Modeling pedestrian crowd behavior based on a cognitive model of social comparison theory

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Abstract Modeling crowd behavior is an important challenge for cognitive modelers. Models of crowd behavior facilitate analysis and prediction of human group behavior, where people are close geographically or logically, and are affected by each other's presence and actions. Existing models of crowd behavior, in a variety of fields, leave many open challenges. In particular, psychology models often offer only qualitative description, and do not easily permit algorithmic replication, while computer science models are often not tied to cognitive theory and often focus only on a specific phenomenon (e.g., flocking, bi-directional pedestrian movement), and thus must be switched depending on the goals of the simulation. We propose a novel model of crowd behavior, based on Festinger's Social Comparison Theory (SCT), a social psychology theory known and expanded since the early 1950's. We propose a concrete algorithmic framework for SCT, and evaluate its implementations in several pedestrian movement phenomena such as creation of lanes in bidirectional movement and movement in groups with and without obstacle. Compared to popular models from the literature, the SCT model was shown to provide improved results. We also evaluate the SCT model on general pedestrian movement, and validate the model against human pedestrian behavior. The results show that SCT generates behavior more intune with human crowd behavior then existing non-cognitive models.

Keywords Cognitive modeling \cdot Modeling pedestrian crowd behavior \cdot Model of social comparison theory

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1 Introduction

Modeling crowd behavior is an important challenge for cognitive modelers. Models of crowd behavior facilitate analysis and prediction of the behavior of groups of people, who are in close geographical or logical states, and are affected by each other's presence and actions. Accurate models of crowd behavior are sought in training simulations (Thalmann 2001), safety decision-support systems (Braun et al. 2003), traffic management (Helbing and Molnar 1997; Rymill and Dodgson 2005), business and organizational science.

Existing models of crowd behavior, in a variety of fields, leave many open challenges. In social sciences and psychology, models often offer only qualitative description, and do not easily permit algorithmic replication. In computer science, models are often not tied to specific cognitive science theories. Moreover, existing computer science models often focus only on a specific phenomenon (e.g., flocking, bidirectional pedestrian movement), and thus must be switched depending on the goals of the simulation.

We propose a novel model of crowd behavior, based on Social Comparison Theory (*SCT*) (Festinger 1954), a social psychology theory that has been continuously evolving since the 1950s. The key ideas in this theory is that humans, lacking objective means to evaluate their state, compare themselves to others that are similar. We believe that social comparison is a general cognitive process underlying the social behavior of each individual in crowd.

In social psychology there are several views on the mechanisms underlying individual behavior when the individual is a part of a crowd. However, to the best to our knowledge, *social comparison theory* has never been connected to crowd behavior phenomena. We believe that it can provide an explanation for the social behaviors that are exhibited in crowds. The basis for our belief is that social comparison theory may account for characteristics of crowd behavior, noted by social psychology scientists such as Le Bon (1895) and others:

- Imitation. One implication of SCT is the formation of homogeneous groups. Festinger notes (Festinger 1954, p. 135): "The drive for self evaluation is a force acting on persons to belong to groups, to associate with others. People, then, tend to move into groups which, in their judgment, hold opinions which agree with their own".
- Contagion. Using social comparison, people may adopt others' behaviors. Festinger writes (Festinger 1954, p. 124): "The existence of a discrepancy in a group with respect to opinions or abilities will lead to action on the part of members of that group to reduce the discrepancy".

While inspired by SCT, we remain deeply grounded in computer science; we propose a concrete algorithmic framework for SCT, and evaluate its implementations in several crowd behavior scenarios. We quantitatively compare the performance of SCT crowd behavior model with popular models in the literature, and show that SCT generates behavior more in-tune with human crowd behavior. We evaluate the use of SCT model in generation of specific pedestrian movement phenomena such as creation of lanes in bidirectional movement and movement in groups with and without obstacles. Moreover, we also evaluate the SCT model on general pedestrian behavior where individuals, pairs and small groups are all walking on the sidewalk in bidirectional fashion with different speed. We compare this behavior to human pedestrian behavior.

In pedestrian movement generation, the SCT model accounts for group formation in pedestrians that are inter-related, a phenomenon unaccounted for by previous models; and where previous techniques apply, SCT shows improved results. In addition the SCT model accounts for group behavior in the presence of obstacles, modeling the selection of group members to bypass obstacles in the same direction as other members of the group.

The biggest challenge in modeling crowd behaviors is in their evaluation process. Unfortunately, only a handful of existing models of crowd behavior have been evaluated against real-world human crowd data. The main difficulty is lack of human data to use in evaluation of models.

We provide qualitative evaluation of the SCT model, as well as others, against human pedestrian behavior. We report on web-based experiment where 39 human subjects compared the behavior generated from the different models to movies of real-world pedestrians. The results demonstrate that the SCT model is superior to others in its fidelity to human pedestrian behavior.

2 Background and motivation

Crowds are defined as large groups of people (agents) who are in similar or closelyrelated states (logical or geographical). Examples of crowds include pedestrians, audiences in theaters or sports stadiums, people in demonstrations and stock-market investors.

Social psychology literature provides several views on the emergence of crowds and the mechanisms underlying its behaviors. These views can inspire computational models, but are unfortunately too abstract to be used algorithmically. In contrast, computational crowd models tend to focus on specific crowd behaviors (e.g, flocking). A common theme in all of them is the generation of behavior from the aggregation of many local rules of interaction (e.g., Rymill and Dodgson 2005; Reynolds 1987, 1999; Kretz 2007).

Social psychology A phenomenon observed with crowds, and discovered early in crowd behavior research is that people in crowds act similar to one another, often acting in a coordinated fashion, as if governed by a single mind (Le Bon 1895; Allport 1924; Blumer 1939; Berk 1974). However, this coordination is achieved with little or no verbal communication.

