A NON-CONTAGIOUS RANDOM SPREAD OF MARKETING MESSAGES

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Abstract. In this paper we attempt to find a solution to the problem in the field of marketing communications of how to insure that a certain area will be sufficiently covered by the launching of items containing communication materials, when an immediate effect is required. It was found that it is possible to determine the necessary number of items for a given density of area coverage. The method comes from the theory of stochastic geometry and random tessellations. We give a mathematical formula based on two parameters; the first is the maximal distance between the resulting location of any launched item and any point on the area, and the second is the confidence probability. The model can be used in circumstances for which there exists an interest in communicating rapidly with a population segment located in a certain area in order to trigger immediate action.

1. Introduction

One of the main preoccupations of marketers has always been to have their messages reach the target public. Lack of knowledge, understanding or favorable attitude toward their product means few sales, or little action (in case of non-commercial marketing). Communication to a target public can go through primary sources (advertising, salespeople ...) or social sources, also called informal channels (word-of-mouth, network diffusion ...). In the modern context of mass markets, much of the communication between marketers and target audiences is being achieved through advertising. As a consequence, an early concern of mass marketers was to find the media plan that would maximize impact among target customers. Since the 1960's (with the increasing use of computers), much research has dealt with media planning. At the beginning, it was mainly based on linear programming or other basic models (Georges (2003), Zangwill (1965), Mevik et al. (1966), Bass et al, (1966), Hofmans (1966), Stasch (1967), Brown (1967), Charnes et al. (1968), Gensch (1968)). In the 70's, qualitative factors were incorporated into the various models (Gensch (1970), Scissors 1971), Kaplan et al. (1971), Catry et al. (1973), Mogg et al. (1974), Aaker 1975), Bogart (1975), Doyle et al. (1975), Locander et al., (1978), Cannon et al. (1980), Hornik (1980), Simon et al. (1980). In the 80's, improvements were proposed for existing models (Cannon et al. (1982), Zufryden (1982), Scissors (1983), Cannon (1984), Naples (1984), Cannon (1985), Leckenby et al. (1986), Rust et al. (1986)). Then simulations and non-linear models became popular, while further refinements of existing models were suggested (Cannon (1988), Danaher 1991, Farris 1991, Rathnam et al. (1992), Abe (1996), (1997), Ephron, (1998), Naik et al. (1998), Ephron et al. (2002), Cannon et al. (2002), Pilotta et al. (2004), Coulter et al. (2005), Vakratsas et al. (2005), Tellis et al. (2006)). By the late 1990's Web Media Planning became a subject of interest for researchers (Leckenby et al. (1998), Cheong et al. (2010), Pergelova et al. (2010)). Also, the issue of the effectiveness of specific media vehicles was then addressed (Prendergast et al. (1999), Taylor et al. (2006), Lancaster et al. (2008), Rubinson (2009), Wakolbinger et al. (2009), Collins et al. (2010), Newstead et al. (2010), Smith et al. (2010), Yim et al. (2010)).

One of the characteristics of media planning models is that they are not limited to immediate effects. First, many of the media involved (billboards, newspapers, magazines ...) provide to the inserted messages a certain life length, and might trigger exposure for several hours, days or even weeks after having been placed. Second, there is often a cumulative effect over time. This is true for most media and is especially important when dealing with low involvement products. Thus, one sole advertisement will not always produce the desired effect. It is the repetition of exposures, and maybe the combined use of several media that will do the trick.

A great deal of research has also been done on the issue of social channels[1]. Recent articles have attempted to better understand the phenomenon (Hartmannet al. (2008), deMatos et al. (2008), Lam et al. (2009), Colliander et al. (2011)). Much emphasis has also been put on the impact of Online Social Interaction (Awad et al. (2008), Chung et al. (2010), Daniasa et al. (2010), Kozinetz et al. (2010), Trusov et al. (2010), Chen et al. (2011)). The interaction between marketers' communication efforts and social channels has also been the focus of investigation (Mason (2008), Villanueva et al. (2008), Bhagat et al. (2009), Godes et al. (2009), Trusov et al. (2009), Hensel et al. (2010), Li et al. (2011), Zubcsek et al. (2011). What characterizes social channels is their contagious nature. This means that the message a recipient is exposed to, will often be transmitted further by him to others, who themselves will transfer it further and so forth. There are however circumstances when marketers cannot rely on retransmission on the one hand, and want their communication to have an instant and immediate effect, on the other hand. In the non-commercial field, this could occur when it is crucial to signal something without delay to a population living in a certain area, for example in case of an approaching tsunami, tab water being suddenly polluted etc. In those circumstances, the use of flyers (preferably self-destructive) sent by airplanes or helicopters might be seriously considered. Such an air operation might also be

[1]For a review of literature until 2008 on Word-of-Mouth Communication coming from the mathematical theory of stochastic geometry, see E. Merzbach and J. Picard (2009).

useful in the commercial field to announce a special surprise promotional event, to promote a perfume through the launching from the air of small tabs with micro-encapsulated flavor. In such cases, the arising question is how many flyers or tabs should be launched in order to obtain certain coverage over a specific area.

