Dynamic Algorithms: 89526
Bar-Ilan University — December 6, 2009. Due: 2 weeks
Liam Roditty — Handout 2

Problem Set 2

1. Dynamic minimum spanning tree

   Given a weighted undirected graph $G(V, E)$ you are required to maintain the minimum spanning tree of G while the edge weights can be increased or decreased. Assume that each time when the weight of a tree edge e is increased you are given also the minimum edge in the cut that is induced by the vertex partition of $T \setminus e$.

   Provide an algorithm and analyze its running time.

2. Topology trees.

   Show that it is possible to merge and split topology trees after the addition or the deletion of a minimum spanning tree edge in $O(\log n)$ time.

3. Non-Fixed graph.

   In the class we assumed that the input graph in Frederickson's algorithm is fixed and updates are done only to the weights. Assume now that edges are deleted and inserted to the graph. Show that it is possible to maintain the minimum spanning tree in $O(\sqrt{m_t} \log m_t)$ worst case time, where m_t is the number of edges currently in the graph.

4. Sparsification

   A connected undirected graph is 2-connected if the removal of any edge from the graph does not partition the graph into two connected components. Present a sparse strong witness to this property. Then, present a decremental algorithm that supports only queries of the following type: Does the current graph 2-connected?