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# **RESOLVING UNCERTAINTY: A UNIFIED OVERVIEW OF RABBINIC METHODS**

#### INTRODUCTION

he Talmud frequently discusses situations in which a legal decision must be made in the face of uncertainty with regard to the facts. Such discussions invoke a number of different principles but at least some of these principles are not given explicit formal definition and the relationships between them are not spelled out. In this paper, I will formally define each of the variety of rabbinic methods used to resolve (or dispel) empirical uncertainty and place them in the context of certain now well-understood probabilistic concepts. In this way, I hope to present a unified overview of rabbinic laws concerning uncertainty.

The modern theory of probability is twice removed from rabbinic laws concerning uncertainty. First, in its current form the theory of probability is simply the study of a particular class of functions and is not concerned with assigning probabilities to real-world events. Second, even if on the basis of certain stipulations the theory is applied to actual events, it remains descriptive and not prescriptive. Nevertheless, certain philosophical issues which have arisen as a result of attempts to explicate the meanings of probabilistic statements are highly relevant to a proper understanding of rabbinic approaches to uncertainty. I will use ideas taken from the study of foundations of probability where these ideas seem helpful but will try to refrain from belaboring the analogy for its own sake.

One historical point needs to be emphasized. The modern theory of probability has its roots in the work of Pascal and others in the seven-teenth century. It would be utterly anachronistic to attribute to *Tanna'im* and *Amora'im* any foreknowledge of these developments. Moreover, doing so does not purchase any explanatory power with regard to rabbinic

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approaches to uncertainty. At the same time, the claim that the ancients were bereft of any systematic thinking with regard to uncertainty is both arrogant and demonstrably false. I will use modern ideas about the foundations of probability as a starting point for identifying which probabilistic insights do and do not lie at the root of rabbinic pronouncements on such matters.

Nevertheless, my approach in this article is unabashedly ahistorical: rather than chart a chronological progression of ideas or identify conflicting schools of thought, I will attempt to harmonize a broad range of sources. Where a Tanna'itic or Amora'ic source permits multiple interpretations, I will not outline all views but rather select the most straightforward or consensual interpretation. Likewise, I will relate to the central ideas discussed in the vast post-talmudic literature devoted to rabbinic laws concerning uncertainty but, for the sake of offering as straightforward and unified a treatment as possible, I will cite opinions of the commentators in an extremely selective manner.<sup>1</sup> The fact that I marshal the support of a particular commentator regarding a particular point should in no way be taken to mean that I can claim such support regarding related points.

In the first part of this article, I will use the distinction between *rubba de-ita kamman* and *rubba de-leta kamman* to motivate a discussion of distinct definitions of probability. This will lay the groundwork for the explication of a number of thorny rabbinic concepts involving uncertainty and indeterminacy.

### INTERPRETATIONS OF PROBABILITY

The Gemara in Hullin 11a-11b interprets the verse "aharei rabbim lehattot" to mean that decisions of a bet din are decided by majority (to be precise, a majority of two is required to convict). This is then generalized to a principle of rubba de-ita kamman (henceforth RDIK; literally: a majority which is in front of us), which includes other cases such as that of "nine stores," in which a piece of meat is found in the street and all that is known is that it comes from one of ten stores, nine of which sell kosher meat. In such cases we apply the principle that kol de-parish merubba parish (henceforth parish; literally: that which is removed, was removed from the majority). The Gemara states that this inference covers only the principle of RDIK, of which the majority rule in the Sanhedrin and "nine stores" are offered as typical examples, but not rubba de-leta kamman (henceforth RDLK; literally: a majority which is

not in front of us). The Gemara offers a number of examples of RDLK where the majority is followed because it would be impossible to perform *mitsvot* or adjudicate cases without doing so (but concludes that precisely because of that impossibility these cases can't serve as a basis from which to infer a general principle of RDLK). Several cases of RDLK that are illustrative are that the husband of one's mother (at the time of conception) may be presumed to be one's father, that a child may be presumed to be potentially fertile and that a murder victim may be presumed not to have been suffering from a prior life-threatening condition.

What is the difference between RDIK and RDLK? Although the names are suggestive, the *Gemara* offers no explicit definition of RDIK and RDLK and no rationale for treating them differently. We might, however, shed considerable light on the distinction by considering an interesting philosophical debate dating back to the 1920's which covers similar conceptual territory. The rest of this section will consist of a slightly lengthy diversion through that territory.

Let's consider carefully what exactly we mean when we say that the probability of some event is p/q. Early (seventeenth and eighteenth century) work in probability was motivated to a large extent by games of chance (coins, cards, dice). Thus when somebody said that "the probability of the event H is p/q," it was understood that what was meant was that the event H obtained in p out of q equally likely possible outcomes. Thus, for example, when we say the probability that the sum of two throws of a die will be exactly six is 5/36, we mean that there are 36 equally likely possible throws and five of them have the desired property. Similarly, in the case of the found meat, there are ten possible sources for the meat and nine of them are kosher, so we might say that the probability that the meat is kosher is 9/10. This understanding of probabilistic statements is usually called the "classical" interpretation.

What is interesting for our purposes is that the classical interpretation turns out to be inadequate as a definition of probability. This became obvious once insurance companies began using probability theory to compute actuarial tables. What does it mean to say that "the probability that a healthy 40-year-old man will live to the age of 70 is p/q"? What are the q equally likely possible outcomes, p of which find our insure celebrating his seventieth birthday? No such thing. This led philosophers such as Reichenbach and von Mises<sup>2</sup> to suggest the "frequentist" interpretation of probability: the statement that "the probability that a healthy 40-year-old man will live to the age of 70 is p/q"

means that of the potentially infinite class of hypothetical healthy 40year-old men, the proportion who will see 70 is p/q.

It is important to understand that according to each of these interpretations, the classical and the frequentist, there is always some subjective aspect in assigning a probability to an event. In the case of classical probability this subjective element is rather benign: we need to define the underlying "equally likely" cases, or what is called in formal parlance the "sample space." For example, in the case of "nine stores" we might just as plausibly use as our sample space the three shopping malls in which the stores are concentrated, or perhaps the 10,000 pieces of meat that are unequally distributed among the stores. The choice of which sample space is most appropriate is ultimately a matter that must simply be stipulated. It is tempting to imagine that the "right" sample space is the one in which the various elements are equally probable. But obviously this formulation is circular since it is the very notion of probability that we are trying to define. To be sure, in many cases there is a rather obvious first choice of sample space. For example, in tossing a die, we would naturally identify the six possible faces as our sample space. This intuition rests on some sort of "indifference principle" (why should one face be more likely than another?). But such indifference principles have proved remarkably resistant to precise formulation. Ultimately, the assignment of sample space is a matter of stipulation.

