Schopenhauer against Hegel or On the Nature of Mass

L.P.Horwitz[a,b] and *Ari Belenkiy*[b]

a) Raymond and Beverly Sackler Faculty of Exact Sciences, School of Physics, Tel Aviv University, Ramat Aviv 69978, Israel
b) Physics Department, Bar-Ilan University, Ramat Gan 52900, Israel

To Royal Danish Academy of Science and Philosophy

Abstract. Inspired by ideas of the new physics of 20th century we try to answer to Schopenhauer's critique of Hegel's several seemingly confused statements on the nature of mass.

We discuss a mass change that has its origin in the action of forces on an object. This phenomenon, well-known in the context of quantum field theory (mass renormalization), has been put into a systematic framework in both Classical and Quantum mechanics by Stueckelberg. We employ this framework to resolve the conflict of opinions between Schopenhauer and Hegel.

We show that Hegel, Kant and Schopenhauer demonstrated remarkable prescience in their views as seen from the perspective of ideas of 20th century physics.

It is not difficult to repeat what everyone says now. The challenge is to say what everyone will say in 30 years. (Arthur Schopenhauer)

Email: larry@post.tau.ac.il
belenkiy@albert.ph.biu.ac.il

1. Introduction

In the Introduction to his work "On the Two Major Problems of Ethics" and infuriated by rejection of his essay "On the Foundation of Morality" by Danish Academy of Science with a remark that "the author of the essay speaks negatively about "summus philosophus", Schopenhauer attempts to prove that Hegel understands little in sciences.

For this task Schopenhauer scrutinized a few statements in the "Foundation of Sciences", which he called, the "Bible of Hegelianism". In the chapter on Physics, paragraph 293, Hegel criticized "porosity" as the physical reality behind the notion of weight. He considered the weight of a body as an indication of its mass (specific gravity) and dismissed, as a consequence, the nature and structure of the material. As an example he described the behavior of a metal bar, which leaves the state of equilibrium when magnetized. Hegel's exact words were:

An example of the existent specification of gravity is furnished by the following phenomenon: when an iron bar, evenly balanced on its fulcrum, is magnetized, it loses its equilibrium and shows itself to be heavier on one pole than at the other. Here the one part of the magnet becomes heavier without changing its volume; therefore matter, without increase in mass, has became specifically heavier.

Schopenhauer pointed out what seemed to be a contradiction in Hegel's reasoning, a somewhat awkward logic. As he sarcastically remarked, *from two positive predicates in the second figure one cannot make any conclusions.* That is, one cannot necessarily conclude the inverse of implication.

Another of Hegel's statements (ibid, paragraph 269) was that *gravity immediately contradicts the inertia law because, as a consequence of gravity, matter is trying to exit from itself and enter another matter.*

Schopenhauer treated this as blatant misunderstanding of the causality law. According to him, both gravity and inertia are examples of that law and if gravity contradicts inertia it also contradicts causality so we have *action without a reason.*

The same harsh treatment was given to the phrase from paragraph 298 where Hegel says that *a common error is a belief that matter is positive, absolute, eternal.*

Schopenhauer indignantly replied that *matter is not appearing or dying like everything else, but indestructible and non-appearing; it always keeps its existence and its quality cannot increase or decrease. This a priori knowledge is as firm and doubtless as every mathematical truth. We, no doubt, are unable of imagining appearance and death of the matter: this is not allowed by the form of our mind. To reject this and announce it as an error means to betray your own mind.*

In an Introduction to the second edition of his work, written 20 years later, Schopenhauer concludes:

All of them [philosophers of Hegel's school] strongly fell in public opinion; Hegel, particularly, is quickly going toward the derision awaiting him at posterity. For the last 20 years opinion about him reached three quarters of the level which 20 years ago caused the Royal Danish Academy **tam justum et gravem offensionem**.

2. Hegel on Gravitation

We know that Schopenhauer's last statement did not become true. Let us see why.