Le Bon (1895) emphasized a view of crowd behaviors as controlled by a "Collective Mind", and observed that an individual who becomes a part of the crowd is transformed into becoming identical to the others in the crowd. Le Bon explains the homogeneous behavior by two processes: (i) *Imitation*, where people in a crowd imitate each other; and (ii) *Contagion*, where people in a crowd behave differently from how they typically would, individually.

Blumer (1939) explains this coordinated crowd behavior occurres through "circular reaction" process which underlying each individual who is participates in collective behavior. According to Blumer "circular reaction" is: "a type of interstimulation wherein the response of one individual reproduces the stimulation that has come from another individual and in being reflected back to this individual reinforces the stimulation."

According to Allport, crowd behavior is a product of the behavior of likeminded individuals. Allport's explanation of crowd homogeneous behavior is that similar people act in similar ways; otherwise they would not be a part of the same group. However, individual behavior affected by the behavior of his surrounding, thus, according to Allport, "the individual in the crowd behaves just as he would behave alone, only more so."

Turner and Killian (1972) investigated Emergent-norm Theory, which hypothesizes that crowd members indeed imitate each other, but also create new norms for the crowd as the dynamics of the situation dictate. Thus while crowds are not entirely predictable, their collective behavior is a function of the decision-making processes of their members.

Berk (1974) explanation of crowd behavior is based on decision making theory. According to decision-theory, each individual always tries to maximize his or reward and minimize costs. Berk argues that crowd behaviors are no exception, and that they should be understood from a game-theoretic perspective. He explained coordinated behavior of crowds as consistent with agents using a minimax strategy where the greater the number of participants that engage in specific action, the less will be an individual cost for engaging in this action. Thus, each individual will selects the action of the majority.

Different theories provide different explanations as to what drives individual behavior when the individual is a part of a crowd. However, all theories agree that when an individual is part of a crowd, his or her individual behavior is affected by others.

We based our work on Social Comparison Theory (Festinger 1954), which (to the best of our knowledge) has never been applied to modeling crowd behavior. Nevertheless, as we show in the previous section, key elements of the theory are at the very least compatible with those theories discussed above. Previously, Carley and Newell (1994) have examined the implications of SCT on computational agents and their sociability. We base our work on their observations.

Computational models Work on computer modeling of collective behavior has been carried out in other branches of science, in particular for modeling and simulation. Inspired by different science fields, researchers have been developing computational models for simulation of collective behavior. However, only a few models have been validated against human data (Daamen and Hoogendoorn 2003; Kretz 2007; Helbing 2001) Indeed, there exists only limited quantitative data on the behavior of human crowds at a resolution which permits accurate modeling. Moreover, a key problem with these models is that the algorithms they provide change with the crowd phenomenon modeled.

Reynolds (1987) simulated *bird flocking* using simple, individual-local rules, which interact to create coherent collective movement. There are only three rules:

Avoid collision with neighbors, match velocity with neighbors, and stay close to the center of gravity of all neighbors. Each simulated bird is treated as a particle, attracted and repelled by others. Tu and Terzopoulos (1994) simulated motion of artificial fish that addressed individual goals. Like Reynolds' "boids", the artificial fish are autonomous creatures which have simple behaviors and together are able to create a more complex, collective behavior.

Henderson compared pedestrian movement to gaskinetic fluids. Based on experiments on real human crowds, he showed in Henderson (1971) that crowd distribution is compatible with Maxwell-Boltzmann's distribution. Henderson (1974) developed a pedestrian movement model based on Maxwell-Boltzmann theory. Since each person has mass and velocity, the crowd may be likened to liquid gas and under some assumption, the Maxwell-Boltzmann theory may be applied. Based on Boltzmannlike equations, Helbing (1993, 2001) developed a general behavior model for simulation of crowd dynamics. The proposed model takes into account social forces caused by interaction between the individuals and external or spontaneous forces which are caused by the physical environment.

Helbing et al. (2001, 1997, 2001) observed phenomena of self-organization in collective motion which can be caused by interaction among pedestrians. By self-organization, it means that there are some behavioral phenomena which were not planned: for example, creation of lane formation in pedestrian movement. These lanes are created as a result of pedestrians moving against the flow. The number of lanes that are created cannot be planned. It depends on the width of the street and on pedestrian density.

Adriana Brown et al. (2003) examined how individual characteristics impact crowd evacuation. They expanded Helbing's physical model by adding to each agent individual parameters, such as dependence level and altruism level. According to the model, there will be a creation of groups which are combined from altruism and dependent agents. By changing these attributes, they examined crowd evacuation by measuring the flow of people passing the door per second, and population distribution in the flow.

Blue and Adler (2000) proposed a different approach to model collective dynamics. They used Cellular Automata (CA) in order to simulate collective behaviors, in particular pedestrian movement. The focus is again on local interactions: Each simulated pedestrian is controlled by an automaton, which decides on its next action or behavior, based on its local neighborhoods. Blue and Adler showed that this simple rule results in the formation of lanes in movement, similarly to those formed in human pedestrian movement (Wolff 1973).

Toyama et al. (2006) expanded the cellular automata model by adding different pedestrian characteristics, such as speed, gender, repulsion level, etc. The model was examined on bi-directional pedestrian movement behavior and on evacuation behavior. The main problem with this approach is that each collective behavior is simulated with different CA model. For example, CA for simulation of pedestrian behavior has different set of rules than the CA for evacuation behavior.

Kretz (2007) proposes the Floor field-and-Agent based Simulation Tool model (F.A.S.T) which is a discrete-space and discrete-time model for pedestrian motion. The F.A.S.T model can be classified as an extension of Probabilistic Cellular Au-

tomata (PCA). The F.A.S.T model has been validated against human data. In particular, the model simulation results of evacuation scenario was compared to results of evacuation exercise at a primary school.

Daamen and Hoogendoorn (2003) performed empirical experiments on human crowds, in particular in terms of movement as pedestrians. In these experiments, participants were asked to walk through a monitored area, in both directions. Their movements were recorded. One conclusion was that "During capacity conditions, two trails or lanes are formed: pedestrians tend to walk diagonally behind each other, thereby reducing the head ways and thus maximizing the use of the infrastructure supply".