If in the case of classical probability, assigning a probability to an event requires a bit of judgment, in the case of frequentist probability such an assignment is fraught with judgment. Think of the example in which we wish to determine the probability that a particular child is potentially fertile (actually in the situation described in the Gemara we wish only to determine that this probability is greater than 1/2). We wish to do so by invoking some rule that says: there is some reference class A in which this child is a member and the expected proportion of members of A which are potentially fertile is p/q. This expected frequency is in turn determined by our past experience with members of class A and the frequency of fertility they exhibited. But what class A is appropriate? Should A be the class of all young mammals, or all human children, or perhaps the class of all children who share this child's medical history, or the class of children who share this child's medical history and genetic stock? If we define the class too broadly we run the risk that our experience with the class is irrelevant to the particular child in question. If we define it too narrowly we run the risk that our experience with the class is too limited to provide any reliable information with regard to the

class in general. And if we define it bizarrely (say, the class consisting of this child and all major household appliances), the results are, well, bizarre. The selection of the reference class A, as well as the determination that our experience with samples from that class is sufficient to project some statistical law onto the whole class, are matters of judgment.

Consider now the extreme case of a probabilistic statement such as "the probability that the United States will attack North Korea within two months is 60%." The problem with such statements is that the events in question belong to no natural class since the ensemble of relevant facts renders the case unique. It is implausible that we mean to say that in 60% of cases like this an attack occurs, because there aren't any cases quite "like this." Since according to the frequentist interpretation every probabilistic statement must refer to some class, these statements are utterly meaningless within the frequentist framework and indeed are rejected as such by von Mises and others.

One attempt to salvage such statements as meaningful has involved yet another interpretation of probability, the "subjectivist" interpretation. According to this interpretation, the statement that the probability of some event is p/q is taken to reflect the degree of certainty with which some rational observer is convinced of the correctness of the statement, as might be reflected in a betting strategy. Unlike the previous interpretations, such an interpretation does not require the identification of any relevant class. For example, for someone to say that the probability that the United States will attack North Korea within two months is 60% is simply to say that they regard as fair either side of a bet with 3:2 odds in favor of such an attack occurring.

To summarize, there are at least three different kinds of probabilistic statements: classical, frequentist, and subjective.<sup>3</sup> For each type, any instance of such a statement is meaningful only to the extent that at least one potentially fuzzy factor can be plausibly defined. In the classical case this factor is a sample set, in the frequentist case it is a reference class, and in the subjectivist case it is simply the strength of a hunch.

In the following sections, we will see how various rabbinic methods can be best understood in relation to these different types of probabilistic statements. Moreover, we will see that different ways of resolving the fuzzy aspects of probabilistic statements can neatly account for certain apparent anomalies. In the next section, we will explain differences between the conditions and consequences of RDIK, on the one hand, and those of RDLK, on the other. After that we will clarify when RDIK is applied and when a converse rule (*kavu'a*) is applied and will eluci-

date the difference between *safek* (uncertainty) and indeterminacy. Finally, we will discuss the mechanics of *sefek sefeka* and contrast it with cases of asymmetric *sefekot* where the asymmetry is ineffective (*en safek motsi mi-yedei vaddai*).

## RUBBA DE-ITA KAMMAN AND RUBBA DE-LETA KAMMAN

We will define the principle of RDIK more precisely in the next section but for now it is enough to define it roughly as follows: A random object taken from a set a majority of the members of which have property P, may be presumed to have property P. As so defined, the principle does not require any (but perhaps the most naive) probabilistic notions. Nevertheless, it is evident that the classical interpretation is fully adequate for a probabilistic formulation of RDIK: RDIK amounts to specifying the members of the set as a sample space and following the result with probability greater than 1/2. Note that RDIK refers specifically to a set of q concrete objects, p of which have some property, while the classical definition of probability refers more generally to q possible outcomes (which may be abstract).

The classical interpretation is, however, clearly irrelevant to the examples of RDLK we have seen. The frequentist interpretation, on the other hand, squares with RDLK perfectly.<sup>4</sup> Simply put, all examples of RDLK are statistical laws: most children born to married women are fathered by their husbands, most children are ultimately fertile, most people are not about to die, etc.

The identification of RDIK with the classical interpretation and RDLK with the frequentist interpretation will help us clear up a number of difficulties.<sup>5</sup> Let us begin with the question of which is stronger, RDLK or RDIK. *Aharonim* have marshaled proofs for each possibility, the most salient of which follow.

The strength of RDLK relative to RDIK can be clearly seen in the following: It is well established that we don't convict in capital cases based on mere likelihood (*Sanhedrin* 38a). Thus, consider the case of an abandoned baby boy, called an *asufi*, whose mother is one of a given large set of women one of whom is a non-Jew. In this case, there is a RDIK in favor of the child's Jewish maternity. While such a child may be regarded as a Jew for certain purposes, a woman who eventually marries him cannot be convicted of adultery, "*she-en horgin al ha-safek*" (Rambam, *Issurei Bi'a* 15:27; see also *Makhshirin* 2:7, *Ketubot* 15a).

Nevertheless, consider another case of uncertain maternity, in which a woman has a relationship with a child that is typical of that of mother and son but, as is generally the case, there are no witnesses to the birth. In this case, there is a RDLK in favor of the woman's maternity. If she and the "son" are witnessed having sexual relations, they can be convicted for incest "she-sokelin ve-sorefin al ha-hazaka" (Kiddushin 80a, Rambam, Issurei Bi'a 1:20). Clearly, RDLK in these cases is stronger than RDIK.

There are other cases, however, in which RDIK appears to be stronger than RDLK. For example, R. Meir holds that haishinan le*mi'uta*—a rov does not trump a contrary hazaka de-me-ikara unless it is an overwhelming rov (Yevamot 119b). Thus, for example, dough of *teruma* that was last known to be *tahor* but was found in the proximity of a child who is tamei cannot be burned, according to R. Meir, on the basis of a RDLK that children typically pick at dough in their vicinity (Kiddushin 80a). Tosafot (Yevamot 67b, s.v. en hosheshin; Yevamot 119a, s.v. kegon; see also Mordekhai on Hullin, ha-Zero'a 737) argue that this principle holds only with regard to RDLK but RDIK always trumps a hazaka de-me-ikara. Moreover, according to R. Yohanan, in the case of the teruma dough even the rabbis who disagree with R. Meir would concede that the teruma can't be burned on the basis of this RDLK. Nevertheless, they would not so concede in a case of RDIK (see Kiddushin 80a, Rashi s.v. im rov). Thus, in these cases RDLK is weaker than RDIK.6

We might be able to reach a definitive answer regarding which is stronger, RDIK or RDLK, by explaining away one or the other set of proofs. But to do so would be to answer the wrong question. To understand the crucial difference between RDIK and RDLK, let's recall the difference between the classical interpretation of probability and the frequentist interpretation.

In the case of classical probability, the part that is left to judgment is rather limited. Typically, a rather straightforward sample space is taken for granted. Once that's taken care of, assigning a probability is a simple matter of calculation. (In fact, in the limited case of RDIK, the cases need only be counted.) In the case of frequentist probability, however, selecting a reference class and then estimating frequencies within the class requires a substantial investment of judgment. With what confidence can we assert that for some class A the event in question occurs with some sufficiently high frequency? Answering this question, even loosely, is inevitably a matter of judgment.

The crucial difference, then, between RDIK and RDLK is that, while RDIK is a counting principle that can be applied on an ad hoc basis, every RDLK is a general statistical law that can only be applied if it has received rabbinic sanction. Since RDLK is always a product of rabbinic judgment, it stands to reason that this judgment is exercised variably. The apparent contradiction regarding the relative strengths of RDIK and RDLK simply reflects the fact that different applications of RDLK are assigned different strengths (both in terms of the strengths of the laws themselves and in terms of the strength of the evidence for the laws). On the other hand, applications of RDIK, which do not—so to speak—pass through rabbinic hands, are all treated in a uniform manner.