Describing the experiment with a bar Hegel suggested two different explanations. First, he said that Kant would suggest that magnetic influence could change the configuration between atoms:

Kant has already opposed "intensity" to the quantitative determination of amount and instead of explaining the different densities of bodies which occupy the same volume by assuming that the heavier body contains more particles, he has assumed that in the heavier body the same number of particles fill the space to a greater degree: in this way, he founded a so-called "dynamic physics". The determination of an intensive quantum would, at least, be just as correct as that of an extensive quantum.

As we see, Kant suggested an explanation, related to the topology of the atomic configuration (see our discussion on Kant below), whereas Hegel argued that the phenomenon can be explained by two other ways: a) "extensive" (more particles) and b) "intensive" (particles become heavier). Moreover, Hegel tried to address a question of fundamental importance, the mechanism by which a particle becomes "heavier“, attributing this effect to what might be called, in the language of contemporary physics, *dynamical interaction.* This appears to be an attempt to understand relations between forces of [electro-]magnetism and gravity. Attempts to unify forces have dominated the particle physics of the 20th century. [1]

The major point of the discussion was to understand the nature of mass. [2] We will see that approaches of all three philosophers have their representatives in the thought of the 20th century. We will also show that the two other questions raised by Schopenhauer could be answered in the framework of new 20th century ideas.

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[1] The first attempt to integrate gravity and electromagnetism was made in the 1920s by the German physicist Kaluza [Kaluza 1921] and the Swedish mathematician Klein [Klein 1926]. It required a 5-dimensional space-time. Their attempt was not successful but modern string theory (see, e.g., [Greene 1999]) has made important use of their basic idea.

[2] For a comprehensive discussion of the historical development of the concept of mass see Jammer [1999].

3. Hegel's Argument in Light of Classical Physics

Hegel observes that a magnetized rod turns, with one pole toward the Earth and the other away. We would understand this in terms of classical physics as the action of a small vertical component of the earth's magnetic field. Hegel sees this, however, as an increase in weight of the north end of the bar and a decrease in mass in the south end.

Example 5.1 (below) shows that there can be assigned to the two ends of the bar a different mass due to the work done by the forces of magnetization. The motion of the bar in the magnetic field is quite analogous to its motion in the gravitational field. [3] The magnetic force on a magnetic pole is of very different (larger) magnitude from the gravitational force on a given mass, but the qualitative idea that Hegel arrived at is affirmed by Example 5.1. Moreover, the " equivalence principle" implies that the electromagnetic field itself falls freely in a gravitational field, [4] and thus appears as a component of the gravitational mass (indistinguishable from inertial mass).

F. Wilczek [Wilczek 2000], in his article in "Physics Today", has emphasized the idea that mass is associated with the state of a system, resulting from the effectiveness of the interaction between its constituents. In the case of magnetism, through interaction between electron spins, as in Wilczek's case, the interaction between particles is mediated by fields. He quotes in this article Wheeler's idea, as deducing mass as a secondary property, i.e., in addition to some intrinsic "non-interacting" mass (which might in fact be zero).

In 1924, in a classic of the relativity theory, Max Born [Born 1962, p. 286] wrote:

> *"Matter has two fundamental qualities: inertia, measured by its mass, and the capability of performing work, measured by its energy. These two are strictly proportional to one another. Whenever electric or magnetic fields or other effects lead to intense accumulation of energy, they are accompanied by inertia. Electrons and atoms are examples of extreme accumulation of energy."*

Born [p. 287] further proceeds with a concrete example. He considers three tiny particles: mesons. Two of them: π_+ and π_-, charged mesons, have mass $273m_e$ while the neutral π_0 meson has mass $264m_e$. The difference, according to Born, is due to carrying electric charge, or due to electric energy.

4. Schopenhauer vs. Hegel

Schopenhauer criticized Hegel's analysis, interpreting the response of a magnetized rod as a mass change. He argued that it is inconceivable that the mass could change, that it is

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[3] In this approximation both are constant forces, obtained from a linear potential.

