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Optimal collective dichotomous choice under quota constraints*

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Summary. This paper presents optimal collective dichotomous choices under quota constraints. We focus on committees that have to decide whether to accept or reject a set of projects under quota constraints. We provide a method for optimal ranking of projects which is suitable for solving this problem. The main result generalizes a number of earlier results in the subject. To outline the applicability of our method, we demonstrate its usage in the area of information filtering.

Keywords and Phrases: Decision-making, Committees, Optimal ranking, Quota.

JEL Classification Numbers: D70, D81.

1 Introduction

This paper focuses on committees that face dichotomous choices, such as accepting or rejecting investment projects. Optimal group decision making in a fixed size committee that faces uncertain dichotomous choices, has been extensively studied in [8], [9], [12], [13], [14], [16], [17], [18], [19], [20], [21], [22], and [15], as well as many others. Recently, a general dichotomous choice framework and the derived optimal decision rule was presented in [3]. However, these papers did not study the issue of decision making under constraints, which is more relevant in economic settings.

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The purpose of the current paper is to present optimal collective dichotomous choices under quota constraints in the general dichotomous choice framework. Choice under quota constraints is a problem which is often encountered in economic settings, but can be found in other areas as well. The quota constraint is when a committee can accept only a subset of projects which are being considered. For example, consider an investment firm which has to decide on a set of projects which is offered. Due to budget or capacity constraints only a subset of these projects can be implemented. The task of the shareholders, which serve as the committee, is to find under the constraints, the subset of projects which gives the maximal net utility to the firm. As a second example consider a university that due to capacity constraint can accept only a limited number of students. We suggest a method for an optimal ranking of projects, which is suitable in solving problems of this kind. This method generalizes the earlier result of [3].

In addition, we expand the application of the collective dichotomous model by developing an approach to information filtering problems.

The outline includes the general dichotomous choice model introduced in the next section. In Section 3 we present the optimal collective decision making process with quota contraints of projects to be approved. In Section 4 we present special cases and related literature. Appllication of the model to information filtering is presented in Section 5. The last section contains a brief summary.

2 The model

There are N projects (tasks), $\mathcal{N} = \{t_1, t_2, \ldots, t_N\}$, and the committee must decide for every project whether to accept or reject it. There are n members in a committee, whose task is to accept or reject projects (tasks). The decision makers share a common objective – maximizing the committee's expected profit. The collective decision is based on the decisions of the individual members. There are two types of projects, good ones (1) and bad ones (-1). Let s_i denote the state of a project t_i. $s_i = 1$ and $s_i = -1$ are referred to as the two possible states of nature with respect to project t_i, indicating that t_i is a good or bad project, respectively. Therefore, for each project there are two possibilities for making a correct collective decision, namely: (1/1) – accept a good project, and (-1/-1) – reject a bad project. The two possibilities of making an incorrect collective decision are (1/-1), (-1/1).[1]

The profit associated with the acceptance (1) of a good project t_i is denoted by $B_i(1 : 1)$. The profit assoicated with the rejection (-1) of a good project t_i is denoted by $B_i(-1 : 1)$, where $B_i(1 : 1) > B_i(-1 : 1)$. Similarly, the profits of the two remaining possibilities for project t_i are denoted by $B_i(-1 : -1)$ and $B_i(1 : -1)$ where $B_i(-1 : -1) > B_i(1 : -1)$.[2] Note that these profits can differ for

[1] Note that the outcome for project t_i is a pair in which the right-hand term denotes the state of nature (s_i) and the left-hand term denotes the collective decision.

