Problem Set 1

1. Decremental single source shortest paths.

   Assume that you are given as a black box a data structure A which is capable to maintain a **single** source shortest paths tree in weighted (non-negative) directed graphs and to answer queries on the distance from the source to any other vertex of the graph.

   Present an algorithm that given a weighted directed graph $G = (V, E)$ computes its distance matrix only by using the data structure A.

2. Decremental reachability in acyclic graphs.

   Let $G = (V, E)$ be a directed acyclic graph and let $v \in V$. A reachability tree of v is a tree that contains paths to all the vertices that are reachable from v in G.

   Present a data structure for maintaining a decremental reachability tree from v whose total update time is $O(|E|)$.

3. Decremental reachability in general graphs.

   Let $G = (V, E)$ be a directed graph (that might contain cycles). Present a data structure for maintaining a decremental reachability tree from $v \in V$ whose total update time is $O(|E| \cdot |V|)$.