[4] See A. Harpaz and N. Soker, "Radiation From an Electric Charge", be published in *Foundations of Physics*.

as rigorous as a mathematical theorem that mass is fixed. This view was, in fact, held by E. Wigner [1939], one of the giants of modern physics, in his seminal paper on the Poincaré group. The Poincaré group is considered to describe the fundamental symmetry of the motion of a closed total system (e.g., a free particle, or an N-body system with no external forces), including both the translation symmetry in space-time characteristic of a net force free environment as well as the Lorentz transformations characteristic of Einstein's special relativity. Although in the non-relativistic world of Galileo and Newton the mass of an isolated object is indeed constrained mathematically [Mukunda and Sudarshan 1974](thus realizing Schopenhauer's assertion), the Poincaré group has a fundamentally different structure which admits a continuum of masses. Wigner nevertheless asserted, as Schopenhauer, that "an elementary particle is an irreducible representation of the Poincaré group". Different masses are associated with different representations, and not considered, in this view, as a dynamical variable. As we shall see in our examples, the achievement of manifest Lorentz covariance in the theory results in a smooth changes of mass due to the action of forces, [5] and hence we are *not* dealing with an irreducible representation of the Poincaré group.

Schopenhauer, moreover, argues that the non-conservation of mass would correspond to a violation of causality. Hegel, on the other hand, permits mass "to exit from itself." We see striking evidence of this phenomena, according to some interpretations, in the dynamics of a double star, where the frequency of rotation of the system changes due to the conjectured emission of gravitational waves, carrying mass from one star to another (see, for example, [Kates 1980]). A more well-established and striking example of the conversion of mass is in nuclear fission, where mass is directly converted to radiation energy. Schopenhauer evidently had in mind the second law of Newton, relating the causal time evolution of a system to the force acting on it by a coefficient called "the inertial mass". The dynamical evolution generated in Newton's mechanics has reached a pinnacle of elegance in the formulation of Hamilton and Lagrange [6] where one clearly sees the causal nature of mechanics through the development implied by first order differential equations in time (the concept of *initial conditions* sufficing for the establishment of the motion for all future time is the essence of causality). One sees, with Schopenhauer, that the mass plays a fundamental role in the structure of this causal theory.

Our examples show, and it is a quite general property of Stueckelberg's [1941] manifestly covariant form of mechanics (which is explained below) that the same Hamiltonian structure holds in a relativistic theory in which the mass is variable and depends on the state of the system. The absolute causality of this theory holds in evolution of the universal time τ rather than in t, the time of an event observed in the laboratory, subject to dynamical variations which render it, in many circumstances, useless as a parameter of evolution. An extreme example was given by Stueckelberg [Stueckelberg 1941] as the starting point of his thinking on this subject. The possibility of a curvature of the world-line, the space-time evolution

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[5] The effective mass changes induced by motion in Special Relativity depend on velocity and are not relevant here.

[6] See, e.g., [Gantmacher 1975].

of a particle, inducing an eventual *backward* evolution in t (as a function of increasing τ) invalidates the use of t as a parameter of the motion; there is no well-defined function $\mathbf{x}(t)$ in this case (see Fig. 1).

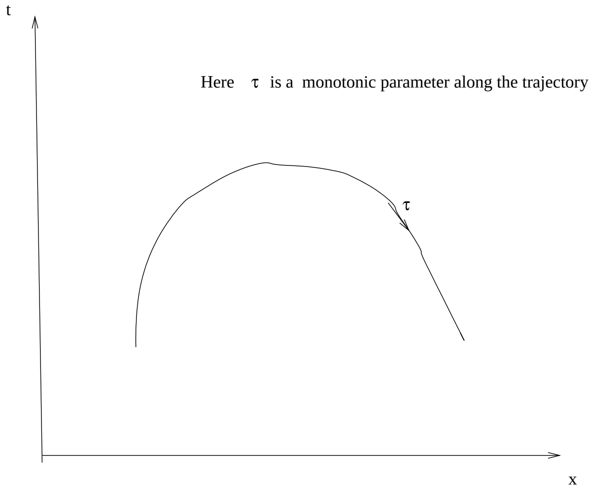

Figure 1. Change of time direction with respect to τ.