More specifically, the question could be posed in a more precise manner: How many flyers or tabs or coupons should be thrown over a certain area in order to make sure with $X\%$ likelihood (the probability p) that no point in the given area should be more than t meters away from the fallen item?

This problem is related to the necessity of instantaneous effect of a communication in the absence of contagious possibility that does not seem to have been dealt with in previous research. The following model which is derived from the mathematical theory of stochastic geometry should contribute to its solution.

The main idea is to develop the concept of random tessellations which are important models of stochastic geometry. They have been used in various fields of science and technology, e.g., in biology, forestry, and material sciences (for further examples see Stoyan-Stoyan (1994), for a description of plane or spatial mosaic-like structures. However, it has not previously been used in the marketing sciences.

Among the different kinds of random tessellations, the well-known Voronoi tessellation is a simple but nevertheless very useful model with seems to be suitable for modelling mosaic-like structures resulting from growth processes, for example, of crystals. Up to now the Voronoi tessellation with respect to a homogeneous Poisson point process (Poisson-Voronoi tessellation) has been investigated very extensively, see for example, Merzbach (1988), Stoyan-Stoyan (1994), and Stoyan-Kendall-Mecke (1987). The only parameter of this model is the intensity $\lambda > 0$, the mean number of points of the underling Poisson point process per unit area or volume, respectively. The knowledge from analytical calculations concerning this model is limited almost exclusively to mean values of geometric characteristics. In recent papers, e.g., Lorz (1990) and Muche (1993), analytical methods and numerical integration were combined in order to obtain higher moments and covariances or the chord length distribution. Alternatively, the distributions of geometric characteristics can be determined by Monte Carlo methods. Clearly, such simulations are very time-consuming. Hinde and Miles 1980, performed large-scaled simulation studies of the plane Poisson-Voronoi tesellation. A small-scale Monte Carolo study concerning the spatial Poisson-Voronoi tessellation were carried out by Quine and Watson (1984) (2,500 cells) and probably others.

2. The mathematical model

The main idea is to use the concept of random tessellation from the theory of stochastic geometry. More precisely, we will use a special case of random tessellation called the Voronoi tessellation or Voronoi mosaic.

In order to properly describe the notion of Voronoi mosaic, we need the concept of d-dimensional point process, and the simplest point process is the homogeneous Poisson point process (hereafter referred to as the Poisson process). It is characterized by the following properties: (see Merzbach (1988) and references there).

1) The random number of points in an arbitrary bounded region B in space is Poisson distributed with parameter $\lambda V(B)$, where $\lambda > 0$ and $V(B)$ denotes the volume of B.
2) The random numbers of points in arbitrary disjoint, bounded regions $B_1, \dots B_n$, $n = 2, 3, \dots$, are stochastically independent.

The only parameter involved in this model is one of scale, the so-called intensity λ, the mean number of points per unit volume.

In order to simulate a Poisson process with parameter λ within a bounded region B, first we have to generate a pseudo-random number N, Poisson distributed with parameter $\lambda V(B)$, and secondly, to generate N independent points, identically uniformly distributed in B. (see Lorz (1990)).

The homogeneous Poisson point process is sometimes called a completely random field.

We can consider also a inhomogeneous Poisson process, where in property 1) we replace $\lambda V(B)$ by an arbitrary intensity. For example, in the two-dimensional space $\mathbb{R}^2$, the following intensities are frequently used:

$$\lambda(x, y) = \lambda x, \quad x > 0, \quad y \in \mathbb{R}$$
$$\lambda(x, y) = \lambda(1 + \sin(x)), \quad z, y \in \mathbb{R}.$$

The intensity function

$$\lambda(x, y) = \lambda \exp\big\{ -(x^2 + y^2)\big\}, \quad x, y \in \mathbb{R}$$

describes a point field, where the intensity function is symmetric and decreases with the distance from the centre.

A Voronoi mosaic in the d-dimensional space, $\mathbb{R}^d$, is determined by a set $\phi \subseteq \mathbb{R}^d$ of points ξ_i, $i = 1, 2, \dots$ and the definition

$$C(\xi_i) = \{\xi \in \mathbb{R}^d : \|\xi - \xi_i\|_p < \|\xi - \xi_j\|_p \quad \text{for all} \quad \xi_j \neq \xi_i, \xi_j \in \phi\}$$

for the cell $C(\xi_i)$ is assigned to the point $\xi_i \in \phi$. The system of all cells forms the Voronoi mosaic.

If we consider only a bounded region of the space, then the number of points ξ_i is generally finite.