[2] Note that one can make a reasonable assumption that the profit from rejecting a project is zero. Thus, profits are obtained only from projects that have been accepted. Formally, $B(-1 : 1) = B(-1 : -1) = 0$. This clearly is a special case of the problem solved in this paper.

different projects. Let $B_i(1) = B_i(1:1) - B_i(-1:1)$ be the positive net profit when $s_i = 1$. $B_i(-1) = B_i(-1:-1) - B_i(1:-1)$ is the positive net profit when $s_i = -1$. Let α_i be the a priori probability that project t_i, which the committee faces, is a good one $0 < \alpha_i < 1$. Again, this probability can differ for different projects. Let us denote by x_j^i, $x_j^i \in \{1, -1\}$ the decision of committee member j concerning project t_i. $x_j^i = 1$ and $x_j^i = -1$ denote, respectively, acceptance and rejection of project t_i by committee member j. The vector $\overline{x}^i = (x_1^i, \ldots, x_n^i)$ is referred to as the decision profile of the committee members for project t_i. The decisional skill of member j concerning project t_i is characterized by the probabilities p_{ji}^1 and p_{ji}^2 that he accepts a good project and rejects a bad project, respectively. That is, $p_{ji}^1 = Pr(1:1)_{ji}$, and $p_{ji}^2 = Pr(-1:-1)_{ji}$. The probabilities $(1 - p_{ji}^1)$ and $(1 - p_{ji}^2)$ can be interpreted as Type I and Type II errors entailed in individual j's decision regarding project t_i. We assume that each individual has some, but not perfect, decisional skills, $0 < p_{ji}^1 < 1$, $0 < p_{ji}^2 < 1$, $p_{ji}^1 > (1 - p_{ji}^2)$, and that decisional skills are statistically independent across individuals.[3] Note that the assumption that the decisions across individuals are independent is plausible and rational, because the paper discusses the optimal collective rule (process) and as shown in [1] and in [2], independent decisions are rational behavior (i.e., constitute a Nash equilibrium) when the optimal collective rule is used.[4]

The committee's decision regarding project t_i is a function of the profile of decisions $\overline{x}^i$ that is associated with this project. Optimization of the committee's collective decision making process with respect to each project in particular by selection an aggregation rule (a function that assigns 1 or -1, acceptance or rejection of a project t_i, to any profile of decisions $\overline{x}^i$ in $\Omega = \{1, -1\}^n$) that maximizes expected payoffs has been studied in [3].[5] We focus on the optimizaton of the committee's collective decision making process when there are constraints on the number of projects that can be accepted.

3 The optimal collective decision making process under quota constraints

The committee has to decide on the approval of a subset of K projects from the set of N projects due to budget or capacity constraint, such that the expected profit is maximized. Choice under quota constraints is relevant with respect to wide domain of economic problems and classification problems. For example,

[3] $p_{ji}^1 > (1 - p_{ji}^2)$ means that for any project t_i a committee member j is more likely to accept a good project than a bad project, i.e., that the simple average of his decisional skills in the two states of nature exceeds 1/2.

[4] Recently, several authors have examined the choice model under rational behavior, relaxing the assumption that decision makers vote non-stategically, for example, [5], [6], [7], [10], [11] and [24]. But as mentioned above, when dealing with the optimal collective rule, independent decisions are rational behavior. Although this is not a unique equilibrium (for example, if all decision makers decide 1 regardless of the information they have, such a strategy profile is also an equilibrium).

[5] Note that the present discussion pertains to the most general dichotomous model [3]. A similar discussion could have been developed for the reduced models that have been presented in the past such as the assumption of a symmetric model ([12], [22]), or the assumption of homogeneous individuals ([16], [17], [20], [21]).

optimally deciding on a subset of projects being considered by an investment firm that has a budget constraint or accepting a limited numbers of students to a college. The committee's decision regarding a project is made by means of an aggregative decision rule. An aggregative decision rule is a function f that assigns, under the constraint, 1 to projects which should be approved and -1 to projects which should be rejected. The rule f is a function of any combination of N decision profiles (note that each profile relates to a separate project).[6]