The curving world-line is interpreted as particle-antiparticle annihilation (one sees the forward and backward going branches at the same time t in the laboratory).

In the statistical theory of ensembles of relativistic systems, it is possible to show that a Boltzmann type H-theorem holds (see [Horwitz et al. 1984]), where in analogy to the result that the entropy of an adiabatically isolated system increases monotonically in t in non-relativistic statistical mechanics, the relativistic measure of entropy increases as a function of τ, maintaining the notion of causality (through the second law of thermodynamics), in the framework of a theory in which mass is dynamically determined, as well. To the extent that Boltzmann's theory establishes an *arrow of time* [7] the relativistic transport theory establishes an *arrow of* τ.

5. Our Variant of the Dispute and an Example.

As we see this explanation requires the notion of equivalence of energy and mass, an external notion which Hegel did not use explicitly. Besides, this explanation does not say of what "matter exits from itself" means. If we want to stay with the notion of mass and

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[7] See Popper's discussion on Boltzmann's ideas in [Popper 1974, p. 156-7]. The Zermelo criticism based on Poincaré cycles is not relevant since there are unbounded degrees of freedom in the thermodynamic limit, but Boltzmann's demonstrations are far from being rigorous in any case. The global problem has not yet been resolved.

find an explanation of the above expression then we have to consider a theory other than the classical Galilean theory. We shall study a relativistic dynamics, which does take into account the equivalence of mass and energy.

Such a theory comes from the work, as mentioned above, of the Swiss physicist E.C.G. Stueckelberg [Stueckelberg 1941] who postulated that the spacetime motion of a charged particle can develop in a way that can be interpreted as "pair annihilation". This phenomenon emerges when the forces are such that the space-time trajectory turns backward in time; the segment of the trajectory running backward in time is interpreted as the antiparticle. [8] A trajectory of this type can no longer be parametrized by t (it is not well-defined function) and Stueckelberg therefore introduced an invariant parameter τ. In terms of this parameter of evolution (actually corresponding to the Newtonian time, playing here its essential role in relativity), he wrote Hamilton equations for motion and a generalized Schrödinger type of equation with τ as the evolution parameter.

The Hamiltonian of the theory is written in manifestly covariant form. This structure is distinguished by the fact that, along with the position of an event $\mathbf{x}$, the time of its detection t is considered to be an observable, i.e. a dynamical variable. As a consequence, its canonical conjugate E, the energy, is also a dynamical variable independent of $\mathbf{p}$, and hence the particle is not restricted to a definite *a priori* mass, according to Einstein's relation between energy, mass and momentum (as explained in Born [p. 294]):

$$m = \frac{1}{c}\sqrt{\frac{E^2}{c^2} - \mathbf{p}^2}.$$

This quantity is measurable, and reduces in the non-relativistic limit to the Newtonian (fixed) mass of the particle. The quantity m is considered as a dynamical variable subject to change due to forces acting on the system.

The Hamiltonian, the Lorentz invariant generator of evolution in τ, has the form (for potential models)

$$K = \frac{p^\mu p_\mu}{2M} + V(x),$$

where $p^\mu = (E/c, \mathbf{p})$, $\mu = 0,1,2,3$ (the time component has index 0 and we use the signature -,+,+,+), M is a quantity intrinsic to the particle with dimension of mass, and

$$p^\mu p_\mu = -(E/c)^2 + \mathbf{p}^2 = -m^2c^2$$

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[8] A notion introduced again in 1948 by R.P. Feynman [Feynman 1948] in his perturbative construction of quantum field theory. In [Feynman 1950] he used the invariant parametrization to construct a path integral for a covariant quantum theory of Stueckelberg's form. J.S. Schwinger [Schwinger 1951] arrived at a similar formulation (called "proper time method") by an ingenious construction of the Green's function for standard quantum field theory.

is proportional to the square of the measured mass. Since K is absolutely conserved during the motion induced by the Hamilton equations (the dot corresponds to derivative with respect to τ):

$$\dot{x}^\mu = \frac{\partial K}{\partial p_\mu}; \quad \dot{p}^\mu = \frac{\partial K}{\partial x_\mu}$$

we see that

$$p^\mu p_\mu = 2M(K - V(x))$$

is a quantity that varies in space-time through the dependence of V on x.

