March 1, 2011 Differential geometry 88-826 Homework 2

1. In the Euclidean plane, let p be a point other than the origin. Consider the polar coordinates (r, θ), and the 1-forms dr and $rd\theta$. They give an orthonormal basis for the cotangent plane

$$T_p^*$$

at p. Find an orthonormal basis for the *tangent* plane T_p, by modifying the basis $\frac{\partial}{\partial r}, \frac{\partial}{\partial \theta}$.

2. Let

$$S^1 \subset \mathbb{R}^2 = \mathbb{C}$$

be the unit circle. Let TS^1 be its tangent bundle (eged hameshik). Construct an explicit map from TS^1 to the Cartesian product

$$S^1 \times \mathbb{R}$$

which is one-to-one and onto.

3. The squaring function $f : \mathbb{R} \to \mathbb{R}$ is given by

$$y = x^2.$$

(a) Describe the associated trivial bundle

$$(E, B, \pi).$$

(b) Give an explicit description of the section of the bundle E corresponding to the squaring function.

4. Let $\tau_{a,b}$ be the conformal parameter of the torus of revolution defined as the set of points in the plane satisfying

$$(x-a)^2 + y^2 = b^2,$$

where $b < a$. Calculate the following limits.

(a) $\lim_{s\to 0} \tau_{a,s}$ for fixed $a > 0$.
(b) $\lim_{s\to(a^-)} \tau_{a,s}$ for fixed $a > 0$.
(c) $\lim_{s\to\infty} \tau_{s,b}$ for fixed $b > 0$.
(d) $\lim_{s\to\infty} \tau_{2s,s}$.