To formaly define the function f, we need to present for each project the conditional probabilities of collectively accepting a good project or rejecting a bad one. For each project t_i, we divide the set of all combinations of the decision profiles into $X_i(1/f)$ and $X_i(-1/f)$, for which the decision rule assigns 1 and -1, respectively, to project t_i. A combination of decision profiles is given by $(\overline{x}^1, \overline{x}^2, \overline{x}^3, \ldots, \overline{x}^N)$, where $\overline{x}^i = (x_1^i, x_2^i, \ldots, x_n^i)$, such that

$$X_i(1/f) = \{(\overline{x}^1, \overline{x}^2, \ldots, \overline{x}^N) | f_i(\overline{x}^1, \overline{x}^2, \ldots, \overline{x}^N) = 1\}$$

and

$$X_i(-1/f) = \{(\overline{x}^1, \overline{x}^2, \ldots, \overline{x}^N) | (f_i(\overline{x}^1, \overline{x}^2, \ldots, \overline{x}^N) = -1\},$$

where f_i is the collective decision for project t_i.

We present the correct collective probability from the decision maker's decisions for project t_i by $\Pi_i(f:1)$ and $\Pi_i(f:-1)$, in the two states of nature, $s_i = 1$ and $s_i = -1$,

$$\Pi_i(f:1) = P_r\{(\overline{x}^1, \overline{x}^2, \ldots, \overline{x}^N) \in X_i(1/f) | s_i = 1\}$$

$$\Pi_i(f:-1) = P_r\{(\overline{x}^1, \overline{x}^2, \ldots, \overline{x}^N) \in X_i(-1/f) | s_i = -1\}$$

and

$$1 - \Pi_i(f:1) = P_r\{(\overline{x}^1, \overline{x}^2, \ldots, \overline{x}^N) \in X_i(-1/f) | s_i = 1\}$$

$$1 - \Pi_i(f:-1) = P_r\{(\overline{x}^1, \overline{x}^2, \ldots, \overline{x}^N) \in X_i(1/f) | s_i = -1\}.$$

Below we use the following non-standard notation to reduce the number of indices.

Notation For any expression M, such that $M_i = M(d_i, y_i, z_i, \ldots)$, M_i is denoted by $\{M(d, y, z, \ldots)\}_i$.

The problem on which we focus is the maximization of the expected profit from N projects over the set, F, where F is the set of aggregative decision rules which take into account the quota constraint. That is,

$$Max_{f \in F} \sum_{i=1}^{N} E_i,$$

where E_i is the expected profit from project t_i, i.e.,

[6] We cannot assign 1 or -1 to an individual decision profile of a project, because the collective decision depends upon the other profiles of the other projects. Thus, we need to rely on the combination of all N profiles.

$$E_i = \{\alpha[B(1:1)\Pi(f:1) + B(-1:1)(1 - \Pi(f:1))] + (1-\alpha)[B(-1:-1)\Pi(f:-1) + B(1:-1)(1-\Pi(f:-1))]\},$$

or

$$E_i = \{\alpha B(1)\Pi(f:1) + (1-\alpha)B(-1)\Pi(f:-1) + (\alpha B(-1:1) + (1-\alpha)B(1:-1))\}_i.$$

Denote by $g_i(\overline{x}/1)$ and $g_i(\overline{x}/-1)$ the probabilities to obtain the decision profile $\overline{x}^i$, given $s_i = 1$ and $s_i = -1$, respectively, for project t_i. That is,

$$g_i(\overline{x}/1) = \left\{ \prod_{j\in A(x)} p_j^1 \prod_{j\in R(x)} (1-p_j^1) \right\}_i$$

$$g_i(\overline{x}/-1) = \left\{ \prod_{j\in R(x)} p_j^2 \prod_{j\in A(x)} (1-p_j^2) \right\}_i,$$

where $A_i(x)$ and $R_i(x)$ represent the partition of the group members, such that $j \in A_i(x)$ if $x_j^i = 1$ and $j \in R_i(x)$ if $x_j^i = -1$.