In all of these previous works above, the behavior of crowds in every domain of study (pedestrian movement, flocking, evacuation, etc.) is computed using a different algorithm, yet the actions and perceptions remain largely invariant (e.g., distances to others, occupied spaces versus empty spaces, goal locations, etc.). Instead, the computation itself changes between modeled behaviors.

For instance, many models for crowd behavior utilize cellular-automata (CA), which differ between domains. One CA model for pedestrian movement (Blue and Adler 2000) uses a set of 6 IF-THEN rules which work in parallel for all cells, to simulate the movement of pedestrians in cells. The rules utilize knowledge of the occupancy in adjacent (rules 1, 3 in Blue and Adler 2000) and farther cells (rule 2), as well as of the distance to oncoming pedestrians in the same lane (rules 4, 6). The rules set the forward velocity and position of the entities, by using a set of non-deterministic choices (sub-rules 5a, 5b, 5c), biased by distributions which differ depending on the environmental settings (e.g., choose from a uniform 50%/50% split distribution if two nearby cells are occupied, or from a 10%/80%/10% distribution when three cells are available). Another CA model for evacuation (Tissera et al. 2007) uses knowledge of adjacent cells and distances to exits, and sets the position of the entities. Thus the actions and perceptions of each entity are similar to those used in the pedestrian model. But the algorithmic computation of the new position is done in two deterministic rules (Tissera et al. 2007, pp. 17), which involve no arbitrary choices at all.

In contrast to these previous investigations, we seek a *single cognitive mechanism*—a single algorithm—that, when executed by individuals, would give rise to different crowd behaviors, depending on the perceptions and actions available to the agents. This single algorithm would account for different crowd phenomena, by virtue of the actions and perceptions available to each individual. In this paper, we focus on various pedestrian phenomena. Elsewhere, we describe the use of the same model for other crowd behaviors (Fridman and Kaminka 2009; Fridman 2007).

A successful introduction of a single cognitive mechanism which accounts for multiple crowd phenomena would allow exploration of its role within cognitive architectures and within unified theories of cognition. Indeed, in Fridman and Kaminka (2011), we describe the implementation of a general SCT mechanism in the Soar cognitive architecture (Newell 1990). Such a model would also change the way crowds are simulated in practice today, in applications of computer science. Rather than rely on labor-intensive programming of crowd behaviors for every new domain of application, we would be able to re-use the crowd behavior module across applications, and thus save significant resources.

3 A model of social comparison

We took Festinger's social comparison theory (SCT) (Festinger 1954) as inspiration for the social skills necessary for our agent in order to be able to exhibit crowd behavior. According to social comparison theory, when lacking objective means for appraisal of their opinions and capabilities, people compare their opinions and capabilities to those of others that are similar to them. They then attempt to correct any differences found. This section shows how SCT can be turned into a concrete algorithm, to be used for generating crowd behavior.

3.1 Festinger's social comparison theory

Festinger presents social comparison theory (SCT) as an explicit set of axioms. The following subset of axioms (re-worded) are particularly relevant (see also Festinger 1954; Carley and Newell 1994 for additional discussion):

- When lacking objective means for evaluation, agents compare their state to that of others;
- Comparison increases with similarity;
- Agents take steps to reduce differences to the objects of comparison.

We propose a concrete algorithmic framework for SCT that can be executed by an agent. Moreover, we propose the use of SCT algorithmic framework for modeling crowd behaviors.

3.2 An SCT algorithm

In order to build algorithmic framework for SCT, each observed agent A_i is taken to be a tuple of k state features $A \equiv \langle f_1^A, \dots, f_k^A \rangle$. Each feature f_j^i of agent A_i $(1 \le j \le k)$ corresponds to a dimension, such that agent A_i is represented by a point in a k-dimensional space, where the various dimensions correspond to state features (such as location in x, y coordinates, color, heading, etc.)

For each such agent, we calculate a similarity value $Sim(A_{me}, A_o)$, which measures the similarity between the observed agent A_o and the agent carrying out the comparison process A_{me} . The agent with the highest such value is selected. If its similarity is between given maximum and minimum values, then this triggers actions by the comparing agent to reduce the discrepancy.

The process is described in Algorithm 1, which is executed by the comparing agent.

Each agent A_i executes Algorithm 1. In line 2 and 3, for each observed agent $A_o \in O$, we calculate a similarity value $Sim(A_{me}, A_o)$, which measures the similarity between the observed agent A_o and the agent carrying out the comparison process (A_{me}) (Equation (1)). We model each agent as an ordered set of features, where similarity can be calculated for each feature independently of the others. We measure similarity between agents independently along each dimension. The similarities in different dimensions are functions $s_{f_i}(f_i^{A_{me}}, f_i^{A_o}) : f_i \times f_i \mapsto [0, 1]$. The function s_{f_i} defines the similarity in feature f_i between the two agents A_{me} and A_o . A value of

Algorithm 1 Argmax SCT $(O, A_{me}, S_{min}, S_{max})$

1: $S \leftarrow \emptyset$ 2: for all $A_o \in O$ do 3: if $S_{min} < Sim(A_{me}, A_o) < S_{max}$ then 4: $S \rightarrow S \cup A_o$ 5: $A_c \leftarrow \arg\max_{A_c \in S}(Sim(A_{me}, A_o))$ 6: $D \leftarrow$ differences between me and agent A_c 7: $a \leftarrow$ SelectAction(D) 8: Apply action a with its Gain (Equation 2) to minimize differences in D.

0 indicates complete dissimilarity. A value of 1 indicates complete similarity. For instance, one commonly used feature denotes normalized Euclidean distance, inverted: A value of 0 means that the agents are as far apart as possible. A value of 1 means that they are positioned in the same location.