Consequently, if you've seen one RDIK you've seen them all. Unless there is some countervailing principle which prevents its application, RDIK is a decision procedure which resolves, but does not dispel, uncertainty in favor of the majority regardless of whether p/q is .99 or .51. That is, in applying the principle of RDIK we acknowledge that there is uncertainty but the RDIK allows us to decide in favor of the majority much in the way that a majority vote settles a case in court. Invoking RDIK is not sufficient, however, to achieve the degree of certainty necessary to establish the facts of a capital case.

Unlike RDIK, however, there are various types of RDLK. There are three types of decision rules and, depending on rabbinic judgment, RDLK can be any one of them.

—The middle type is the one we have seen in the case of RDIK—a resolution procedure. These are often referred to as *"hakhra'a."*<sup>7</sup> An example of an RDLK of this type is that most births are not of healthy males (*Hullin* 77b).

—There are stronger decision rules which simply render irrelevant the minority possibility—some examples of RDLK are treated as certainties in the sense that we proceed as if the uncertainty has not simply been resolved but rather has been dispelled altogether (or as some would have it, there is no *ledat ha-safek*). These are often referred to as "*berur*." It is about these that we say "*sokelin al ha-hazakot*"—in capital cases certainty is required and these examples of RDLK, unlike any example of RDIK, do indeed provide certainty for legal purposes.<sup>8</sup>

—Finally, there are weaker decision rules which are merely "defaults" in the sense that they are applied only as last-resort tie-

breakers when no more substantive decision rule is available. These are often referred to as "*hanhaga*." The typical example of a default rule in halakha is *hazaka de-me-ikara* (with the possible exception of *hezkat mamon*—possession—which is regarded as a substantive desideratum and not a mere default).<sup>9</sup> In some cases, RDLK is established merely as a default rule so that at most it can neutralize,<sup>10</sup> but not defeat, another default rule such as *hazaka de-me-ikara*. For R. Meir, most cases of RDLK are of this variety.

## RUBBA DE-ITA KAMMAN AND KAVU'A

Let us now return to the principle of RDIK and attempt to define it more precisely. We have already seen that according to the *Gemara* (*Hullin* 11a), this principle covers both the case of majority vote in the Sanhedrin and that of "nine stores" where the meat is found on the street. Moreover, the *Gemara* often invokes the related, though clearly not identical, principle of *bittul be-rov*: a mixture of permitted and forbidden objects may sometimes be assigned the status of the majority. Although the *Gemara* does not specify the source of this principle, most commentators follow the opinion of Rashi (*Gittin* 54b, s.v. *lo ya'alu*) that it is derived from "*aharei rabbim le-hattot*" as well.

The generalization from the case of majority vote to cases such as "nine stores" is not inevitable—the case of voting involves legal, not empirical uncertainty. As R. Elchonon Wasserman (*Kunteres Divrei Soferim* 5:7) puts it: if Eliyahu ha-Navi declared the questionable piece of meat to have come from the minority we could take his word for it, but if he ruled in accord with the minority position in the Sanhedrin we would ignore him (as in the case of *tanuro shel akhnai* in *Bava Metsia* 59b). Similarly, it has been argued (*Sha'arei Yosher* 3:4) that the extension to *bittul be-rov* does not seem, on the face of it, to be one that can be glibly asserted. Clearly, the principle the *Gemara* wishes to base on *aharei rabbim le-hattot* is sufficiently general that it covers all of the above cases.

Before we consider what this principle might be, let's consider the remarkably similar situation with regard to another decision principle, namely, *kol kavu'a ke-mehetsa al mehetsa dami* (henceforth *kavu'a*; literally: that which is fixed is as half and half). Like RDIK, the case identified in the *Gemara* as the "source" case of *kavu'a* is a technical, legal question. Someone throws a stone into an assembly of nine Israelites and one

Canaanite, intending to kill whichever person the stone happens to hit. The question is whether this unspecific intention is sufficient intention to kill an Israelite to warrant conviction for murder. The Rabbis apply the principle of *kavu'a* to determine that the Israelite majority does not render the intention sufficient (*Ketubot* 15a). What exactly the principle might be requires explanation. But note that in this case there is no doubt that the actual victim was indeed an Israelite and not a Canaanite. The issue under discussion is only whether the intention to kill an Israelite. Thus, there is no uncertainty regarding any of the facts of this case and no decision-method for resolving empirical uncertainty is called for.

The *Gemara* then cites as the classic example of *kavu'a*, the parallel case to that of "nine stores" that we considered above: "If there are nine stores which sell kosher meat and one which sells non-kosher meat and someone took [meat] from one of them but he doesn't know from which one he took, the meat is forbidden."

The parallelism between RDIK and *kavu'a* is remarkable. In both, the "source" case involves court procedures and includes no elements of actual uncertainty, and in both the standard case is a version of "nine stores," in which the central issue is apparently one of uncertainty. This suggests that RDIK and *kavu'a* do not directly concern uncertainty, but rather are dual principles regarding mixed sets that cover cases of uncertainty as a by-product.

The principle of RDIK might thus be formulated this way:

Given a set of objects the majority of which have the property P and the rest of which have the property not-P, we may, under certain circumstances, regard the set itself and/or any object in the set as having property P.

The principle of *kavu'a* is the opposite of this:

Given a set of objects some of which have the property P and the rest of which have the property not-P, we may, under certain circumstances, regard the set itself, and consequently any object in the set, as being neither P nor not-P but rather a third status. We can call this status hybrid, or perhaps, indeterminate.

It is important to note that RDIK comes in two varieties: RDIK can assign a single status to the entire mixed set (as in the case of *bittul*) or it might assign a status directly to an individual object in the set (as in *parish*). *Kavu'a*, on the other hand, comes in only one variety: a hybrid

status must be assigned to a set and then only indirectly to an individual item in the set. When *kavu'a* is invoked, each individual item in the set loses its individual identity and is regarded simply as a fragment of an irreducibly mixed entity. It is not treated as an individual of uncertain status but rather as a part of a set that is certainly mixed.

Given this, we are ready to answer the central question: When do we apply RDIK and when do we apply *kavu'a*?

Roughly speaking, the idea is that when an object is being judged in isolation, it must be assigned a status appropriate to an individual object; when it is judged only as part of a set, it can be assigned some new status appropriate for a set. *Kavu'a* can only be invoked in the latter case. To see this distinction very starkly, consider two scenarios in each of which we have before us a box containing nine white balls and one black ball.

Scenario 1: I reach into the box, pull out one ball without showing it to you and ask: What is the color of this ball?

Scenario 2: I don't reach into the box, but instead ask: What is the color of a random ball in this box?

In the first case, if you were to answer, say, "black," your answer would be either true or false, but either way would be an appropriate response to the question that was asked. There is a determinate answer to the question, although this answer is unknown to you. In the second case, the answer "black" (or "white") is neither true nor false, since there is no determinate answer to the question. You could say nothing more specific than that the box contains both white and black balls.