For a given decision rule f,

$$\Pi_i(f:1) = \sum_{(\overline{x}^1,\overline{x}^2,\ldots,\overline{x}^N)\in X_i(1/f)} g_i(\overline{x}/1)\cdot \prod_{j=1,j\neq i}^{N} \{\alpha g(\overline{x}/1) + (1-\alpha)g(\overline{x}/-1)\}_j$$

$$\Pi_i(f:-1) = \sum_{(\overline{x}^1,\overline{x}^2,\ldots,\overline{x}^N)\in X_i(-1/f)} g_i(\overline{x}/-1)\cdot \prod_{j=1,j\neq i}^{N} \{\alpha g(\overline{x}/1) + (1-\alpha)g(\overline{x}/-1)\}_j.$$

In writing $\Pi_i(f:1)$, we assume that the state of nature is 1 only for project t_i. We do not know the state of nature of the other projects, so we consider the two states of nature.

Similarly, for $\Pi_i(f:-1)$, the state of nature is assumed to be -1 only for project t_i, while for the other projects, both states of nature are considered.

Notice that the solution of the maximization problem on which we focus is also the solution of the following problem:

$$Max_f \sum_{i=1}^{N} \{\alpha B(1)\Pi(f:1) + (1-\alpha)B(-1)\Pi(f:-1) + C\}_i$$

where C is a constant, or

$$Max_f \sum_{i=1}^{N}(\alpha_i B_i(1) \sum_{(\overline{x}^1,\overline{x}^2,\ldots,\overline{x}^N)\in X_i(1/f)} g_i(\overline{x}/1) \cdot \prod_{j=1,j\neq i}^{N} \{\alpha g(\overline{x}/1) + (1-\alpha)g(\overline{x}/-1)\}_j + (1-\alpha)_i B_i(-1) \sum_{(\overline{x}^1,\overline{x}^2,\ldots,\overline{x}^N)\in X_i(-1/f)} g_i(\overline{x}/-1) \cdot \prod_{j=1,j\neq i}^{N} \{\alpha g(\overline{x}/1) + (1-\alpha)g(\overline{x}/-1)\}_j + C_i).$$

To simplify, instead of selecting an aggregative decision rule f, we can optimally rank the projects for any combination of decision profiles. We propose assigning 1 to the K best projects, i.e., the K projects with the highest optimal ranking.

Theorem 1 *The optimal rule sets* $f_i = 1$ *for the* K *projects which give the* K *highest* ΔE_i^* *where*[7]

$$\Delta E_i^* = \{\frac{\alpha B(1)g(\overline{x}/1) - (1-\alpha)B(-1)g(\overline{x}/-1)}{\alpha g(\overline{x}/1) + (1-\alpha)g(\overline{x}/-1)}\}_i.$$

Proof As mentioned earlier, let us focus on the solution to the problem:

$$Max_f \sum_{i=1}^{N}(\alpha_i B_i(1) \sum_{(\overline{x}^1,\overline{x}^2,\ldots,\overline{x}^N)\in X_i(1/f)} g_i(\overline{x}/1) \cdot \prod_{j=1,j\neq i}^{N} \{\alpha g(\overline{x}/1) + (1-\alpha)g(\overline{x}/-1)\}_j + (1-\alpha)_i B_i(-1) \sum_{(\overline{x}^1,\overline{x}^2,\ldots,\overline{x}^N)\in X_i(-1/f)} g_i(\overline{x}/-1) \cdot \prod_{j=1,j\neq i}^{N} \{\alpha g(\overline{x}/1) + (1-\alpha)g(\overline{x}/-1)\}_j + C_i).$$

For a given combination of decision profiles, the expected opportunity profit, i.e., the profit from accepting project t_i rather than rejecting it, is:

$$\Delta E_i = \{\alpha B(1)g(\overline{x}/1)\}_i \cdot \prod_{j=1,j\neq i}^{N} \{\alpha g(\overline{x}/1) + (1-\alpha)g(\overline{x}/-1)\}_j - \{(1-\alpha)B(-1)g(\overline{x}/-1)\}_i \prod_{j=1,j\neq i}^{N} \{\alpha g(\overline{x}/1) + (1-\alpha)g(\overline{x}/-1)\}_j.$$

[7] However, in cases where only projects with $\Delta E_i^* > 0$ can be implemented, e.g., the investment firm, the optimal rule sets $f_i = 1$ only for a subset from the K best projects which satisfy $\Delta E_i^* > 0$.