From the form of K given above and the Hamiltonian equations, we see that $\dot{x}^\mu = p^\mu / M$ and therefore

$$c^2 dt^2 - d\mathbf{x}^2 = -\frac{p^\mu p_\mu}{M^2} d\tau^2 = \frac{m^2}{M^2} d\tau^2.$$

For $m = M$ the interval of the invariant *proper time* [9]

$$ds^2 = c^2 dt^2 - d\mathbf{x}^2$$

is equal to the invariant parameter interval $d\tau^2$. The condition $m = M$ is called the "mass shell" condition.

Example 5.1

We give a simple example, which reflects Hegel's association of the mass with the state of a system, for a single particle falling in a linear (local gravitational type uniform, or electric or magnetic) potential field. Here (see [Burokovsky et al. 1996])

$$K = \frac{p^\mu p_\mu}{2M} + Mgz$$

where $z = x_3$. (We fix the direction of the force along the z-axis and write it, for example, as a uniform gravitational force with constant g of the dimension of acceleration).

The Hamiltonian equations are:

$$\dot{x}^\mu = \frac{\partial K}{\partial p_\mu} = \frac{p^\mu}{M}$$

$$\dot{p}^\mu = \frac{\partial K}{\partial x_\mu},$$

and therefore

$$\dot{p}^0 = 0, \dot{p}^1 = 0, \dot{p}^2 = 0$$

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[9] A Lorentz transformation to the rest frame of the particle puts $d\mathbf{x}^2 = 0$ and hence the invariant ds^2 becomes equal to the interval of time measured in that frame. It therefore is called the "proper time".

and

$$-Mg = \dot{p}^3 = M\ddot{x}^3 = M\ddot{z}.$$

From this we get

$$z = -\frac{1}{2}Mg\tau^2 + z_0 + \dot{z}_0\tau,$$

coinciding with the usual non-relativistic problem, but with τ instead of t. Because $\frac{dt}{d\tau} = \frac{p^0}{Mc}$ and $p^0 = E/c = const$ we have the new relation

$$t = \frac{E}{Mc^2}\tau + t_0.$$

Now from $p^3 = M\dot{z}$ we obtain

$$p^3 = -Mg\tau + M\dot{z}_0.$$

The particle mass is given by

$$m^2 = {p^0}^2 - \mathbf{p^2} = E^2/c^2 - {p^1}^2 - {p^2}^2 - (M\dot{z}_0 - Mg\tau)^2 =$$

$$E^2/c^2 - {p^1}^2 - {p^2}^2 - {p_0^3}^2 - M^2g^2\tau^2 + 2M^2\dot{z}_0 g\tau =$$

$$m_0^2 - M^2g^2(\tau^2 - \frac{2\dot{z}_0\tau}{g}).$$

For $\tau \to 0$

$$m_0 = E^2/c^2 - {p^1}^2 - {p^2}^2 - {p_0^3}^2$$

represents the initial mass of the particle.

The total mass goes to zero when

$$\tau_0^2 - \frac{2z_0\tau_0}{g} = \frac{m_0^2}{M^2g^2}.$$

or at

$$\tau_0 = \frac{\dot{z}_0}{g} + \sqrt{\frac{\dot{z}_0^2}{g^2} + \frac{m_0^2}{M^2g^2}}.$$

We see that the force field, e.g. gravity, removes the mass; the particle trajectory eventually reaches the light cone, a geometrical surface in space-time corresponding to motion on the degenerate hyperbola of zero mass.