Obviously, the case of the stone-thrower considered above is analogous to scenario 2—asking about the status of an unspecified member of the group is like asking about the color of an unspecified ball. The appropriate level at which to assign status in this case is the level of the set, not the level of the individual, and the set is indeed mixed. This is the sort of case in which *kavu'a* can be invoked.

By contrast, a piece of meat that is found in the street is clearly analogous to scenario 1—the status of a particular item is in question. This is the kind of case in which RDIK is invoked. Now admittedly, the case of a piece of meat bought in one of the stores might plausibly be regarded as analogous to scenario 1 since the act of buying could be considered analogous to pulling out a specific ball. However, the rabbinic principle is, somewhat counter-intuitively, otherwise: apparently, the critical moment is the one prior to actually encountering the piece

in question. When the piece is found on the street, it is judged as an individual because prior to the moment that it is found, it is already no longer "in the set." When the piece of meat in question is bought in the store, prior to its being bought it is indeed "in the set."

The distinction between *kavu'a* and RDIK might be restated in terms of the issue of sample space selection considered above. RDIK assumes the "standard" sample space. In the case of the meat found in the street, that sample space is the set of stores. But *kavu'a* entails the selection of a non-standard, but entirely sensible, sample space: the single element consisting of the entire set of stores. This single item is mixed.

Let us now spell out in detail the precise method for determining when to apply RDIK and when to apply *kavu'a*.

1. First, there are a number of cases in which *kavu'a* cannot be invoked because a hybrid status is inappropriate.

—In the case of a vote in Sanhedrin which is, by definition, a mechanism for rendering a decision.

—If uncertainty regarding the status of an individual object which belonged to the set arose only after the object had been isolated from the set (*parish*), then it is this object alone which must be assigned some status. While a member of a set consisting of objects some of which are P and some of which are not-P can be assigned a hybrid status as part of the set, an individual object being assigned a status on its own cannot. Thus, we need to choose either P or not-P for this object and we choose the majority of the set from which it comes. For example, in the case of "nine stores" in which the meat is found on the street, the isolated piece of meat is assigned either the status "kosher" or the status "non-kosher."

—Similarly, if the set is somehow "incohesive," so that each object in it is regarded as having left the set, we apply RDIK and not *kavu'a*. Thus, for example, a set of travelers passing through a town do not constitute a set for purposes of *kavu'a*, while the residents of the town do (*Ketubot* 15b; and compare Yoma 84b).<sup>11</sup>

—Finally, if it is not certain that the set contains any objects that are, say, not-P, then the set is said not to satisfy the condition of *ithazek issura*. Such a set cannot be assigned a hybrid status and RDIK is invoked rather than *kavu'a*. Thus, the *Tosefta* (*Taharot* 6:3) already considers a case in which we are given a mixture of

ten loaves, including one loaf that is *tamei*, that is eaten in two rounds of five loaves each. Those who eat in the first round are tameh because at that point the set certainly contains one *tamei* loaf, but those in the second round are *tahor* because by then the set might not contain a *tamei* loaf. Similarly, a mixture of objects including one that has been used for idolatry is forbidden for use but if one of the objects is destroyed the mixture is permitted for use (*Zevahim* 74a as understood by Rambam, *Avoda Zara* 7:10).<sup>12</sup>

To summarize: in all cases in which we are not assigning a status to a mixed set, kavu'a is not invoked but rather RDIK.<sup>13</sup> Note that although in these cases the membership of the doubtful item in the set, or the cohesiveness of the set itself, may be inadequate for invoking kavu'a, this does not diminish the relevance of the set for purposes of RDIK. Thus, for example, even though the piece of meat found on the street cannot be assigned a hybrid status because it is not part of the set, the fact that the meat is known to have originated in the set still renders the composition of the set (i.e., the majority) relevant to determining the status of the piece.

2. When the above rule does not apply, so that at issue is the status of a mixed set, we first check if the principle of *bittul* can be applied. If *bittul* can be applied, the set is no longer regarded as a mixed set but rather as a uniform set, and *kavu'a* is inapplicable. Thus, for example, a mixed set consisting of one (non-distinguishable) non-kosher piece of meat and more than one kosher pieces is regarded as a permissible set.<sup>14</sup>

3. Finally, there are a number of cases in which we are dealing with a mixed set but *bittul* is not applicable:

—First, if the objects in the set are each identifiable as either P or not-P (*nikkar bimkomo*). For example, in "nine stores" the status of each store is known, it is only the origin of a particular piece of meat that is in doubt. Clearly, in such a case, we can't define the set as either P or as not-P; as a set it is both.

—Second, if individual objects in the set are each regarded as sufficiently significant that the status of each cannot be subordinated to the status of the set (*hashivi ve-lo beteli*) or if *bittul* is inapplicable for any other reason. Thus, given a herd of oxen including one that has been sentenced to death and is forbidden for use, we can't invoke *bittul* due to the significance of living creatures, and

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hence we invoke kavu'a by default (Zevahim 73b).

—Third, if the set includes an equal number of objects that are P as are not-P. In such a case *bittul* is obviously not possible.

In each of these cases<sup>15</sup> we are dealing with an irreducibly mixed set and the principle of *kavu'a* is invoked: the set is assigned a new hybrid status (P and not-P) as are individual objects drawn from the set.

#### KAVU'A AND SAFEK

The crucial distinction between *safek*, which reflects uncertainty regarding an individual object, and *kavu'a*, which is a definite hybrid status assigned to a set, cannot be over-emphasized. When *kavu'a* is invoked, it is the definite mixed status of the entire set that concerns us and not the uncertain status of any individual item in the set. It is generally the failure to appreciate this distinction which leads to the conclusion that *kavu'a* is completely counter-intuitive.

Let's consider for a moment the alternative, more common, explication of kavu'a as merely a leveling of the playing field in which the case is treated as a symmetric *safek*. On this understanding, which I reject, the sample space would contain two elements: kosher and nonkosher. According to my explanation, in cases of kavu'a, the sample space consists of a single element: the entire mixed set. Might not the phrase *mehetsa al mehetsa* suggest that the rule is in fact that we assign each status a probability of 1/2, that is, that we have a sample space consisting of two elements? Why do I reject this possibility?

First of all, because such a rule would be arbitrary and the one I argue for is perfectly sensible. Moreover, the notion that *mehetsa al mehetsa* refers to a probability of 1/2 is utterly anachronistic. The assignment of probabilities to the range [0,1], so that 1/2 is in the middle, is a relatively recent convention. The phrase *mehetsa al mehetsa* certainly refers to set composition and not to probability. Specifically, it refers to the third case in Rule 3 of the *kavu'a*/RDIK rules above in which *kavu'a* applies to a mixed set that includes an equal number of objects that are P as are not-P. The point of the phrase *kol kavu'a kemehetsa al mehetsa dami* is that in all cases that satisfy the conditions for *kavu'a*, RDIK is not invoked just as it is obviously not invoked in the case where there is no majority.