The partition of the combinations into $X_i(1/f)$ and $X_i(-1/f)$ for each t_i is determined by the control variable, namely by rule f. For any combination of decision profiles, assign 1 to the K projects with the highest ΔE_i, thus ensuring that f maximizes the expected profit from the N projects.

We can write ΔE_i in the following way:

$$\Delta E_i = \prod_{j=1, j\neq i}^{N} \{\alpha g(\overline{x}/1) + (1-\alpha) g(\overline{x}/-1)\}_j \cdot \{\alpha B(1) g(\overline{x}/1) - (1-\alpha) B(-1) g(\overline{x}/-1)\}_i$$

or

$$\Delta E_i = \prod_{j=1}^{N} \{\alpha g(\overline{x}/1) + (1-\alpha) g(\overline{x}/-1)\}_j \cdot \left\{ \frac{\alpha B(1) g(\overline{x}/1) - (1-\alpha) B(-1) g(\overline{x}/-1)}{\alpha g(\overline{x}/1) + (1-\alpha) g(\overline{x}/-1)} \right\}_i .$$

The first part of this ΔE_i formula, i.e., $\prod_{j=1}^{N}\{\alpha g(\overline{x}/1) + (1-\alpha) g(\overline{x}/-1)\}$ is the same for all the projects. Consequently, the ranking between the projects can be determined only by the last part of the formula, and the optimal ranking is therefore determined by

$$\Delta E_i^* = \left\{ \frac{\alpha B(1) g(\overline{x}/1) - (1-\alpha) B(-1) g(\overline{x}/-1)}{\alpha g(\overline{x}/1) + (1-\alpha) g(\overline{x}/-1)} \right\}_i .$$

□

Another way to present the proof can be found in the Appendix.

The intuition behind the theorem is that optimal ranking should be based on the comparison of the opportunity profit of the various projects. The opportunity profit, i.e., the profit from accepting a project rather than rejecting it, measures the relative desirability of different projects, where the project with the highest opportunity profit is the best one.

4 Special cases and related literature

To further interpret our main result, we now relate to the relevant literature and demonstrate its usage by presenting special cases.

Convergence of the decision rule in the absence of constraints

When $N = 1$ or when $N \geq 1$ and K is unrestricted, our problem is reduced to the special setting analyzed by past literature. Namely, the problem is equivalent to that of choosing the optimal rule for one project where there is no constraint.

As in [3], our problem is reduced to:

$$Max_f E = \alpha B(1)\Pi(f:1) + (1-\alpha)B(-1)\Pi(f:-1) + C$$

where

$$\Pi(f:1) = \sum_{\overline{x}\in X(1/f)} g(\overline{x}/1), \quad \Pi(f:-1) = \sum_{\overline{x}\in X(-1/f)} g(\overline{x}/-1),$$

where $X(1/f)$ and $X(-1/f)$ are the sets of all decision profiles to which f assigns 1 and -1, respectively. $g(\overline{x}/1)$ and $g(\overline{x}/-1)$ are the probablities for obtaining this profile in the two states of nature.

For every profile of decisions, the optimal rule $\hat{f}$ assigns 1 if the opportunity profit is positive, so that

$$\alpha B(1)g(\overline{x}/1) > (1-\alpha)B(-1)g(\overline{x}/-1)$$

or

$$\hat{f} = \ln \frac{\alpha B(1)g(\overline{x}/1)}{(1-\alpha)B(-1)g(\overline{x}/-1)} > 0.$$

This is equivalent to $\Delta E^* > 0$.

Note that $\hat{f}$ turns out to be a weighted qualified majority rule:

$$\hat{f} = \sum_{j=1}^{n} W_j^* x_j + \psi + \gamma + \delta$$

where

$$W_j^* = \frac{\beta_j^1 + \beta_j^2}{2}, \ \beta_j^1 = \ln \frac{p_j^1}{1-p_j^1}, \ \beta_j^2 = \ln \frac{p_j^2}{1-p_j^2}, \ \psi = \sum_{j=1}^{n} \frac{1}{2} \ln \frac{p_j^1(1-p_j^1)}{p_j^2(1-p_j^2)},$$

$$\gamma = \ln \frac{\alpha}{1-\alpha}, \ \delta = \ln \frac{B(1)}{B(-1)}.$$

For further interpretation of this optimal rule, see [3].

