May 6, 2020

Differential geometry 88-826 homework set 3

1. Consider the 1-dimensional manifold $S^1 = \{e^{i\theta}\} \subseteq \mathbb{C}$. Consider the Riemannian metric of S^1 expressed by a 1×1 matrix with respect to the coordinate θ. Since θ is the arclength parameter, the vector field $\frac{\partial}{\partial\theta}$ along S^1 has unit length.

(a) Conclude that the matrix of the Riemannian metric is the 1×1 matrix with coefficient 1;
(b) find the coefficient of the Riemannian metric with respect to a new coordinate defined by the stereographic projection of the circle to $\mathbb{R}$.

2. Let $C > 0$ be a positive real number. Calculate the Gaussian curvature of the Riemannian metric $\frac{C^2}{y^2}(dx^2 + dy^2)$ using formula (3.4.2) on page 32 of the choveret of the course.

3. Consider the torus of revolution $(x - a)^2 + y^2 = b^2$ where $0 < b < a$ and let τ be its conformal parameter as in Section 3.12 of the choveret of the course.

(a) Let $a = 1$ and find the limit of τ as $b \to 0$;
(b) let $b = 1$ and find the limit of τ as $a \to \infty$;
(c) find a torus of revolution with conformal parameter $\tau = i$.

4. Consider the lattice $L \subseteq \mathbb{C}$ spanned by the roots of the polynomial $z^3 - 1$. Find the conformal parameter $\tau(\mathbb{C}/L)$.