Finally, there are important halakhic differences between cases that

are deemed *safek* and cases where *kavu'a* is applied. For example, if a person had before him two indistinguishable pieces of meat, one kosher and one non-kosher—a case of kavu'a—and he ate one of them, he is obligated to bring an *asham taluy*. But if he had before him one piece, possibly kosher but possibly non-kosher—a case of *safek*—he is not so obligated (Rambam, Shegagot 8:2, based on Keritut 17b).<sup>16</sup> Similarly, if a mouse takes a piece from a mixed pile of pieces of *hamets* and of matsa, in a manner such that the principle of kavu'a would apply, into a house which has been inspected for *Pesah*, the house must be reinspected. But if it took a single piece of which has an even chance of being *hamets* or *matsa* into the house—this is a *safek*—the house need not be re-inspected (*Pesahim* 9a as understood by Rambam, *Hamets u*-Matsa 2:10-11). In the case of safek, we can presume that an inspected house remains free of hamets since one possible resolution of the uncertainty regarding the subsequent events is consistent with this presumption. In the case of *kavu'a*, however, there is no uncertainty to resolve. Rather, some object of known mixed status has certainly been brought into the house; this is enough to nullify the presumption.

Now that we have established that cases of *kavu'a* are not cases of *safek*, which cases are in fact *safek*? The status of an object is *safek* when it is not judged as part of a set (so that *kavu'a* and *bittul* do not apply) and it has not been removed from a set with a majority (so that *parish* does not apply)<sup>17</sup> and it does not belong to some reference class for which some statistical law is known (so that RDLK does not apply). A clean example of *safek* is one in which a piece of meat is found in the street and might have come from one of two stores, one kosher and one non-kosher.

Actually, few cases of *safek* are that neatly symmetric. In the following sections, we will deal with two types of asymmetric *safek*, *safek haragil* and *sefek sefeka*, and see which types of asymmetries matter and which don't.

## EN SAFEK MOTSI MI-YEDEI VADDAI

In several places in the *Gemara* we find the principle *en safek motsi mi*yedei vaddai. For example, in Avoda Zara 41b, Resh Lakish argues that if an idol is found broken we can assume that its owner renounces it and thus it is no longer forbidden to make use of it. R. Yohanan rejects this argument on the grounds that it was certainly (vaddai) initially an

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idol but only possibly (*safek*) renounced and thus on the basis of the principle *en safek motsi mi-yedei vaddai* we may not use it. *Tosafot* (s.v. *ve-en safek*; *Hullin* 10a, s.v. *taval ve-ala*) points out that the *safek* referred to in such cases is in fact a "*safek ha-ragil*"—that is, it is more likely than not that the idol was renounced. According to *Tosafot*, then, at least some cases of *en safek motsi mi-yedei vaddai* are actually cases of a majority failing against a *hazaka de-me-ikara*. But then it would appear that these cases contradict the widely accepted rule (against the view of R. Meir) that a majority defeats *hazaka de-me-ikara* (*Nidda* 18b). Why are these cases treated differently?

We can answer this question by answering a more elementary one: what exactly is a *safek ha-ragil* and why is it not simply called *rov*? R. Osher Weiss (*Minhat Osher*, *Bereshit* 58) suggests that what Tosafot call a *safek ha-ragil* is specifically a case in which some event does not belong to any recognized reference class covered by a statistical law. In such cases, for which von Mises would argue that the notion of probability is undefined, the principle of RDLK does not apply. Rather, the *safek* is *ragil* only in the sense that a typical observer might find one possibility to be subjectively more likely than not. But this subjective probability is not relevant; a *safek ha-ragil* is, for legal purposes, no kind of majority at all.<sup>18</sup>

Simply put, the term *safek* refers not to cases where two possibilities are known to be equiprobable, but rather to cases in which two possibilities exist and neither RDIK nor RDLK are applicable. We might think of a *safek* as a case with a sample space consisting of two opposite elements.

#### SEFEK SEFEKA

Not every case in which both RDIK and RDLK fail necessarily results in two possibilities. Recall that in our comparison of RDIK with the classical interpretation of probability above, there was a glaring gap. The sample spaces in RDIK were limited to concrete objects in a set. What happens when an object is not drawn from a set of objects in some proportion (so that RDIK in the usual sense does not apply), but there are more than two possibilities for assigning its status. Does there exist some extension of RDIK to such abstract sample spaces? According to one opinion, *sefek sefeka* is just such an extension.

The principle of *sefek sefeka* is this: If a particular prohibition holds only if both conditions A and B hold, and in fact both A and B are in doubt, then we can assume that the prohibition does not hold. Rashba

in his Responsa (1:401) states that sefek sefeka operates "like a rov." There are two main branches of thought regarding the nature of this analogy. According to one approach, sefek sefeka is like rov in its effect but not in its mechanics. Thus according to Ra'ah (Bedek ha-Bayyit on Torat ha-Bayyit 4:2) the principle of sefek sefeka may simply arise naturally from the iterative application of the rules for handling a single safek; the first safek reduces the issur to a de-rabbanan and the second renders it permitted (see also Sha'arei Yosher 1:19). A related interpretation, ascribed to R. Yosef Dov Soloveichik, is that we regard a safek between permitted and only possibly forbidden as if there is no ledat ha-safek. Consequently, even in instances of sefek sefeka in which it might be possible to ascertain the actual facts, one is not obliged to make a special effort to do so (Rosh on Avoda Zara, En Ma'amidin, 35, and Rema, Yoreh De'a 110:9). Understood this way, sefek sefeka has the effect of one type of RDLK: in each, there is no ledat ha-safek.

A second approach to *sefek sefeka*, the one of primary interest for our purposes, extends the analogy to *rov* to include the mechanism through which *sefek sefeka* works. Thus R. Y. S. Natanzon (*Responsa Sho'el u-Meshiv* 1:196 and many other places) elaborates that *sefek sefeka* can be thought of as an extension of the idea of RDIK in which the sample space consists of the set of possible truth assignments to the various individual conditions.<sup>19</sup> For example, a newlywed woman who is found to have had sexual relations prior to marriage would be forbidden to her husband if she had done so (a) subsequent to betrothal, and (b) consensually (*Ketubot* 9a). Of the four possibilities (subsequent/ consensual, subsequent/non-consensual, prior/consensual, prior/nonconsensual), only one renders her forbidden. Consequently, if we are in doubt as to which of these four possibilities is indeed the case, we can follow the majority and she would not be forbidden to her husband.

If indeed *sefek sefeka* is a type of *rov*, in its mechanics as well as its effect, it is certainly not RDLK since many examples of *sefek sefeka*, such as the one above, are not amenable to explanation in terms of RDLK.<sup>20</sup> Moreover, there are differences between *sefek sefeka* and RDLK even in terms of effect. We have already seen that R. Meir's principle of *haishinan le-mi'uta* applies only to RDLK and not to RDIK. But in *Nidda* 59b we find that (according to R. Yohanan) R. Meir invokes a *sefek sefeka* against *hazaka de-me-ikara* and appears unconcerned with the principle of *haishinan le-mi'uta*. This is consistent with *sefek sefeka* as an extension of RDIK but not as RDLK.<sup>21</sup>

The most intriguing aspect of the identification of sefek sefeka with

RDIK is the consequent link to the classical interpretation of probability. The issues that are raised by the commentators regarding restrictions to the application of *sefek sefeka* are remarkably similar to those subsequently raised with regard to the classical interpretation of probability. Consider, for example, the statement that "the probability that the sum of two throws of a die will be exactly six is 5/36." The classical interpretation of this statement is that in a sample space consisting of 36 possible throws of the dice, five have the desired condition. But, as we have seen earlier, this choice of sample space is not inevitable; it must be stipulated. Admittedly, this particular sample space is intuitively sensible based on two hard-to-pin-down "indifference principles."