To see how each mass is associated with a single sheeted hyperbola, consider the relation

$$E^2/c^2 = \mathbf{p}^2 + m^2c^2,$$

or,

$$-p_\mu p^\mu = m^2c^2 = E^2/c^2 - \mathbf{p}^2 \geq 0,$$

the equation for a hyperbola in the space coordinatized by $(E/c, \mathbf{p})$. For $m \to 0$, this hyperbola degenerates to a cone with apex at the origin. It is this cone that forms a limiting boundary of the Lorentz transformation of a point contained in the interior of the cone (light-like, with physical mass), corresponding to the the energy and momentum of a particle. Since for a relativistic generating function of the motion which has quadratic kinetic term $p^\mu p_\mu/2M$, one has the relation

$$\dot{x}^\mu = \frac{p^\mu}{M},$$

the light cone configuration $p_\mu p^\mu = 0$ then corresponds to

$$dx^\mu dx_\mu = -c^2dt^2 + d\mathbf{x}^2 = 0,$$

so that $|d\mathbf{x}|/dt = c$, a configuration giving rise to the term "light cone", corresponding to motion along a cone in space-time.

In terms of t, the solution obtained above for $z(\tau)$ becomes

$$z(t) = -\frac{1}{2}gM(\frac{Mc^2}{E})^2(t-t_0)^2 + v_0^3(t-t_0) + z_0.$$

If $E = Mc^2$ we get usual formula with M as mass.

6. Hegel and Kant and an Example

As we saw above, Hegel attributed to Kant an idea that

the configuration of a system can have an effect on its mass.

Though this idea cannot be found in Kant explicitly Hegel could deduce it from examination of the Propositions XI and XII of Kant's "Physical Monadology". In *Proposition XI* Kant [p. 63] first notices that

The force of inertia of a body (which is, of course, called its mass) is, however, the sum of the forces of inertia of all the elements of which it is composed.

Later, in *Proposition XII*, Kant [pp. 64-5] gives a description of what looks very much like modern "string theory":

Therefore, to explain the infinite diversity of densities, each specific to a kind of medium, for example, eather, air, water and gold, one would have to indulge an exaggerated passion for conjecture. One would have to fabricate a rash and arbitrary account of the structure itself of the elements - than which nothing is less accessible to the understanding - imagining *it to have the form sometimes of the thinnest bubbles, sometimes of branches and winding coils. For, in this way, matter can be thought of as distended in a wondrous fashion, and an immense space as filled with very little matter. But consider what reasons militate against such views.*

These tiny fibres which are of immeasurable slenderness, or the minute bubbles which, under immeasurably thin skins, contain a vacuum which is, relative to the matter which they contain, enormous, must eventually be ground down by the continuous collision and friction of the bodies. In this way, the minutely ground particles would eventually fill the interstitial empty spaces; and thus the space of the world would everywhere become paralysed by an overwhelming inertia, and all motions would in short time be brought to rest.

The last part of Kant's statement can be associated with the large energies which are created in the breaking of the strings. This idea emerges in modern physics in the topological properties of quantum field theories associated with the identification of the physical vacuum and the states of instantons. This idea also finds its expression in the Stueckelberg's framework in which the mass of a system depends on its state (its configuration and topology).

Example 6.1

Extending Stueckelberg's idea to the description of many-body systems, Horwitz and Piron [Horwitz and Piron 1973] postulated that the parameter τ is universal, as the Newtonian time in the classical mechanics. It is then possible to consider action at a distance by a potential $V(x_1 - x_2)$, where x_1 and x_2 are the events along the two respective world lines at equal τ. As for the time parameter of non-relativistic mechanics, τ establishes the correlation between different points of a mechanical system. One can then write the generator of motion for the two-body system with interaction potentials, as

$$K = \frac{(-\frac{E_1^2}{c^2} + \mathbf{p_1}^2)}{2M_1} + \frac{(-\frac{E_2^2}{c^2} + \mathbf{p_2}^2)}{2M_2} + V(x_1 - x_2),$$

where the spacetime points x_1, x_2 are at the same τ. Let us introduce two pairs of canonical variables. The first pair is a mutual center of gravity