To determine the overall similarity between two agents, we use a weighted sum over the functions s_{f_i} . With each feature f_i , we associate a weight $w_i \ge 0$. The similarity between two agents is then given by Eq. 1 below.

$$Sim(A_{me}, A_o) \equiv \sum_{j=1}^k s_{f_j}(f_j^{A_{me}}, f_j^{A_i}) \cdot w_j \tag{1}$$

For each calculated similarity value, we check in line 3 if it is bounded by S_{min} and S_{max} , and in line 5 we pick the agent A_c that maximizes the similarity, but still falls within the bounds. S_{min} denotes values that are too dissimilar, and the associated agents are ignored. Festinger writes (Festinger 1954): "When a discrepancy exists with respect to opinions or abilities there will be tendencies to cease comparing one-self with those in the group who are very different from oneself". Respectively, there is also an upper bound on similarity S_{max} , which prevents the agent from trying to minimize differences where they are not meaningful or helpful. For instance, without this upper bound, an agent that is stuck in a location may compare itself to others, and prefer those that are similarly stuck in place.

In line 6, we determine the list of features (f_i, w_i) which cause the differences between A_{me} and the selected agent A_c (list of features with $f_i < 1$). We order these features in an increasing order of weight w_i , such that the first feature to trigger corrective action is the one with the lowest weight. Thus, the correction order increases from lowest weight to the highest one. The reason for this ordering is intuitive, and we admittedly did not find evidence for it (or against it) in the literature and the experiments (see Sect. 5.1.1).

Finally, in step 7 of the algorithm, the comparing agent A_{me} takes corrective action (*a*) on the selected feature. Note that we assume here that every feature has one associated corrective actions that minimize gaps in it, to a target agent, independently of other features. Festinger writes (Festinger 1954): "The stronger the attraction to the group the stronger will be the pressure toward uniformity concerning abilities and opinions within that group". To model this, we use a gain function *Gain* (see

(2)), which translates into the amount of effort or power invested in the action. For instance, for movement, the gain function would translate into velocity; the greater the gain, the greater the velocity.

$$Gain(Sim(A_{me}, A_c)) \equiv \frac{S_{max} - S_{min}}{S_{max} - Sim(A_{me}, A_c)}$$
(2)

4 Modeling pedestrian movement

The coordinated behavior of crowds has often been investigated in the context of pedestrian dynamics. Pedestrian motion (direction and velocity) is affected not only by physical elements (e.g., the sidewalk), but also by the motion of other pedestrians. Wolff (1973) noted that pedestrians have a hight degree of cooperation and coordination without which, walking on sidewalk would be impossible.

One of the major problems in modeling crowd behaviors in general is the lack of human data that models can be compared with. The difficulty in creation this data is that crowd experiments are complex and very costly since they have to be in large scale. Moreover, there is also a difficulty in analyzing these experiments in order to receive the numerical data. Defining the appropriate measurements for crowd behaviors is probably one of the biggest challenges for researches in crowd behaviors experiments. However, there are fortunately few domains that accepted measurements do exist and some numerical and qualitative data are available.

One example is pedestrian domain where there exist some qualitative data. The biggest challenge in pedestrian behaviors is defining appropriate measurements. One of the common and most explored phenomena in pedestrians is that there is a creation of lanes in bidirectional movement. There are a few human crowd experiments on creation of lanes (Wolff 1973; Daamen and Hoogendoorn 2003). However, the main conclusion is that lanes in human pedestrian movement are formed in bidirectional movement. The more agents organize into lanes, the less their need to change lanes in the future. In pedestrian domains, the commonly used quantitative measures are thus the number of lane changes and flow. However, this ignores more general pedestrian behavior evaluation, such as grouping.

We explore the use of SCT in generating pedestrian movements in different settings (individual, groups, with and without obstacles) and compare its performance to known models. Our goal is to explore if SCT model can account for common pedestrian behavior phenomenons like lane formations in bidirectional movement, and movement in groups, with and without obstacles.

To implement the model for pedestrians movement experiments, we used NetLogo (Wilensky 1999). We simulated a sidewalk where agents can move in a circular fashion from east to west, or in the opposite direction. Each agent has limited vision distance (beyond this distance it cannot see). It also has a cone-shaped field-of-view of 120 degrees. Each agent initially moves with a default walking velocity (in our case, 0.1). Agents are not allowed to move through other agents, and thus no two agents can occupy the same space.



Fig. 2 Lane formations-experiment end-results

Figure 1 shows the NetLogo sidewalk environment, in an initial state where simulated pedestrians are randomly placed about. Each small triangle is a simulated pedestrian, able to move left-to-right or right-to-left. Pedestrians exiting the sidewalk on any side appear on the other side, heading in the same direction. Figure 2 shows an end-result from one of the experiments (described below), where lanes have formed.

Each agent has a set of features and its corresponding weight. For simulating pedestrian movement, we used the following features and weights:

Walking direction (weight: 2). Agents can move in two opposite directions, east and west. Represent agents' goal orientation.

Color (weight: 3). Each agent has a color (blue, pink, red, green, etc.). Represent agents' relation, agents with similar color consider related to each other.

Distance (weight 1). Each agent has a position, given in global coordinates.

The similarities in different features (s_{f_i}) are calculated as follows. $s_{f_{color}} = 1$ if the color is the same, 0 otherwise. $s_{f_{direction}} = 1$ if direction is the same, 0 otherwise. and finally, $s_{f_{distance}} = \frac{1}{dist}$, where *dist* is the Euclidean distance between the positions of the agents.

The rationale for feature priorities, as represented in their weights, follows from our intuition and common experience as to how pedestrians act. Positional difference (distance) is the easiest difference to correct, and the least indicative of a similarity between pedestrians. Direction, which in this simulation indicates an agents' goal orientation, is more indicative of a similarity between agents. Color, which we defined to be the same for agents of the same group, even more so. If an agent sees two others, one in the same direction as it (and far away), and the other very close to it (but in the opposite direction), it will calculate greater similarity to the first agent, and try to minimize the distance to it (this may cause a lane change).