-P1. For each toss, the six possibilities are plausibly symmetric.

-P2. The two tosses are plausibly independent—that is, the possibilities for each toss remain plausibly symmetric regardless of the result of the other toss.

But critics of the classical interpretation have pointed out that any attempt to regard the choice of a sample space based on such principles as anything more than mere stipulations inevitably runs aground on at least one of the following two problems:

-Q1. Primitive claims of plausible symmetry must not be confused with precise claims of equiprobability. To do so would be to invite infinite regress by introducing probability into the definition of probability.

-Q2. The same problem sometimes permits different plausible symmetries that may lead to contradictory conclusions.<sup>22</sup>

Consider now the central issues regarding the applicability of *sefek sefeka*. Recall that the idea is that both A and B are necessary conditions and both are in doubt. *Rishonim* have limited the applicability of *sefek sefeka* to cases in which two "indifference principles" can be invoked:

-P1'. A and not-A, and B and not-B, respectively, are plausibly symmetric in the sense that rov can't be applied to either one of them (*Ketubot* 9a, Tosafot s.v. *ve-i ba-it*).

-P2'. A and B must be independent ("*sefek sefeka ha-mitha-pechet*").<sup>23</sup>

Thus, for example, if there were a firm rule that most cases of sexual

relations are consensual, our newlyweds' marriage above could not be saved by *sefek sefeka*. Moreover, the *sefek sefeka* works only because the question of consent is regarded as independent of the question of timing (prior to or subsequent to betrothal), in the sense that resolving one question would not resolve the other.

In fact, *Rishonim* are at pains to point out that the plausible symmetries underlying the definition of *sefek sefeka* must be regarded as mere rabbinic stipulations:

-Q1'. Rivash (*Responsum* 372) points out that symmetry in these cases should not be confused with equiprobability.

-Q2'. Tosafot note that by formulating the problem differently we might find plausible symmetry between A&B and not-(A&B) thus destroying the *sefek sefeka* (e.g. "*shem ones had hu*," *Ketubot* 9a, Tosafot s.v. *ve-i ba'it*).

Continuing with our example, no claim is made that the probability that relations were consensual is precisely \_. It is enough that there are two possibilities, consensual or non-consensual, and no firm rule rendering one more likely. Moreover, as Tosafot notes, the possibility that the relations were non-consensual could in principle be counted as multiple possibilities by distinguishing violent rape from statutory rape. It is a matter of stipulation that we do not do so but rather bundle all cases of rape under a single label.

In short, attempts to explicate *sefek sefeka*, like attempts to explicate the classical interpretation of probability, contend with the need for some fuzzy symmetry criterion, more primitive than equiprobability, on the basis of which to choose a sample space.<sup>24</sup> This parallelism makes it tempting to see in *sefek sefeka* precisely the kind of extension of RDIK required to complete the analogy with the classical interpretation of probability.

Since very few cases of *sefek sefeka* are actually considered in the *Gemara*, it is unclear how broadly we might apply it. In the most restrictive interpretation, *sefek sefeka* would only apply where two conditions are required and each condition permits precisely two possibilities: yes or no. All the cases in the *Gemara* are of this type.<sup>25</sup> In a broader interpretation, the breakdown to independent conditions might just be a short-cut for counting elementary cases to determine if a majority of them permit or forbid. For example, suppose widgets came in three colors (red, white, blue), and three sizes (small, medium, large), and that a widget is

forbidden precisely if it is red OR blue AND medium OR large. Then, by the broader interpretation, a particular widget of unknown color and size might be permitted since only 4 of 9 possibilities are forbidden. By a narrower interpretation, uncertainty regarding color and uncertainty regarding size would each be resolved independently in favor of a 2 of 3 majority to forbid and the widget would be forbidden.

According to the broader interpretation, constraint P1' above should be interpreted as strictly analogous to P1: for purposes of case counting, the cases must be plausibly symmetric but there need not necessarily be precisely two possibilities within each constituent *safek*. Note that even according to this broad interpretation, *sefek sefeka* does not provide a blanket license for multiplying numerical probabilities. It merely offers a shortcut for counting cases. As explained at great length by the *Shakh* (*Yoreh De'a* 110:9, *Dinei Sefek Sefeka*), where case counting is inapplicable, such as when the constituent sefekot involve conceptually incommensurable aspects of the situation, *sefek sefeka* does not apply.

#### SUMMARY

To summarize, rabbinic methods for resolving uncertainty regarding the status (P or not-P) of some object are non-numerical but rather yield one of the following outputs:

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-certainly P / not-P
-probably P / not-P
-uncertain (safek)
-hybrid (mehetsa al mehetsa )
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These methods can be roughly summarized by the following procedure:

1. If the object is known to belong to some fixed set containing objects that are P and others that are not-P, then use the rules outlined above to apply either RDIK or *kavu'a*. When RDIK is invoked, the conclusion is either probably P or probably not-P. When *kavu'a* is invoked, the conclusion is *mehetsa al mehetsa*.

2. If the object is not known to belong to some fixed set, but does belong to some class about which we have some recognized statistical law (RDLK) concerning the property P, then assign its status according to that law. Depending on the strength assigned to the law, this status might be regarded as certain or as merely probable.

3. When the necessary conditions for P or not-P to hold can, according to rabbinic judgment, be most naturally formulated in terms of a multiplicity of plausibly independent symmetric *sefekot*, we resolve in accord with the majority of theoretically possible instances (*sefek sefeka*). This conclusion is regarded as probable.

4. If none of the above allows resolution, the status of the object is *safek*. In such cases, second-order default rules might be invoked to determine a course of action. These second-order rules involve the nature (*tum'a*, *yuhesin*, *mamanot*, or *issurin*) and severity (*deoraita* or *de-rabbanan*) of the prohibition in question and various presumptions (*hazakot*), a detailed discussion of which is beyond the scope of this article.<sup>26</sup>

#### NOTES

- 1. This literature includes a number of classical book-length treatments, such as Shev Shema'at'ta, Sha'arei Torah, and Sha'arei Yosher. In addition, Penei Yehoshua often makes reference to an unpublished monograph he wrote on this topic called *Kelal Gadol*. Some recent contributions to the classical literature that are especially noteworthy can be found in the writings of R. Osher Weiss. See especially his Minhat Osher on Bereshit, Chapter 58 and Shi'ur on Parashat Mishpatim (5763, transcript distributed by Makhon Minhat Osher). An important contemporary book that treats this topic from a historical perspective and covers a number of sources discussed in this paper in detail is Nachum L. Rabinovitch, Probability and Statistical Inference in Ancient and Medieval Jewish Literature (University of Toronto, 1973). Contemporary articles devoted to probabilistic aspects of rabbinic methods for handling uncertainty are too numerous to list. Three articles which I have found exceptionally useful appeared in Higayon (Vol. 4): L. Moscovitz, "On the Principles of Majority and Ithazek Issura" (Hebrew); N. Taylor "The Definition of Rov in Halakha" (Hebrew); Y. Werblowsky, "Rov and Probability." My gratitude to all these authors. Two important papers by J. Beck and V. Shtern, "The Talmudic Concepts of Safek and Sefek Sefeka" and "The Talmudic Concept of Zil Batar Ruba," which I became aware of only as this article was completed, appeared in Otam beMoadam (Bulletin of the Young Israel of Brookline), Nissan 5761 and Tishrei 5762, respectively. Finally, I am grateful to a good number of friends and colleagues who made very helpful comments on earlier drafts of this article. I am not listing them by name only for fear of inadvertent omission.
- See R. von Mises, *Probability, Statistics and Truth (Dover*, 1957), originally published in German (Springer, 1928).
- 3. To be sure, these three do not constitute an exhaustive list of all interpretations that have been suggested. Other interpretations, such as the "logical"

interpretation, purport to subsume one or more of these. Certainly, for our purposes these three will suffice.

