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| Advanced Data Structures 1 | Moshe Lewenstein |
| Exercise 4 | Submit by 18.1.2006 |

Include your name, ID number, and exercise number. **Solutions:** Try to solve the problem on your own. You may also discuss the problem with others. However, (1) you **must** write the answer on your own and in your own words, (2) write down the names **next to each answer** of all the people you collaborated with on each question (this does not reduce points, but is intended to increase honesty).

1. Consider the following heuristic for the list-accessing problem: On the $i$'th access, if $i$ is odd we do MTF (move the accessed item to the front), and if $i$ is even, we just access the item, but do not move it to the front (the list remains the same.

   Prove that the given heuristic is 3-competitive in the dynamic model as defined in class.

2. **Prove the following:** Suppose we are given two BSTs $T_1$ and $T_2$ on the same $n$ keys. Show that one can transform $T_1$ into $T_2$ using at most $2n + O(1)$ rotations.

   *Hint:* There is a simple, short, and non-messy solution to this question—and probably a lot of messy ones. Try to find the simple one.

3. Assume that the dynamic optimality conjecture is true. Prove that the time it takes to splay the nodes of a binary tree one by one in symmetric order is $O(n)$ where $n$ is the number of nodes in the tree

4. Let $T$ be a tree, and say we have just splayed an item $x$ in $T$. For a node $v$ in $T$ we denote by $d_v$ the depth of $v$ before splaying $x$, and by $d'_v$ the depth of $v$ after splaying $x$. Prove or disprove the following claim: $\forall v \in T : d'_z \leq d_z + 2$

5. Consider the move to root heuristic with $zig$ operations only on a tree containing the integers 1 to $n$ in symmetric order. Assume $i < j$.

   (a) Prove that $i$ is an ancestor of $j$ if the most recent request for $i$ has occurred since the most recent request to any of $i+1, ..., j$. Similarly, $j$ is an ancestor of $i$ if the most recent request for $j$ has occurred since the most recent request to any of $i, ..., j-1$.

   (b) Prove that any tree $T$ containing these items in symmetric order can be produced from any other tree containing these items in symmetric order by a sequence of at most $n$ requests under the move to root heuristic.

   (c) For any $m$ sufficiently large show a sequence of accesses such that serving it with the move to root heuristic takes $\Omega(n)$ time on average per access. (the initial tree is arbitrary).