Given a k-partition $\pi = \{S_1, \dots, S_k\}$ and a set $M \subseteq N$, the remaining incomplete partition π_{-M} after removing M is defined as $\{S'_1, \dots, S'_k\}$, where $S'_i = S_i \setminus M$. Let $W_{-m}(\pi)$ denote the minimum value after removing at most m nodes, i.e., it is defined as:

$$W_{-m}(\pi) = \min_{M \subseteq N, |M| \leq m} \{W(\pi_{-M})\}.$$

To obtain $W_{-m}(\pi) \neq -\infty$, every $S \in \pi$ needs to contain at least $m+1$ nodes, so that no node-subset of π_{-M} is emptied.

2 COMPLEXITY OF MAX-MIN-K-PARTITION

We studied computational complexity for the max-min defender's problem and found two intricate results that are detailed in [14]. The standard verification problem itself turns out to be coNP-complete, which intricates one more level in the polynomial hierarchy (PH). We indeed show that Max-Min-k-Partition (given a max-min k-partition instance, does a k-partition π s.t. $W_{-m}(\pi) \geq \theta$ exist?) is complete for class Σ_2^P, even in two cases:

(a) when $k = 2$ for arbitrary link weights $w \lessgtr 0$, or
(b) when $k = 3$ for non-negative link weights $w \geq 0$.

Though we don't know for $k = 2$ and $w \geq 0$, these results match what is known on MaxCut [15] (amounts to Min-2-Cut with $w \lessgtr 0$, and NP-complete) and Min-3-Cut [9] (NP-complete when one node is fixed in each node-subset), but one level higher in PH.

3 ITERATED BEST RESPONSE ALGORITHM

Usually, in the second level of PH, instances become very quickly intractable (e.g., above $n \geq 20$ in [20]). Thus, we introduce an algorithm that we call Iterated Best Response algorithm (IBR) for solving a max-min k-partition problem. The idea is to start from a random (new) k-partition π, and then iterate the following loop:

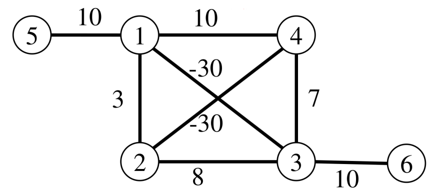
(1) Attacker response: $M \leftarrow \arg\min_{M' \subseteq N, |M'| \leq m} W(\pi_{-M'})$.
(2) Defender response: find $\hat{\pi}$, optimal k-partition of $N \setminus M$ s.t. $\mathrm{df}(\hat{\pi}) \leq |M|$ (where $\mathrm{df}(\hat{\pi}) = \sum_{C \in \hat{\pi}} \min\{|C| - m - 1, 0\}$), and complete $\hat{\pi}$ into a k-partition π of N s.t. $\forall C \in \pi, |C| \geq m+1$.

An outer loop may run this best-response dynamics several times.

This algorithm provides a run-time absolute bound for the approximation error. Let lb be the maximum value $W_{-m}(\pi)$ found so far for any k-partition π, that is the value of the currently best known solution. Let ub be the minimum value $W(\hat{\pi})$ found so far. Then the solution returned by the algorithm is within an additive $ub - lb$ of the optimum. Denoting $\mathrm{OPT} = \max_\pi \{W_{-m}(\pi)\}$, it means:

$$\mathrm{OPT} - W_{-m}(\pi) \leq ub - lb.$$

Consider the example in Figure 1. Assume $k = 2$ and $m = 1$. Due to negative links, $\{(1,2,5),(3,4,6)\}$ and $\{(1,4,5),(2,3,6)\}$ are the only meaningful 2-partitions. Also, removing 1 or 3 is always better than removing other nodes. For these meaningful actions of defender/attacker, a payoff matrix is given as the table in Figure 1 ($M = \emptyset$ means no attack). Assume $\pi = \{(1,2,5),(3,4,6)\}$ is chosen at first. Then, the best response of the attacker is $M = \{3\}$. Then, lb is updated to 13. Then, the defender chooses $\hat{\pi}_{-M} = \{(1,4,5),(2,6)\}$, which is an optimal partition of $\{1,2,4,5,6\}$. Then, $W(\hat{\pi}_{-M}) = 20$ is used to update ub, i.e., as long as the attacker chooses $\{3\}$, the value of the defender is at most 20. Next, the defender chooses $\{(1,4,5),(2,3,6)\}$, which subsumes $W(\hat{\pi}_{-M})$. The best response of the attacker is $M = \{1\}$ and lb is updated to 18. The defender



	attacker		
defender	$\emptyset$	$\{1\}$	$\{3\}$
$\{(1,2,5),(3,4,6)\}$	30	17	13
$\{(1,4,5),(2,3,6)\}$	38	18	20

Figure 1: Example ($n = 6$, $k = 2$, and $m = 1$) and payoff matrix

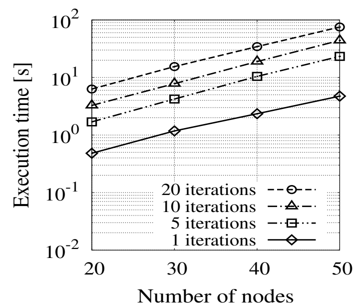


Figure 2: Evaluation results (real life network)

chooses $\hat{\pi}_{-M} = \{(4,5),(2,3,6)\}$, which is an optimal partition of $\{2,3,4,5,6\}$. Then, $W(\hat{\pi}_{-M}) = 18$ is used to update ub. Now, $lb = ub$ holds and IBR terminates.

4 EXPERIMENTAL EVALUATION

We experimentally evaluate the performance of IBR. All the tests were run on a machine: an Intel Xeon E5-2680v4 CPU @ 2.40GHz processor with 125.8GB RAM, Ubuntu 16.40 LTS, and a mixed integer programming package Gurobi version 7.5.0. We show experiments based on a real life network called Wikipedia Requests for Adminship (RfA) network [30]. This is a network among Wikipedia users where each link (i,j) has a weight corresponding to the vote of user i towards user j to become an administrator. The weight of a link is given based on the intensity of the sentiment expressed in the vote [17]. The original graph is directed. For a pair of nodes i and j, we create an undirected link with weight $w(i,j) + w(j,i)$. The original graph has about 10,000 nodes and 100,000 links. Based on this original graph, we select a subgraph with n nodes by randomly choosing a root node, then by adding neighboring nodes in a breadth-first manner. In an obtained graph, the probability that a link exists is about 20% (about 90% of them have positive weights). We set the number of removed nodes m to $n/10$.

Figure 2 shows the computation time of IBR by varying n with $k = 3$. We show the results by varying the number of iterations for *outer-loop*. Each data point is an average of 100 problem instances. We can see that IBR can solve problem instances with $n = 50$ within 5 seconds when the number of iterations is 1.

Acknowledgements. This work was partially supported by the Ministry of Science and Technology, Israel, the Japan Science and Technology Agency (JST) Strategic International Collaborative Research Program, and JSPS KAKENHI Grant Number JP17H00761.

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