$$X = (M_1 x_1 + M_2 x_2)/(M_1 + M_2)$$

and its momentum

$$P = p_1 + p_2.$$

Another pair of variables are the "relative" (with respect to each other) coordinate

$$x = x_1 - x_2$$

and "relative" momentum

$$p = (M_2 p_1 - M_1 p_2)/(M_1 + M_2).$$

We now see that

$$K = \frac{P^\mu P_\mu}{2M} + \frac{p^\mu p_\mu}{2M_{reduced}} + V(x)$$

where $M = M_1 + M_2$, the so-called "reduced mass" $M_{reduced} = \frac{M_1 M_2}{M_1 + M_2}$, and we assume $V(x)$ to have support for x spacelike (for $-x_0^2 + \mathbf{x}^2 > 0$).

The variable X is *cyclic* (i.e., missing in the Lagrangian) so its conjugate momentum P is conserved. According to the classical Emmy Noether's theorem (see [Arnol'd 1996]) the system has a symmetry, in this case space-translation invariance. Then one can solve for the orbits by solving the dynamical problem posed by

$$K_{reduced} = \frac{p^\mu p_\mu}{2M_{red}} + V(x),$$

since p_μ and x_μ are canonical variables just as p_1^μ, x_1^μ and p_2^μ, x_2^μ.

The total mass-squared for the two body-system, considered as a whole, is

$$M_{total} = \sqrt{\frac{-P^\mu P^\mu}{c^2}} = \sqrt{\frac{2m(K_{reduced} - K)}{c^2}}.$$

Evaluating K approximately at $x_1 - x_2 \to \infty$ spacelike if $V(x)$ vanishes at spacelike ∞,

$$K \cong \frac{p_1^\mu p_{1,\mu}}{2M_1} + \frac{p_2^\mu p_{2,\mu}}{2M_2} \cong -\frac{1}{2}(M_1 + M_2)c^2 = -\frac{1}{2}Mc^2,$$

if we assume that the particles are close to "mass shell" (so that $m_1 \cong M_1, m_2 \cong M_2$). More deeply bound orbits, for which $K_{reduced}$ is more negative, then result in lower observable total mass for the system.

This example shows, as Kant, that the mass of the system depends on the relative configuration of its constituents.

7. Conclusions and Discussions

Although Schopenhauer was critical of Hegel (at the same time admiring Kant) and Hegel disagreed with Kant's views, we have seen that many of the ideas of these thoughtful and creative men (in some ways like the foresight of Democritus in declaring the atomic structure of matter) have emerged as venerable insights into the views developed in the 20th century.

We have seen that Schopenhauer strongly believed that the mass of an object is an invariable property. Since the mass participates in the relation between force and acceleration (defined geometrically in terms of change of position), generating the causal property through differential equations, it appears that Schopenhauer associated mass with causal structure through the Hamilton equations. We have shown, however, that one can construct a Hamiltonian type theory, in the framework of relativistic mechanics, which maintains classical causal properties, but in which the mass is a dynamical variable determined by the state of the system and the forces.

The perception that apparent causal behavior of the world as we see it is associated with Newtonian type equations, which as we know can be embedded in the symplectic mechanics of Hamilton and Lagrange, plays an essential role in the development of modern analytic mechanics, both classical (Galilean) and relativistic. For such a mechanics, initial values causally determine the future evolution of the system. We have seen that there is a relativistic Hamiltonian mechanics in which the mass is variable but is causal. [10] Hegel's assertion that the mass of a system may not be conserved is therefore not in contradiction with causal evolution.

Hegel perceived that the mass of a system may depend on its state, associated with some intrinsic potential energy, such as magnetization of the rod. Although the experiment he discusses has quantitatively little of this effect he, in fact, persists in this line of thought in his assertion, so vociferously rejected by Schopenhauer, that "matter may exit from itself", indicating that mass may be carried out of a body in a form other than identifiable matter. We have pointed out examples in Section 4 where "matter may exit from itself".