The norm used is the L_p-norm:

$$\|\xi - \xi_i\|_p = \sum_{k=1}^{d} \left(\left| \xi^{(k)} - \xi_i^{(k)} \right|^p \right)^{1/p}, \quad p > 0$$

where

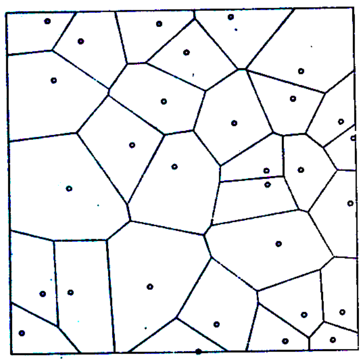
$$\xi = \left(\xi^{(1)}, \ldots, \xi^{(d)}\right) \quad \text{and} \quad \xi_i = \left(\xi_i^{(1)}, \ldots, \xi_i^{(d)}\right).$$

Generally, $p = 2$ is the Euclidean norm.

Let $C_i(t) = C(\xi_i) \cap B(\xi_i, t)$, where $B(\xi_i, t) = \{\xi \in \mathbb{R}^d : \|\xi - \xi_i\|_p < t\}$ is the open ball centered at ξ_i and with radius t.

Assuming the uniform distribution of falling points of the flyers, then the point field ϕ is completely random. Therefore, as a classical result, this point process can be considered as a homogeneous Poisson point field.

The following figure (taken from Stoyan-Kendall-Mecke (1987)) shows an example of a Voronoi mosaic with respect to a homogeneous Poisson point field in the plane $\mathbb{R}^2$.



For any $t \geq 0$, let $\Psi(t) = \bigcup_{i=1}^{\infty} \overline{C_i(t)}$. It is a Boolean model with spherical grains.

The main result is the following.

Theorem. *For any point $x \in \mathbb{R}^d$, and any $t > 0$,*

$$P\{x \in \Psi(t)\} = 1 - \exp\{-\lambda v_d t^d\}$$

where λ is the intensity of the process and v_d is the volume of the unit sphere of $\mathbb{R}^d$.

A proof of this result can be found in Muche (1993). Here we present a simplified proof.

Proof. $P\{x \notin \Psi(t)\}$ is the probability that there are no points to a distance t from a point ξ_i. According to the Poisson distribution, this probability is equal to $\exp\{-\lambda v_d t^d\}$ since the "volume function" is given by $v_d t^d$ (see Stoyan-Kendall-Mecke (1987), p. 66). Then the result follows. $\square$

Remark. The volume of the unit sphere must be computed in accordance with the metric (distance) used in the region. The usual metric is the Euclidean norm. However, in some cases, the distance needs to be computed differently, due to topographic or urban constraints, like rivers, mountains, houses, roads, stairs, etc.

In the Euclidean norm $v_1 = 1$, $v_2 = \pi$ and more generally

$$v_d = \pi^{d/2} / \Gamma\left(\frac{d}{2} + 1\right)$$

where Γ is the Gamma function:

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx.$$

Corollary. *In the d-dimensional space, the intensity λ is given by:*

$$\lambda = \frac{ln(1 - P\{x \in \Psi(t)\})}{-v_d t^d}.$$

For our purpose, the interesting case is the plane $\mathbb{R}^2$, with the Euclidean norm. We then get

$$\lambda = \frac{ln(1 - P\{x \in \Psi(t)\})}{-\pi t^2}.$$

Consequently, it is possible to determine the number of flyers (λ multiplied by the surface of the region) required to cover a certain area, given the confidence probability and the radius of each zone of inference.

3. Numerical results

As an illustration, the following table gives the number of flyers per area unit.

$p \setminus t$	5	10	20
0.80	0.020502	0.005125	0.001281
0.90	0.029332	0.007333	0.001833
0.95	0.038161	0.009540	0.002385
0.99	0.058664	0.014666	0.003666

Here p is the confidence probability and t is the radius of the influence flyer zone. For example, if t is measured in meters and the region is one kilometric square, then multiply the numbers in the table by 10^6. If, for example, the confidence probability is chosen to be 0.80 (permitting an error of 20%), and the flyer influence radius is 10 meters, we need 5125 flyers to cover the entire region.

4. Some extensions

Our model can be generalized in several directions:

1. Shape of the cells: The cells $C_i(t)$ are not generated by balls. For different reasons the influence zone of a flyer is not a circle, but has another shape.
2. Different times: The flyers do not arrive at the same time. In this case, we have a dynamic random tessellation and we need to use vector stochastic processes.
3. Growth rate: Each cell or each flyer has a different growth rate. In other words, the zone of influence is not the same for different objects. In this case, we have to add a random growth coefficient and we get another kind of random tessellation.

5. Conclusions

This article has attempted to find a solution to a problem in the field of marketing communications that was not dealt previously, i.e., how to insure that a certain area will be sufficiently covered by the launching of items containing communication materials, when an immediate effect is required.

It was found that it is possible to determine the necessary number of items for a given density of area coverage.

It suffices to use a mathematical formula based on two parameters; one is the maximal distance between the resulting location of any launched item and any point on the area, and the second is the confidence probability.

The model can be used in circumstances for which there exists an interest in communicating rapidly with a population segment located in a certain area in order to trigger immediate action.

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