Each agent A_{me} calculates $Sim(A_{me}, A_o)$ according to the model. If the chosen feature for closing the gap is distance, then the velocity for movement will be multiplied by the calculated gain $Gain(Sim(A_{me}, A_o))$. For other features (which are binary), the gain is ignored.

To evaluate the SCT model, we contrast it with a popular alternative model, often used in pedestrian crowd research (Blue and Adler 2000; Helbing and Molnar 1997;

Helbing et al. 2001). In this *individual choice* model, each agent chooses lanes arbitrarily if forward movement is blocked. If so, then the agent seeks to move left or right. If both lanes are available, one is chosen arbitrarily. This choice may be biased by culture (Wolff 1973), but we did not utilize such a bias in our experiments. This model was repeatedly shown to produce lane formations and is considered to be a base model for pedestrian movement.

We compare these models as is commonly done in pedestrian movement experiments: We controlled for *crowd density*, calculated as the number of agents divided by the area. We follow the literature in measuring two principal characteristics of pedestrian movement: the total number of *lane changes* (lower numbers indicate improved lane formations), and the *flow* (average speed divided by the space-per-agent).

In the following sections, we evaluate the social comparison model and its implementation in modeling pedestrian movement. The basic movement pattern that our simulated pedestrians follow, stemming from the social comparison model, is as follows: Each agent follows an initially set direction. It chooses moving in this direction, unless blocked. If forward movement is indeed blocked, the agent can change lanes to the left or right. It will choose the lane where there is an agent that is similar to it (if such an agent exists); otherwise, it chooses arbitrarily.

4.1 Experiment 1: independent pedestrians

Our first experiment contrasted the social comparison model with previous models. We begin by examining individual pedestrian movements, where each synthetic pedestrian is independent of others. Each agent had a unique color. Each agent's direction (east or west) and initial position was chosen randomly. We contrasted the social comparison model with that of individual choice, which was shown to produce lane formations (Blue and Adler 2000; Helbing and Molnar 1997; Helbing et al. 2001) and is considered to be a base model for pedestrian models.

In this experiment, we eliminated the Gain component by fixing its value at 1 (see below for experiments examining gain). S_{max} was set at 6, which means that no agent will be too similar, and any dissimilarity triggers an action (other than color which cannot be changed). S_{min} was set at 2, which means that agents that differed only in distance (but not in color or direction) were not considered similar. Each trial was executed for 5000 cycles.

Figure 3 shows lane changes for the individual-choice and social comparison models. The X-axis measures density. The Y-axis measures the number of lane changes during the course of 5000 cycles. Each configuration was repeated 30 times. Figure 4 measures flow for the two models; the X-axis again measures density, the Y-axis measures the flow.

The figures shows that the number of lane changes is significantly lower than that of the individual-choice model (one-tailed *t*-test, p = 0.009), implying that lanes form faster and are maintained longer with the social-comparison models. However, as the flow results show, there are no meaningful differences in flow. In other words, the social comparison model performs better, but with essentially no cost to the flow.



Fig. 3 Independent pedestrians' lane changes



Fig. 4 Independent pedestrians' flow

4.2 Experiment 2: independent pedestrians with varying gain

The next set of experiments explored the performance of the model when the gain component was allowed to vary, per its definition in the social comparison model. We repeated the individual pedestrian experiments, though ignoring color: All agents moving east were colored red, and all agents moving west were colored blue. Because of this, agents really see only two kinds of agents: Those who have similarity of 1 (or less), and those with similarity of 5 (or more). Thus the only way to vary the gain, is to vary the S_{min} and S_{max} values, as they set the enumerator in the gain calculation.

To evaluate the effect of the gain, we contrasted three variants of the social comparison model introduced earlier:

- $S_{max} = 5.5, S_{min} = 5$, i.e., $Gain(Sim(A_{me}, A_o)) = 1$
- $S_{max} = 5.5, S_{min} = 4$, i.e., $Gain(Sim(A_{me}, A_o)) = 3$



(d) g(o) = 7.

Fig. 5 Screen shots, independent pedestrians: varied gains

• $S_{max} = 5.5, S_{min} = 2$, i.e., $Gain(Sim(A_{me}, A_o)) = 7$

Figure 5 shows the initial positions of the agents in one of the trials (5(a)), and typical results after 5000 cycles, with a Gain of 1 (5(b)), Gain of 3 (5(c)), and Gain of 7 (5(d)). The figures show how the increased gain causes the agents to group more closely together, thus forming lanes faster.

Figures 6 and 7 shows lane formation and flow results for the individual and social comparison models. The X-axis in both figures measures density. The Y-axis in Fig. 6 measures the number of lane changes; a higher number indicates more lanes are formed, since agents must still change lanes, having met opposing pedestrians. The Y-axis in Fig. 7 measures flow. Each data-point is an average over 15 trials.

The figures show that the number of lane changes in SCT is significantly lower than that of the individual-choice model (one tailed *t*-test, p = 0.05). This is true even with a gain of 1, which effectively neutralizes the gain in comparison to the individual-choice model. Moreover, the difference with the individual-choice model increases with an increased gain. However, there are essentially no differences in flow. These results support the hypothesis that the use of SCT can lead to quicker lane formations, which would indicate an improved model of crowd behavior.

4.3 Experiment 3: pedestrians in groups

We now move away from considering scenarios previously appearing in the literature, to exploring new types of pedestrian behaviors. In particular, we experiment



Fig. 6 Independent pedestrians with varying gain: lane changes. A lower result is better



Fig. 7 Independent pedestrians with varying gain: flow

with pedestrian movement where the pedestrians belong to different groups internally. This type of situation arises, for instance, in pedestrians that are composed of families and/or friends. The individual-choice model does not account for such behavior, because it does not treat the group in any way. In contrast, we expect our social comparison model to treat groups (agents that belong to the same group would be more similar).

