May 18, 2021

Differential geometry 88-826 homework set 5

1. Consider the following segment of the exterior differential complex on a manifold M:

$$\Omega^1(M) \xrightarrow{d_1} \Omega^2(M) \xrightarrow{d_2} \Omega^3(M).$$

Prove that the segment is exact, i.e., $d_2 \circ d_1(\xi) = 0$ for all 1-forms $\xi \in \Omega^1(M)$.

2. Compute the Gaussian curvature of the metric $f^2(dx^2 + dy^2)$ with conformal factor $f(x, y) = \frac{1}{1+C(x^2+y^2)}$, $C \in \mathbb{R}$.

3. Let $\mathbb{T}^n$ be the n-dimensional torus. Compute the de Rham cohomology group $H^0_{dR}(\mathbb{T}^n)$.

4. Let S^1 be the circle. Compute the de Rham cohomology group $H^1_{dR}(S^1)$.

5. Let $r \in \mathbb{R}$ and let D be the unbounded region

$$D = \{(x, y) \in \mathbb{R}^2 \colon x^2 + y^2 \geq r^2\}$$

endowed with the standard orientation $dx \wedge dy$. Determine if the induced orientation on ∂D is clockwise or counterclockwise.