

$C \in \mathbb{R}^{n \times n}$, $F \in \mathbb{C}^{n \times n}$ $C^T = C$
 Le. C is symmetric for $C^T = C$ and
 Hermitian for $F^H = F$

$$C^T = \{ y \in \mathbb{R}^n \mid G \cdot y^T = 0 \}$$

C -d orthogonal G means G is orthogonal
 $C \in \mathbb{R}^{n \times n}$ is symmetric \Leftrightarrow
 $C^T = C$

$$\dim(C^T) = n - \text{rank}(G) = n - k = n - \dim(C)$$

This is G symmetric
 : y is orthogonal for C

$$\dim(C^T) = n - \dim(C) = k = \dim C$$

\square \square

parity check matrix H is $n-k$ rows
 (B) C is $n \times k$ matrix

$$C^T \cdot H = 0$$

$$H \in \mathbb{R}^{(n-k) \times n} \text{ s.t. } C \subseteq \mathbb{R}^n, \dim C = k$$

$$C = \{ x \in \mathbb{R}^n \mid H \cdot x^T = 0 \}$$

S is $n-k$ rows H is $n-k$ rows, C is k rows

$$H \cdot G^T = 0$$

$G \cdot H^T = 0$
 : means G is orthogonal to H

S is $n-k$ rows $G \in \mathbb{R}^{k \times n}$

H is $n-k$ rows $H \in \mathbb{R}^{(n-k) \times n}$

S is $n-k$ rows C^T is $n-k$ rows

$$C^T = \{ y \in \mathbb{R}^n \mid x \cdot y = 0, \forall x \in C \}$$

orthogonal

rows C are orthogonal $C \cap C^T = \{0\}$
 C is self dual

rows C are orthogonal $C \subseteq C^T$
 (self orthogonal)

$$C = \{ 0000, 1001, 0100, 1101 \}$$

$$C^T = \{ y_1 y_2 y_3 y_4 \in \mathbb{F}_2^4 \mid y_1 + y_4 = 0, y_2 = 0 \}$$

$$= \{ 0000, 0010, 1001, 1011 \}$$

$$(C^T)^T = C, C \text{ is self dual}$$

$$y \in C^T, x \in C, x \cdot y = 0$$

C is self dual, $C \subseteq C^T$
 : means C is orthogonal

C is self dual, $C \subseteq C^T$
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$$\dim C = \dim(C^T)$$

$k = \dim C$