To examine this hypothesis, we carried out experiments in which color is meaningful: Agents belonging the same group have the same color. In these experiments, all agents move in the same direction, again, for 5000 cycles. Gain was allowed to vary per the model, as described above (see (2)). The population contains 150 agents with a different number of colors (we experimented with 5, 10, and 20 color). Walking direction of all agents is West. S_{max} was set at 6.5, and S_{min} was set at 2.

To account for the western cultural intuition that friends (and family) walk sideby-side, rather than in columns, we added another feature: The similarity in position along the x-axis. The revised features and weights are as follows: Direction, with weight 2; Distance, with weight 0.5; Color, with weight 3; X-Coordinate, with weight 1.



(c) After 5000 cycles, social comparison model.

Fig. 8 Screen shots, grouped pedestrian movement

The rationale behind these weights is that the agent will first close the distance gap with the agent selected as most similar, and only then try to locate itself on the same X-Coordinate.

There exists a significant challenge in being able to quantitatively measure the grouping results of the experiments. Normally, a simple clustering measure would do, as all agents of same color would group together. However, due to the initial random positions and the limited visual range of agents, agents of the same color may never group together, instead forming several groups that are far from each other.

Balch (1998) has offered a clustering measure, *hierarchical social entropy*, that can address such cases. The key intuition behind this measure is to iteratively sum entropy over increasing areas. The measure equals 0 when all agents are positioned in the exact same spot, and grows with their spreading around. Thus lower values indicate improved grouping. Balch (1998) provides the details.

Table 1 shows the measurement results for the individual-choice and socialcomparison models. Each row corresponds to the average results over multiple trials, with a different number of colors. The table shows (third column) that the social comparison model provides for significantly improved grouping compared to the individual-choice model (one-tailed *t*-test, p = 0.05). Again, these results support the hypothesis that SCT is an improvement over the basis individual-choice model.

4.4 Experiment 4: groups and obstacles

Our final set of pedestrian movement experiments addresses the response of groups within moving pedestrian crowds to obstacles. Here, we consider an obstacle which to

Individual-choice	Social comparison	
173.2	87.4	
143.3	85.8	
101.5	60.1	
	Individual-choice 173.2 143.3 101.5	

 Table 1
 Grouping measurements of individual-choice and social comparison models. Lower values indicate improved grouping

the agents appears as a long dividing fence whose end is not visible. Thus, they need to decide whether to move together or split. Intuitively, we recognize that such groups will choose to stick together in face of an obstacle (moving together to one side of it), while individual-choice pedestrians choose arbitrarily. We sought to examine whether the social comparison models would account for this behavior.

We created a sidewalk environment as described earlier, but this time with an elongated rectangular obstacle in the middle. When agents approach this obstacle, they must select to move to one of its sides. In the experiments, we allowed 100 agents of two colors (red and blue) to move west from their initial positions. Each agent has the following features: Direction, distance and color (weights: same as in the individual pedestrian experiments). Agents use comparison at all times, and not just when stuck. S_{max} was set at 6.5, S_{min} at 3.

Figure 9 shows the initial random positions of the agents (9(a)), their positions after going moving for a while using the individual-choice model (9(b)), and their positions when moving using the social comparison model (9(c)). The figures show clearly that the social comparison model causes similarly-colored agents to group together on one side of the obstacle, passing it together. In contrast, the individual-choice model has no such effect on the behavior of the agents.

Quantitative analysis again proved challenging, as here no clusters form. We needed, instead, to measure to what degree agents of the same color stay on one side of the obstacle. To do this, we defined virtual "gates" on either side of the obstacle, and monitored agents that move through them. Each trial allowed 100 agents to pass through the gates 10 times (i.e., 10 *waves*). At the end of each wave, we calculated (separately) the entropy of each color as its agents are divided between the two gates. A score of 0 indicates perfect grouping (all agents of same color pass through same gate). A score of 1 indicates perfect lack of grouping (the agents are evenly split between the two groups). The final result of each wave is the average entropy value across the two colors.

Figure 10 shows the average entropy value for each wave, for the ten waves. The results are averaged over 25 trials. The X-axis shows the wave number (1-10). The Y-axis measures the entropy. The figure shows that the entropy value of the social-comparison model quickly goes down from 1 and approaches 0, while it remains around 1 for the individual-choice model. Indeed, after 10 waves, the average entropy value for the social comparison model is 0.131, while it is 0.992 for the individual-choice model.



(c) Final positions, with social-comparison model.

Fig. 9 Screen shots, grouped pedestrians' movement around the obstacle



Fig. 10 Entropy of grouped pedestrians' movement around the obstacle

5 Validation against human data

In this set of experiments, we compare simulated behavior generated by crowd modeling algorithms, to human pedestrian behavior. Unlike in previous experiments where we concentrated on specific phenomena like lane changes or movement in groups, in this experiment we focus on much general pedestrian behavior where groups such as family and friends, couples and individuals are walking with different speeds in bidirectional fashion. Our hypothesis is that generating pedestrian behavior with SCT model is more in tune with human pedestrian behavior, compared to a base model from the literature. We also want to examine the impact of the SCT model components (SCT bounds, correction order and gain function) on the quality of the simulated behavior.



Fig. 11 Human pedestrian behavior

There does not exist a standard methodology of evaluation; some researchers generate accurate behavioral data by engaging crowds in virtual environments (Pelechano et al. 2008), while others do qualitative comparisons of their models' predictions against movies of crowds, i.e., via observation experiments (e.g., Helbing et al. 2000; Kretz 2007). We follow the same approach. Below, we describe the observation experiment we executed to evaluate the SCT model on general pedestrian behavior.

5.1 Comparing generating pedestrians behavior of SCT model to human pedestrian behavior

We used human crowd movies where different pedestrian behavior phenomena are presented (Fig. 11) and created screen-capture movies of different models of the same behavior (Fig. 12). We rely on experiments with human subjects which compare each of the resulting simulated behaviors to human crowd behavior. In addition, the subjects also voted for the most similar and dissimilar simulated behavior.

