April 3, 2022

INFINITESIMAL ANALYSIS    88-503    HOMEWORK SET 3

**Due Date: 25 april '22**

1. Prove that there exists a hyperinteger $H$ divisible by all standard integers $n \in \mathbb{N}$.

2. Show that if a sequence converges in $\mathbb{R}$ then it has exactly one cluster point (nekudat hitztabrut).

3. Suppose that $a_i \geq 0$ for all $i \in \mathbb{N}$. Prove that the series $\sum_1^\infty a_i$ converges iff $\sum_1^n a_i$ is finite for *all* infinite $n$, and that this holds iff $\sum_1^n a_i$ is finite for *some* infinite $n$.

4. Use the hyperreal characterisation of uniform continuity to show that $f(x) = \frac{1}{x}$ is not uniformly continuous on $(0,1)$.