All-Pairs of Shortest paths

1. Let $G(V, E)$ be a **directed** unweighted graph with n nodes. Show that in $O(n^w \log n)$ time it is possible to compute a matrix $C = (c_{ij})$ such that $\delta(i, j) \leq c_{ij} \leq (1+\epsilon)\delta(i, j)$, for any $\epsilon > 0$.

2. The *diameter* of a graph is the longest shortest path in the graph. Let $G(V, E)$ be a **directed** unweighted graph with n nodes. Show that it is possible to find the diameter of G in $O(n^w \log^2 n)$ time.

3. Let A and B be two $n \times n$ boolean matrices. Show how to find witnesses to all entries in AB that have only a single witness. What is the running time of your algorithm.

4. Let A be the the adjacency matrix of an unweighted undirected graph. Suppose that it is possible to compute in $M(n) + O(n^2)$ time a boolean matrix such that entry (i, j) is 1 if and only if the distance between i and j is odd. Show that it is possible to compute the distance matrix of the graph represented by A in $M(n) \log n + O(n^2 \log n)$ time.

5. Show how to implement Seidel's algorithm using only boolean matrix multiplication.