It is also well-known that what appears to be the fundamental ingredient of a complex nucleus (proton or neutron), has a different mass when embedded in the nucleus than it does when free (it may vary, moreover, when found in one nucleus or another). The difference in effective mass may be accounted for by energy stored in the field of interaction, but the description of such an object *as a particle* involves consideration of its mass, as a variable depending on the state of the system. This variability of the mass (including the conversion of one type of particle to another, such as a transformation from neutron to proton) may be associated as well, in part, with the conversion of matter to radiation in the examples discussed above.

A rather striking example of this effect may be found in electron-positron annihilation, which appears to be an obvious counterexample to Schopenhauer's assertion that "matter is absolute, non-destructible, eternal". In this case, two massive particles combine and totally disappear with the emergence of pure electromagnetic radiation. From the point of view of Stueckelberg's theory, one understands this process as an evolution of a single particle from *forward-going* in time to *backward-going* in time (flowing monotonically in τ, see fig. 1). The backward-going particle is interpreted as a forward-going particle with opposite charge; in the laboratory one sees two particles with opposite charge approaching each other and

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[10] "Einstein causality", as often referred to in modern works, refers to light velocity limited transmission of information. It plays an important role in quantum field theory (see [Itzykson and Zuber 1980]) but was not involved in Schopenhauer's criticism.

annihilating. During this evolution, the incoming particle (say, electron) must change its mass, since the trajectory must pass through the light cone. This change of mass is carried by the energy emitted in the electromagnetic radiation. It passes through a region AB (fig. 2) where the object is not observable as matter; it has no rest frame in this region accessible by Lorentz transformation.

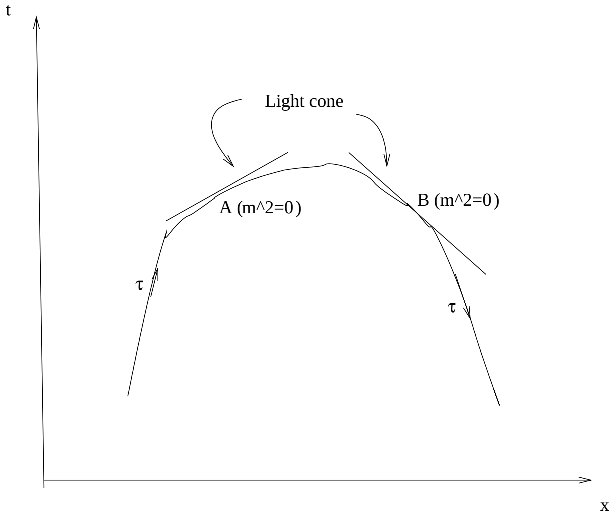

Figure 2. Electron-positron annihilation from Stueckelberg's point of view.

It then crosses the light cone again (at point B) and becomes a particle observable as a positron moving in the direction of positive time. The mass of this object (an event moving in space-time) has varied dramatically in its history. Although this phenomenon appears to involve the utter annihilation of the particle and antiparticle, the picture provided in τ represents the process as the evolution of a single event maintaining its essential identity, in part a realization of Schopenhauer's assertion of conservation, here, the conservation of number of *events*, but not mass. This property is maintained in the quantum theory through conservation of the norm of the wave function, but may not be in the associated quantum field theory, where events may be created and destroyed. But that is another story.

According Hegel, Kant held the idea that a change in mass can be accounted for by a difference in the filling of the space due to a change in configuration. As we have pointed out above, it is remarkable that Kant's view, based on endowing matter with microscopic structure of tiny filaments (or bubbles) in association with configurations that are crucial for the quantity of mass, is qualitively very close to that of modern string theory.

Although it is true that many of the roots of current ideas in science can be found in Greek philosophers: atomism of Democritus [Russell 1945], Plato's idea of symmetry and structure and the underlying notion of energy (fire) of Heraclitus [Heisenberg 1959] - it seems to us that the ideas expressed by Hegel, Schopenhauer and Kant attain a significantly higher level of insight. It was not possible to decide the validity of Hegel's natural philosophy in 1840, and the Danish Academy left this judgement for posterity.

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