, she needs to compare the expected revenue with and without partially (or fully) disclosing this information.

While this paper is given from the auctioneer's perspective, i.e., consider the resulting revenue (which is the expected second price bid), a complementary analysis can be provided from a social welfare perspective in the settings discussed. Naturally, the more information provided to the bidders the higher is the social welfare, as the bidder who wins the item is more likely the one who values it more. Nevertheless, when considering also the auctioneer's expected revenue as an element of the social welfare, different results may be obtained. In particular, we can think of scenarios where a market maker (e.g., a regulator) can interfere and possibly subsidize acts aiming towards obtaining information about the common value, in a way that improves the overall social welfare, in settings where such information is not available for free to the auctioneer. Alternatively, we can think of scenarios where the information is available to the auctioneer and yet the regulator choose to buy such information and disclose it to the bidders (e.g., by hiring an expert that will provide a more accurate estimate of the common value). This analysis is beyond the scope of the paper, however can make use of many parts of the analysis given in section 2.

Finally, we note that while the model assumes a single characteristic of the auctioned item associated with uncertainty, it can easily be extended to the case of multiple characteristics. In this case, the analysis needs to rely on the joint distributions of the different possible value sets given the original distribution of values of the values of each characteristic.

4 Related work

To best of our knowledge, the works most relevant to the model investigated in this paper are Goeree and Offerman, 2003 [7] and Milgrom and Weber, 1982 [14] who consider scenarios where bidders' valuation is a function of a common uncertain value. The main difference between the model given in this paper and their work is that both papers constrain somehow the bidders' valuation function. The first consider the private and the common values to be independent in their contribution to the overall value for the bidder, and consequently assume an additive valuation function. The second, assume that all of the bidders' valuations depend on the common value in the same manner and each bidder's valuation is a symmetric function of the other bidders' signals. In our model, a general valuation function is used, and consequently different results are obtained relating the the usefulness of disclosing information (as for example illustrated in section 3.

The model where the item's value is a combination of private and common value is sometimes referred to as *correlated value model* [17]. However, this term is somehow ambiguous and often refers to different model settings as we hereby illustrate. For example, Eso, 2005 [4] studies an auction model with risk averse bidders where the correlated value stems from the correlation coefficient among the bidders' valuations. Similar model was considered by Wang [18] whose work tries to answer the question: given the cost of a fixed price selling model, how does the distribution of the potential buyers' valuations determine which selling mechanism is preferred, the fixed price or the auction. He found that in case where the buyers valuations' distribution is sufficiently dispersed or when the object's value is sufficiently high, the auction mechanism is preferred.

Many researchers deal with the problem of *uncertainty* in auctions. Most works commonly refer to the uncertainty aspects associated with the bidders. Dyer et al. [3] consider the case where the bidders are uncertain about the number of bidders participating in the auction, which is often the case in online auctions that apply English-like protocols. Parkes [16] and Larson & Sandholm [10] consider the problem where bidders do not know their own private value and need to spend some computation efforts in order to reveal it. They show that indeed there is no correspondence between the classical rational analysis equilibrium and their case where rational bounded agents are considered. Hosam and Khaldoun [8] consider situations where agents are uncertain regarding their task execution, where agents are assumed to have partial control over their resources.

5 Conclusions and Future Research

The auctioneer's decision to disclose information relating to the common value highly affects the expected revenue in hybrid-value environments where the bidders valuations depend on the common value of the auctioned item. The model given in this paper considers general valuation functions of bidders, as opposed to former work in which more restrictive valuation functions are used. Consequently, the preferred strategy for the auctioneer is different than constantly preferring to fully disclose the common value — we show that for the general case there is no rule of thumb for deciding how much information to disclose, if at all, as any small change in one of the model parameters can result in a different preferred strategy. Using the analysis given throughout Section 2, the auctioneer can extract her preferred strategy, based on her estimates of whether or not the bidders are aware of the fact that she holds such information.

Future research, extending the analysis given in this paper, may involve the analysis of the model from the social welfare perspective as discussed in the former section. Additional directions for future research include: (a) Equilibrium analysis of settings where the bidders have some a-priori estimate of the probability that the auctioneer has access to the accurate common value, and similarly the auctioneer has some a-priori estimate for the probability the bidders are aware of the existence of such information; (b) The analysis of settings where only part of the bidders are aware of the fact that the auctioneer has the accurate common value information; and (c) The analysis of settings where the auctioneer herself can only obtain a noisy signal for the common value to begin with (e.g., only some values can be eliminated and some uncertainty remains).

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