ΔE_i^* can be also interpreted as the difference between the expected net profit when the state of nature is 1 and -1, for a given decision profile. This interpretation is formally represented by:

$$\Delta E_i^* = \left\{ B(1)\frac{\alpha g(\overline{x}/1)}{\alpha g(\overline{x}/1) + (1-\alpha)g(\overline{x}/-1)} - B(-1)\frac{(1-\alpha)g(\overline{x}/-1)}{\alpha g(\overline{x}/1) + (1-\alpha)g(\overline{x}/-1)} \right\}_i .$$

Notice that the ratio between the two terms above is equivalent to $\frac{\alpha B(1)g(\overline{x}/1)}{(1-\alpha)B(-1)g(\overline{x}/-1)}$, which gives a different ranking. This explains why the optimal ranking is generally not determined by $\hat{f}$. This rules out a possible hypothesis that the solution to our problem is identical to the solution of the unconstrained problem.

Homogeneous projects with identical committees

Consider the case in which all the projects are characterized by identical a priori probabilities, profits and the decision skills of the members. That is, $\forall_{t_i,t_m} : \alpha_i = \alpha_m$, $B_i(1) = B_m(1)$, $B_i(-1) = B_m(-1)$, and $\forall_j\ p_{ji}^1 = p_{jm}^1$, $p_{ji}^2 = p_{jm}^2$. (In this case, let p_j^1 and p_j^2 be the decision skills of the members.) The optimal ranking is based on weighted decisions, that is, $\sum_{j=1}^{n} W_j^* x_j^i$, where W_j^* is defined above.

The explanation of this formula is as follows:

$$\forall_{t_i,t_m},\ i \neq m \quad : \quad \Delta E_i^* > \Delta E_m^* \Leftrightarrow \frac{g_i(\overline{x}/1)}{g_i(\overline{x}/-1)} > \frac{g_m(\overline{x}/1)}{g_m(\overline{x}/-1)} \Leftrightarrow$$
$$\ln \frac{g_i(\overline{x}/1)}{g_i(\overline{x}/-1)} > \ln \frac{g_m(\overline{x}/1)}{g_m(\overline{x}/-1)} \Leftrightarrow \sum_{j=1}^{n} W_j^* x_j^i > \sum_{j=1}^{n} W_j^* x_j^m.$$

Note that this ranking is the same as the one obtained by $\hat{f}$ because δ, γ and ψ are equivalent for all the projects.

Furthermore, in this case, when the committee members are homogeneous in their decision skills, that is, $\forall_j\ p_j^1 = p^1$, and $p_j^2 = p^2$, and thus $\forall_j\ W_j^* = W^*$, the optimal ranking is determined as would be expected, by the number of members accepting the project, that is, $\sum_{j=1}^{n} x_j^i$.

Homogeneous projects with different committees

When the projects are characterized by the same a priori probability and the same profit, but the decision skills of the members depend upon the projects, the optimal ranking is based on $\sum_{j=1}^{n} W_{ji}^* x_j^i + \psi_i$.[8] Note that this ranking is the same as the one obtained by $\hat{f}$ where γ and δ are the same for all the projects.

The more general case in which ΔE_i^* can give the same ranking as $\hat{f}$ is when the projects are characterized by different parameters except for the profit $B(\cdot : \cdot)$ which is the same for all the projects. (Note that δ is the same for all the projects).

