

Due Date: 7 july '24

1. Consider the LSEQ transformation (see Section 6.9 of the class notes). Apply LSEQ to Ψ_i in the following formulas and determine whether the new formula is true:
 - (1) Ψ_1 is the formula $(\forall r \in (0, 1))(\exists x \in \mathbb{R}) r < x$;
 - (2) Ψ_2 is the formula $(\forall r \in (0, 1))(\exists x \in \mathbb{R}) |\ln r| < x$.
2. The order type of ${}^*\mathbb{N}$ was described in Corollary 5.13.5 of the choveret of the course. Describe an analogous order type for ${}^*\mathbb{Z}$.
3. Let f be a real function defined on an open neighbourhood of $c \in \mathbb{R}$. Let *f be its natural extension. Show that if *f is constant on $hal(c)$, then it is constant on some interval $(c - \varepsilon, c + \varepsilon)$ for appreciable ε .
4. Let f be a real function that is continuous on some interval $A \subseteq \mathbb{R}$. If ${}^*f(x)$ is real for all $x \in {}^*A$, show with the help of the previous exercise that it is constant on A .