5.1.1 Comparison to human crowd

In order to compare to general behavior and not to be connected to specific video clip, we used several video clips of human pedestrian behavior and several screen-captured movies for each model. In the simulated behavior we created three screen-captured movies for each model that was randomly chosen for each subject. In human behavior we used two sets of video clips that were taken from different locations and in different times. The first set of movie clips were taken in the morning in downtown Vancouver, during rush hour. People are mostly walking individually, and only few are moving in small groups. The second set of movie clips were taken in the afternoon in a street that leads to the Eiffel tower in Paris, during leisure time. Most of the pedestrians are moving in small groups with observable mutual relations among them expressed in body language. Each real-world video clip was cut to be one minute long. To generate a one-minute clip in the simulated behaviors, each model was executed for 5000 cycles (6 minutes), and the last minute was used.

We designed a web-based experiment in which subjects could participate in their free time. First we presented a brief description of the experiment. The subjects were told that the purpose of the experiment is to compare each of the simulated behaviors



Fig. 12 Simulated pedestrian behavior

to human crowd behavior. However, the purpose of the simulation is not to simulate each seen pedestrian in the human crowd, but to simulate the general pedestrian behavior. The experiment was carried out in two phases, a training phase that was presented to the subjects after the experiment description, and an experiment phase.

The experiment was carried out using 39 adult subjects (males: 28). Additional 6 subjects were dropped due to technical reasons (such as network problems that prevented them from watching all clips). The subjects were asked to watch the human pedestrian movie that was randomly chosen in each experiment. Then, they were ask to watch screen-captured movie of each model that was also chosen randomly. After each simulated movie, the subjects were ask to rank the seen behavior, by answering the question: To what degree is the seen simulated behavior similar to the previously seen human behavior? (1—not similar, 6—most similar). Subjects could go back and forward, revising their answers until they felt comfortable with them. The order of presentation changed randomly to control for order effects. At the end of the experi-

Component	SCT-B-2-6.5	SCT-B-5-6.5	SCT-H-L	SCT-NoGain
Smax	6.5	6.5	6.5	6.5
Smin	2	5	2	2
Gain	Eq. 2 (func.)	Eq. 2 (func.)	Eq. 2 (func.)	1 (const)
Correction order	L-H	L-H	H-L	H-L
Component	SCT-G-C2	SCT-G-C3	SCT-G-C4.5	
Smax	6.5	6.5	6.5	
Smin	2	2	2	
Gain	2 (const)	3 (const)	4.5 (const)	
Correction order	L-H	L-H	L-H	

 Table 2
 SCT models

ment, we asked the subjects additional two questions: What simulated movie was the most similar to human behavior and what simulated movie was the most dissimilar.

We wanted to examine the impact of the SCT model components on the quality of the simulated pedestrian behavior. In particular, we wanted to examine the impact of SCT bounds (S_{min} and S_{max}), gain function, and correction order on the generated behavior. We define seven models, each emphasizing a different SCT component. The models are explained below, and summarized in Table 2.

First we wanted to examine the impact of SCT bounds on the generated pedestrian behavior. We hypothesize that more narrow bounds will provide more similar behavior to individual model. To examine this hypothesis, we define the following models:

- SCT-B-2-6.5 We set S_{max} to 6.5 (practically: no agent too similar) and S_{min} to 2 (which means that agents that differ only in distance and in X-axis are not consider similar). The gain is calculated according to (2) and the correction order is from the low weight features (distance) to high weigh features. In this domain agents cannot change their color, thus, the last corrected feature is direction. Our hypothesis that this model will provide most similar behavior to human pedestrians.
- SCT-B-5-6.5 We set the S_{min} to 5 which mean that agents that similar at least in color and direction are consider to be similar. Thus, in this model only agents with same color and direction will move together.

Another component that we want to examine is the impact of correction order on simulated pedestrian behavior. In the SCT-H-L model we define the correction order to be from high to low. Our agents cannot change their colors, and in this model if the selected agent is moving in opposite direction, the agent will first change it direction and then will try to close the distance gap.

Finally, we wanted to evaluate the importance of the gain in the model. We define the following models:

- SCT-NoGain. is SCT without the gain function (i.e., gain is constant 1).
- SCT-G-C2. The gain function is constant (2).
- SCT-G-C3. The gain function is constant (3).



- SCT-G-C4.5. The gain function is constant (4.5).

Initially we wanted to compare eight different simulated behaviors to human pedestrian behavior, the individual choice model and seven SCT models. We run a short pilot in which we presented to three subjects the experiment and afterwards ask their opinion. All subjects claimed that the experiment was too long. Moreover, they claimed that SCT-B-2-6.5 model provide very similar behavior to that of SCT-H-L model, and also that models SCT-NoGain, SCT-G-C2, SCT-G-C3 and SCT-G-C4.5 produced similar behavior. Thus, we reduced the number of different models that presented to the subjects. In the experiment phase we compared between four simulated behaviors. We used the Individual-choice model, SCT-B-2-6.5, SCT-B-5-6.5 and one of randomly chosen SCT-NoGain, SCT-G-C3 and SCT-G-C4.5 models. The models SCT-H-L and SCT-G-C2 were used only in the training phase, and their results were not used.

5.2 Results

We first wanted to examine the ranking of the models in comparison to the actual crowd. The results are summarized in Fig. 13. The categories in the X-axis correspond to different models. The Y-axis correspond to grades of the compared models. Each set of bar shows the mean and median results. A higher result indicates improved fidelity, i.e., greater similarity to human pedestrian behavior.

The results clearly demonstrate that the SCT-B-2-6.5 model provide higher results than the compared models. While it may seem that the SCT-B-2-6.5 model results is close to Individual and SCT-B-5-6.5 models results, according to a *t*-test (two-tailed) SCT-B-2-6.5 was found to be significantly different than the Individual model (p = 0.001) and significantly different than SCT-B-5-6.5 (p = 0.03).