[8] W_{ji}^* and ψ_i for project i as previously defined:

$$W_{ji}^* = \frac{\ln \frac{p_{ji}^1}{1-p_{ji}^1} + \ln \frac{p_{ji}^2}{1-p_{ji}^2}}{2}, \quad \psi_i = \sum_{j=1}^{n} \frac{1}{2} \ln \frac{p_{ji}^1(1-p_{ji}^1)}{p_{ji}^2(1-p_{ji}^2)}.$$

Several notes on the interpretation of the general case

In the general case in which the projects are characterized by different profits, there are three sufficient conditions that ensure a higher ranking of project t_i relative to project t_j :

(i) $B_i(1) > B_j(1)$
(ii) $B_j(-1) > B_i(-1)$

and

(iii) $\frac{\alpha_i}{1-\alpha_i} \frac{g_i(\overline{x}/1)}{g_i(\overline{x}/-1)} > \frac{\alpha_j}{1-\alpha_j} \frac{g_j(\overline{x}/1)}{g_j(\overline{x}/-1)}$.[9]

These conditions arise from the following interpretation of the general case result:

$$\begin{aligned} \Delta E_i^* - \Delta E_j^* > 0 \Leftrightarrow \{ & \alpha_i \alpha_j g_i(\overline{x}/1) g_j(\overline{x}/1)[B_i(1) - B_j(1)] + \\ & (1-\alpha_i)(1-\alpha_j) g_i(\overline{x}/-1) g_j(\overline{x}/-1)[B_j(-1) - B_i(-1)] + \\ & \alpha_i(1-\alpha_j) g_i(\overline{x}/1) g_j(\overline{x}/-1)[B_i(1) + B_j(-1)] - \\ & (1-\alpha_i)\alpha_j g_i(\overline{x}/-1) g_j(\overline{x}/1)[B_i(-1) + B_j(1)]\} > 0. \end{aligned}$$

As mentioned above using $\hat{f}$ will not yield the optimal ranking, since the values of the different net profits in the two states of nature are important.

From the above discussion, it is clear that the resullts obtained in the reduced models mentioned in note 5 above can give the optimal ranking when simplifying assumptions are made concerning the differences between the projects. For example, when the projects have identical parameters the results of the reduced models give the optimal ranking.

5 Application of the model to information filtering

Our "constrained" dichotomous choice model can be applied to alternative classification decisions such as information filtering. In recent years, on-line information has become overwhelming, and an urgent need has arisen for systems which can help to filter or search for relevant information. That is, if for example, a user who is limited in time wishes to search through a large set of documents, the system should automatically recommend documents worth reading. Recommendations can be based on other people's opinions on these documents and the similarity between their interests and the user's interests [23]. Note that the recommendations are based on a group of people and not on one person only, because this improves the recommendations concerning information filtering [4].[10] The problem is then to lay out rules which decide which documents should be

[9] Condition (iii) can give the same ranking as $\hat{f}$ minus δ, i.e., $\sum_{j=1}^{n} W_{ji}^* x_j^i + \psi_i + \gamma_i$.

[10] The statement that majorities are more likely to select the correct alternative than any single individual is attributed to [4].

recommended, based on people's opinions about a particular document and the similarity between these people and the user.

This problem can be viewed as decisions made by a committee. Here the projects are the documents, and accepting a project corresponds to recommending a document. A "good project" is a document which is of interest to the user and should be recommended to him, while a bad project is one which is of no interest to the user and should not be recommended. Each member of the committee has to decide whether a particular document is of interest and therefore should be recommended. A committee member's decision skills are based on the similarity between his interests and the user's interests. That is, as the similarity between the user's interests and those of the committee member increase, the decision skills of the committee member increase. That is, there is a higher probability that a document which is of interst to the committee member will also be of interest to the user. In order to find these decision skills, a preliminary step should be taken, where both the user and the committee members are given a set of documents and are asked to indicate, for each document, whether it is of interest to them or not. This information can be used to calculate the decision skills (the probabilities). Note that the decisions and the decision skills of each member are independent across individuals.

When there is a need to make recommendations with respect to a set of documents with which the user is not familar, the opinions of the committee members are used. That is, each committee member has an opinion whether a given document in the set is of interest (i.e., assigning "1" to the document) or whether it is uninteresting (i.e., assigning "-1" to the document). Note that since the number of users is huge, the decisions of each committee member are given, regardless of the user, and hence the collective information can be used for making recommendations for many users. Using the decision vector of all the committee members with respect to each document, as well as their pre-computed decision skills, recommendation can be given based on the method for optimal group decision making.