- 4. See the discussion in Rabinovitch, Chapter 3.
- 5. The case should not be overstated. Certainly the rabbis regarded majority as relevant for resolving uncertainty and certainly they distinguished between two distinct kinds of majority. While it is true that these notions of majority can be neatly embedded in full-blown theories of numerically quantifiable probability, it certainly does not follow that the rabbis were in conscious possession of any such theory. Nor, by the way, is this necessarily a bad thing. Although scholars since Leibniz have occasionally toyed with the idea of using probability theory to numerically quantify evidence for legal purposes, one is hard-pressed to find even a single instance in which such flights of fancy have advanced the cause of justice. See, for example, the epilogue of J. Franklin, *The Science of Conjecture* (Johns Hopkins University, 2001).
- 6. It has also been suggested that the principle *en holkhin be-mamon ahar harov* holds only with regard to RDLK but that RDIK does apply even for *dinei mamonot* (Rashbam, *Bava Batra* 93a s.v. *de-hu gufeh; Terumat ha-Deshen* 314), but it is not clear that this distinction is relevant here. See footnote 9 below.
- 7. The terms I use here, "hakhra'a," "berur," and "hanhaga," are those used in the lengthy discussion in Sha'arei Yosher 3:1-4. Although the distinctions among these three categories are implicit already in the Gemara, explicit conceptualization of them is a relatively recent phenomenon. As a result, there is no consensus with regard to nomenclature. Often, different nomenclature is employed, sometimes even with some of these same terms used in different permutations. R. Hayyim Volozhin (cited in Responsa Nahalat David, 24) notes that the word "hazaka" is used in all three ways.
- 8. To be sure, in capital cases, the act which is grounds for conviction must be established by direct witnesses and no amount of circumstantial evidence is sufficient (see *Sefer ha-Mitsvot*, *Lo Ta'aseh* 290). Even the strongest type of RDLK is applied only towards establishing the necessary background facts (see *Shev Shema'at'ta* 4:8). It should also be noted that legal certainty is required only with regard to the facts of the case; obviously with regard to the procedural matter of voting for conviction, a procedural majority—i.e., RDIK—is sufficient.
- 9. This is the most plausible explanation of Shemuel's oft-cited principle *en* holkhin be-mamon ahar ha-rov. In fact, though, the Gemara applies this principle only twice (Bava Kamma 27a and Bava Batra 92b) and both those cases permit a much narrower interpretation of the principle, namely, that typical market behavior is irrelevant for establishing the intentions of the participants in a specific transaction.

Note also that there is at least one case (*Ketubot* 75b) in which R. Gamliel appears to hold that *hezkat ha-guf* trumps *hezkat mamon. Penei Yehoshua* argues that according to this view *hezkat ha-guf* is also more than a default rule, but other explanations are possible. The case of *hezkat ha-guf* is in any case interesting in a way that bears mentioning: unlike other forms of *hazaka de-me-ikara*, which are purely normative principles, the

principle of *hezkat ha-guf* is rooted in a particular model of the real world, namely, one in which as many facts as possible persist as long as they are not known to have been contradicted. Researchers in artificial intelligence find such models to be especially useful for many technical reasons. Compare, for example, Y. Shoham, *Reasoning About Change* (MIT Press, 1988), Chapter 5 and R. Elchonon Wasserman, *Kovets Sh'iurim, Ketubot* 75b, Paragraph 265.

- 10. To be precise, the duel between such a *rov* and the competing *hazaka de-me-ikara* results in a standoff. See *Yevamot* 119a and *Kiddushin* 80a *Tosafot* s.v. *semokh*. Later we will encounter cases in which a purported *rov* is so dubious that it is simply defeated by a *hazaka de-me-ikara*.
- It is often erroneously thought that mobility (*naidi*) is the converse of *kavu'a* (in the sense of "stationary"). In fact, mobility is simply one possible symptom of the items in question failing to constitute a cohesive set. See M. Koppel, "Inclusion and Exclusion" (Hebrew), *Higayon* 1, pp. 9-11; M. Koppel, "Further Comments on *Rov* and *Kavu'a*" (Hebrew), *Higayon* 4, pp. 49-52; L. Moscovitz, "On the Principles of Majority and *Ithazek Issura*" (Hebrew), *Higayon* 4, pp. 18-48.
- 12. For the same reason, if some—but not all—of the objects in the mixture are mixed with other, permitted objects, the resulting mixture is permitted. While this is not the only possible interpretation of the *Gemara* in *Zevahim*, it is the one offered by the Rambam. Nevertheless, with regard to a seemingly analogous case considered by the *Gemara*, a mixture involving valuable objects, Rambam (*Ma'akhalot Asurot* 16:10) does not permit the second mixture. It has been speculated that in the latter case, where *bittul* fails due to a set property rather than a property of the forbidden object, every object in the set is regarded as forbidden so that the presence of any such object in the second set is sufficient for *ithazek issura*.
- 13. Of course, it is possible in such cases that the set from which the object in question originates includes an equal number of objects that are P as are not-P. Such cases are regarded as *safek*, more about which below.
- 14. On this understanding *bittul* applies to the entire set and is roughly analogous to the case of votes in Sanhedrin from which it is (by some accounts) learned. This reading of *bittul* is also neatly parallel with our understanding of *kavu'a*, which also applies to sets. According to this view, all elements of a mixed set on which *bittul* has been effected (in favor of permissibility) should be permitted even simultaneously and to a single person. In fact, Rosh (*Hullin* 7:37) so rules; *Tosafot Rid* (*Bava Batra* 31b), however, disagrees. It may be that *Tosafot Rid* holds that *bittul* does not apply to the entire set but rather to each individual object in the set separately, along the lines of *parish*. See *Kovets Shiurim*, *Bava Batra*, paragraph 127.
- 15. This reflects the view of Rashba (Hullin 92a) that kavu'a applies any time bittul fails. However, Tosafot (Zevachim 73b s.v. ela, Gittin 64a, s.v. asur) hold that the only authentic cases of kavu'a are nikar bimkomo. All agree, though, that when the failure of bittul is mi-de-rabbanan, as it always is in cases like hashivi, that kavu'a also holds only mi-de-rabbanan.
- 16. The term used in the *Gemara* is *ikba issura*. Rambam, however, uses the term *kavu'a*. Nevertheless, one should not extrapolate too freely from

asham taluy to kavu'a in general. Some cases regarded as kavu'a for purposes of asham taluy, would not ordinarily be considered cases of kavu'a. For example, if a melakha is performed, possibly on Shabbat, possibly on another day, an asham taluy may be brought.