Another hypothesis underlying the experiment is that SCT model with narrower bounds (S_{min} , S_{max}) will provide closer behavior to individual model behavior, but not the same. Indeed, the results demonstrate that SCT-B-5-6.5 lies in between the SCT-B-2-6.5 and individual models. According to *t*-test (two-tailed) SCT-B-5-6.5 was found to be significantly different than SCT-B-2-6.5 (p = 0.03) and significantly different than the Individual model (p = 0.017).

Our last hypothesis was that SCT models without the gain function will provide less similar behavior to human pedestrian behavior. The results clearly demonstrates that SCT-NoGain, SCT-G-C3 and SCT-G-C4.5 models in which the gain is fixed, get the lowest results.



When we ask the subjects: "What simulated behavior was the most similar to human behavior?" The SCT-B-2-6.5 model gets the highest number of votes. To the question: "What simulated behavior was the most dissimilar to human behavior?", the subjects answered with the SCT-NoGain, SCT-G-C3 and SCT-G-C4.5 models. The answers to these two questions are shown in Fig. 14.

6 Discussion

The SCT model, described and evaluated above, stands on two conceptual cognitive science legs. First, it draws a connection between social comparison theory and crowd behavior. Second, it interprets social comparison theory as admitting superficial comparisons, i.e., at the level of visible differences between agents, in addition to cognitive differences (e.g., intentions). We address these two issues below.

Social comparison in crowds To the best of our knowledge, social comparison theory has never been connected to crowd behavior phenomena. However, we believe that social comparison theory may account for some important characteristics of crowd behavior, as it clearly addresses processes in groups, and no limit is placed on group size.

We focus here on one of the primary characteristics of crowds is the similarity between individuals' behaviors. This is explained by a process of *imitation* (Le Bon 1895), convergence of like-minded individuals (Allport 1924), or emerging norms (Turner and Killian 1972).

Social comparison processes can give rise to this phenomenon. Festinger writes (Festinger 1954, p. 124): "The existence of a discrepancy in a group with respect to opinions or abilities will lead to action on the part of members of that group to reduce the discrepancy". Indeed, one implication of SCT is the formation of homogeneous groups. Festinger notes (Festinger 1954, p. 135): "The drive for self evaluation is a force acting on persons to belong to groups, to associate with others. People, then, tend to move into groups which, in their judgment, hold opinions which agree with their own". This quote, in particular, seems to be compatible with Allport (1924).

Do people engage in surface comparisons? Festinger hypothesizes (Festinger 1954, p. 117): "There exists, in the human organism, a drive to evaluate his opinions and

his abilities". Thus a question that emerges with respect to the mechanisms described here is whether in fact the type of surface comparisons are admitted by social comparison theory.

There has been extensive research clarifying the concepts "abilities" and "opinions". Smith and Arnkelsson (2000) explain that ability evaluation refers to person performance at specific task. Festinger himself provided a link between ability and performance: "abilities are of course manifested only through performance which is assumed to depend upon the particular ability" (Festinger 1954, p. 118). He then provide an example: "Thus, if a person evaluates his running ability, he will do so by comparing his time to run some distance with the times that other persons have taken." (Festinger 1954, p. 118).

Moreover, the meaning of opinion comparison, was also extensively investigated during the years since the publication of Festinger (1954). Goethals and Darley (1977) relate this concept to "Related Attributes Hypothesis" meaning people will prefer to compare with others similar to them on attributes that are related to their opinion or performance. Festinger provide the basis for this research claiming: "If persons who are divergent from one's own opinion or ability are perceived as different from oneself on attributes consistent with the divergent, the tendency to narrow the range of comparability becomes stronger" (Festinger 1954, p. 133). Goethals and Klein provide an example which directly admit surface comparisons: "An individual evaluating his or her tennis-playing ability. He or she might compare with others who are about the same age, who have the same degree of recent practice and comparable equipment, and who are the same sex" (Goethals and Klein 2000, p. 25).

There is much evidence that people perform surface comparison in their everyday tasks even when they are walking in the street. For example, people use SCT whether to decide to return a lost wallet (Hornstein et al. 1968). Here is another example: A well-known experiment in social sciences was performed by Milgram et al. (1969). The experiment involves one participant who stood in the middle of a busy street and stared into an empty spot in the sky. The experiments purpose was to examine group pressure. The results showed that when there was only one participant, there were only a few people that passed and briefly glanced up. However, when there were several participants, almost 80 percent of the passers by also stopped and stared into the sky. It thus seems that the application of social comparison theory to explaining crowd behavior is justified.

7 Summary and future work

This paper presented a model proscribing crowd behavior, inspired by Festinger's social comparison theory (Festinger 1954). The model intuitively matches many of the characteristic observations made of human crowd behavior, and was shown to cover several distinct pedestrian crowd phenomena. Moreover, we presented validation of SCT model (and competing models) against human crowd behavior. We evaluate the SCT on pedestrian phenomena and showed that SCT model generated pedestrian behavior more in tune to human pedestrian behavior. We are currently exploring the use of SCT in this and other domains. In our future work we plan to provide quantitative validation of the SCT model against human data by using machine vision algorithms for data analysis. These can be used to provide different quantitative metrics calculated from the video stream such as group formation, density or crowd flow. We plan to compare the metrics received from human crowd video to the ones received from simulated behavior. It will also be interested to consider comparisons against additional models than the base-line (Pelechano et al. 2008).

Moreover, we plan to extend the SCT model to include repelling forces. Thus, each agent should not only be attracted to the similar but also should avoid the dissimilar. We also plan to explore the expression of leadership in social comparison theory, and expand our model to simulate collective behaviors with the influence of leaders. Our main goal that with SCT model we will be able to simulate crowd behaviors with and without leadership influence.

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