In some situations, given a set of N documents, the user may want to obtain a recommendation for only K documents ($K < N$), thus the quota decision rule presented above will be used.

6 Conclusion

This paper presented optimal collective dichotomous choices under quota constraints in the general dichotomous choice framework. We suggested a method for optimal ranking of projects. In addition, we illustrated the application of the collective dichotomous model as an approach to information filtering problems.

There are a number of issues that have not been dealt with here but that would be of interest to pursue. For example, to find the optimal collective dichotomous choices under other constraints, in particular, under partial order constraints, where some projects have to precede others, before the later ones can be approved.

Also, until now the optimal collective decision was a function of the individual decisions in cases where these decisions did not take into consideration the existence of other projects nor the existence of the constraints. It would be interesting to see if and how the optimal collective process and the decision would change as a result of the individuals' decisions when taking these factors into account, and if such a change would be profitable.

Another question discusses the advantages of a method in which each individual ranks each project instead of giving a binary decision. In the example of recommending documents, each individual would need to assign a score to each document. On the basis of these rankings, the decision about which documents to recommend would be made.

Appendix

Our problem can be written in the following way:

$$Max_f \sum_{i=1}^{N} \Bigg[\sum_{(\bar{x}^1,\bar{x}^2,\ldots,\bar{x}^N)\in X_i(1/f)} \left\{ \frac{\alpha B(1) g(\bar{x}/1)}{\alpha g(\bar{x}/1) + (1-\alpha) g(\bar{x}/-1)} \right\}_i \cdot \prod_{j=1}^{N} \{\alpha g(\bar{x}/1) + (1-\alpha) g(\bar{x}/-1)\}_j + \sum_{(\bar{x}^1,\bar{x}^2,\ldots,\bar{x}^N)\in X_i(-1/f)} \left\{ \frac{(1-\alpha) B(-1) g(\bar{x}/-1)}{\alpha g(\bar{x}/1) + (1-\alpha) g(\bar{x}/-1)} \right\}_i \cdot \prod_{j=1}^{N} \{\alpha g(\bar{x}/1) + (1-\alpha) g(\bar{x}/-1)\}_j + C_j \Bigg]$$

or,

$$Max_f \sum_{j\in(\bar{x}^1,\bar{x}^2,\ldots,\bar{x}^N)} \left[\sum_{i\in\mathcal{N}'} a_i + \sum_{i\in\mathcal{N}-\mathcal{N}'} b_i \right] \cdot M_j + \sum_{i=1}^{N} C_i ,$$

where

$$a_i = \left\{ \frac{\alpha B(1) g(\bar{x}/1)}{\alpha g(\bar{x}/1) + (1-\alpha) g(\bar{x}/-1)} \right\}_i$$

$$b_i = \left\{ \frac{(1-\alpha) B(-1) g(\bar{x}/-1)}{\alpha g(\bar{x}/1) + (1-\alpha) g(\bar{x}/-1)} \right\}_i$$

$$M_j = \prod_{j=1}^{N} \{\alpha g(\bar{x}/1) + (1-\alpha) g(\bar{x}/-1)\}_j$$

$$\mathcal{N}' = \{t_i \in \mathcal{N}/f_i = 1\} \quad \mathcal{N}-\mathcal{N}' = \{t_i \in \mathcal{N}/f_i = -1\}.$$

Therefore, the solution of the maximization problem on which we focus is the same solution to the following problem

$$Max_f \sum_{i \in \mathcal{N}'} a_i + \sum_{i \in \mathcal{N} - \mathcal{N}'} b_i$$

for any combination of decision profiles.

Therefore, for a given combination of decision profies, the expected opportunity profit, i.e., the profit from accepting project t_i rather than rejecting it, is $a_i - b_i$ and this is ΔE_i^*.

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