- 17. Recall, though, that RDIK resolves uncertainty but does not dispel it. On this basis, *Shev Shema'at'ta* (2:15) argues that certain talmudic references to *safek*, such as *safek asiri* (*Bava Metsia* 7a) and *safek mamzer* (*Kiddushin* 73a), include cases of RDIK. This claim serves as the starting point for a great deal of "*yeshivishe* Torah" but is not essential for an understanding of the underlying issues.
- 18. Recall that we saw that RDLK comes in at least three varieties (depending on the naturalness of the reference class and the strength of the statistical law), the weakest of which is a simple default rule. We can think of *safek ha-ragil* as a degenerate case of RDLK in which the reference class is not deemed convincing enough to even rate as a default rule. One difference between RDLK as a default rule and *safek ha-ragil* is that RDLK as default neutralizes a *hazaka de-me-ikara* (Yevamot 119a) while the fact that a *safek is safek ha-ragil* is irrelevant in the face of such a *hazaka*. Note, though, that at least on one occasion (Ketubot 9a, s.v. ve-i ba'it), Tosafot conflate RDLK as default with *safek ha-ragil*. Moreover, according to Tosafot, the fact that a *safek is ragil* is not entirely irrelevant: Tosafot (Pesahim 9a, s.v. ve-im timtsi lomar) hold that safek ha-ragil might not be subject to the rule that safek tum'a bi-reshut ha-yahid tamei.
- 19. One disadvantage of this approach is that sometimes the phrase *sefek sefeka* is used with regard to a mixed set which then falls into another set (for example, *Zevahim* 74a). According to this second interpretation of *sefek sefeka*, the overlap in terminology is misleading and such mixture within mixture cases must be analyzed in the context of the rules governing mixed sets discussed above. According to the first approach, the conflation of ordinary kinds of *sefek sefeka* and mixtures within mixtures makes sense: in each the "outer" *safek*/mixture is treated leniently because the *hamur* side fails to satisfy the requirement of *ithazek issura* (see the discussion near footnote 12 above).
- 20. For example, the question of whether a particular woman is more likely to have had relations subsequent to or prior to her betrothal would appear to depend on the circumstances of the case in question; there is no evidence of any general statistical rule that the rabbis applied to all such cases. For more convincing examples of the inapplicability of RDLK, see *Taharot* 6:4 and *Nidda* 33b. It is important to note that there are cases of *sefek sefeka* and pseudo-*sefek sefeka* which happen to also be RDLK. For example, in *Hullin* 77b, Rashi s.v. ve-khol ha-yoledot (perhaps based on J. Yevamot 16:1) formulates the Gemara's claim that most births are not both living and males in terms of *sefek sefeka* even though stillbirths are clearly a minority. The reason that this works is that it happens to also be a RDLK (See Shev Shema'at'ta 1:18). Similarly, Rambam (Para Aduma 9:16) employs the terminology of *sefek sefeka* to explain the Gemara's claim (Hullin 9b) that mei hatat that are found uncovered can be assumed not to have been handled by an agent who was both human and *tamei*. This would contradict the law that *sefek*

sefeka bi-reshut ha-yahid tamei except that it also happens to be a RDLK. See also Bava Kamma 11a, Tosafot s.v. de-eno.

- 21. Still, it must be noted that a majority of possibilities as in sefek sefeka is not always treated in exactly the same way as a majority of concrete objects as in the usual case of RDIK. In Tosefta Taharot 6:2 (as explained in Ketubot 15a), we find that in a case of safek tum'a involving parish (the word used in the Tosefta is nimtsa), we follow the majority even for leniency; only in a case of kavu'a, do we follow the rule that safek tum'a bi-reshut ha-yahid tamei. Nevertheless, in Taharot 6:4 we find that sefek sefeka regarding tum'a bi-reshut ha-yahid is an inadequate basis for leniency. On the face of it, this would suggest that *sefek sefeka* is treated differently than RDIK. See Ahiezer [Yoreh Deah] 2:13. It would appear more appropriate, however, to ascribe this anomaly to the generally exceptional nature of the principle safek tum'a bi-reshut ha-yahid tamei. As is evident from the Tosefta, safek tum'a refers specifically to cases of kavu'a, which, as we have seen, is an atypical use of the term safek. Thus, it would appear that the sefek sefeka cases in the Mishna are regarded as more similar to the Tosefta's kavu'a case than to the RDIK case. In fact, R. Soloveichik argues that the rule that safek tum'a bi-reshut ha-yahid tamei employed in the Mishna applies only when the presence of *tum'a* is certain so that the only uncertainty involves whether it was metamei other things. Thus all cases of safek tum'a for which the rule is relevant may fall under some broader definition of kavu'a (alluded to in footnote 16 above with regard to *asham taluy*). Consequently, the rule is inapplicable in the *nimtsa* case of the *Tosefta* in which RDIK is applied to a single object which might not be *tamei* at all. (It is difficult, however, to reconcile this restriction to the rule with the language of the Mishna where the rule is applied: "safek hayeta sham, safek lo hayeta sham.") See Shiurei ha-Rav Aharon Lichtenstein on Taharot, pp. 167-171. (For another example of the atypicality of the rule safek tum'a bi-reshut ha-yahid *tamei*, see footnote 18 above.)
- 22. A number of interesting examples of this phenomenon, all beyond the scope of this paper, are subsumed under the name "Bertrand's Paradox." See von Mises, p. 77.
- 23. Halikhot Olam, Agur and Rivash attribute this view to Tosafot but without citing a source. Indeed it is not found in the standard Tosafot but can be found in Tosafot Yeshanim, Ketubot 9b (cited in the outer margin in the Vilna Shas). The Shakh on Yoreh De'a 110:9, Gloss 63, points out that in many known cases of sefek sefeka the independence condition fails and that in fact we require only partial independence—namely, that where there is a natural ordering on A and B (say, A is logically prior to B), it is sufficient that the question of B or not-B not be resolved given that A but not vice versa. Thus, for example, we find (Taharot 6:4) a sefek sefeka in which the two conditions are (A) that one entered a room in which there is a tamei object and (B) that one touched the object. Obviously, if B holds then A holds as well. Still, since the question regarding A arises prior to that regarding B, this is a valid sefek sefeka (albeit an ineffective one for unrelated reasons). In this case, there are not four possibilities, as in the standard sefek sefeka, but rather three (the possibility that one touched the object

but was not in the room is eliminated). Still, there is a majority of two out of three.

- 24. See the discussion in Rabinovitch, chapter 5.
- 25. A number of arcane exceptions are considered by later commentators. A summary can be found in E. Cohen, "Introduction to Sefek Sefeka" (Hebrew), Sinai 112 (5753), pp. 273-282.
- 26. See P. Schiffman, "Safek in Halakha and in Law" (Hebrew), Shenaton ha-Mishpat ha-Ivri 1 (5734), pp. 328-352.