Limits on the Power of Indistinguishability Obfuscation and Functional Encryption

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This Talk

A framework for proving impossibility results for commonly-used non-black-box techniques

- Limits on the Power of Indistinguishability Obfuscation
- Limits on the Power of Functional Encryption

Obfuscation

- Makes a program “unintelligible” while preserving its functionality

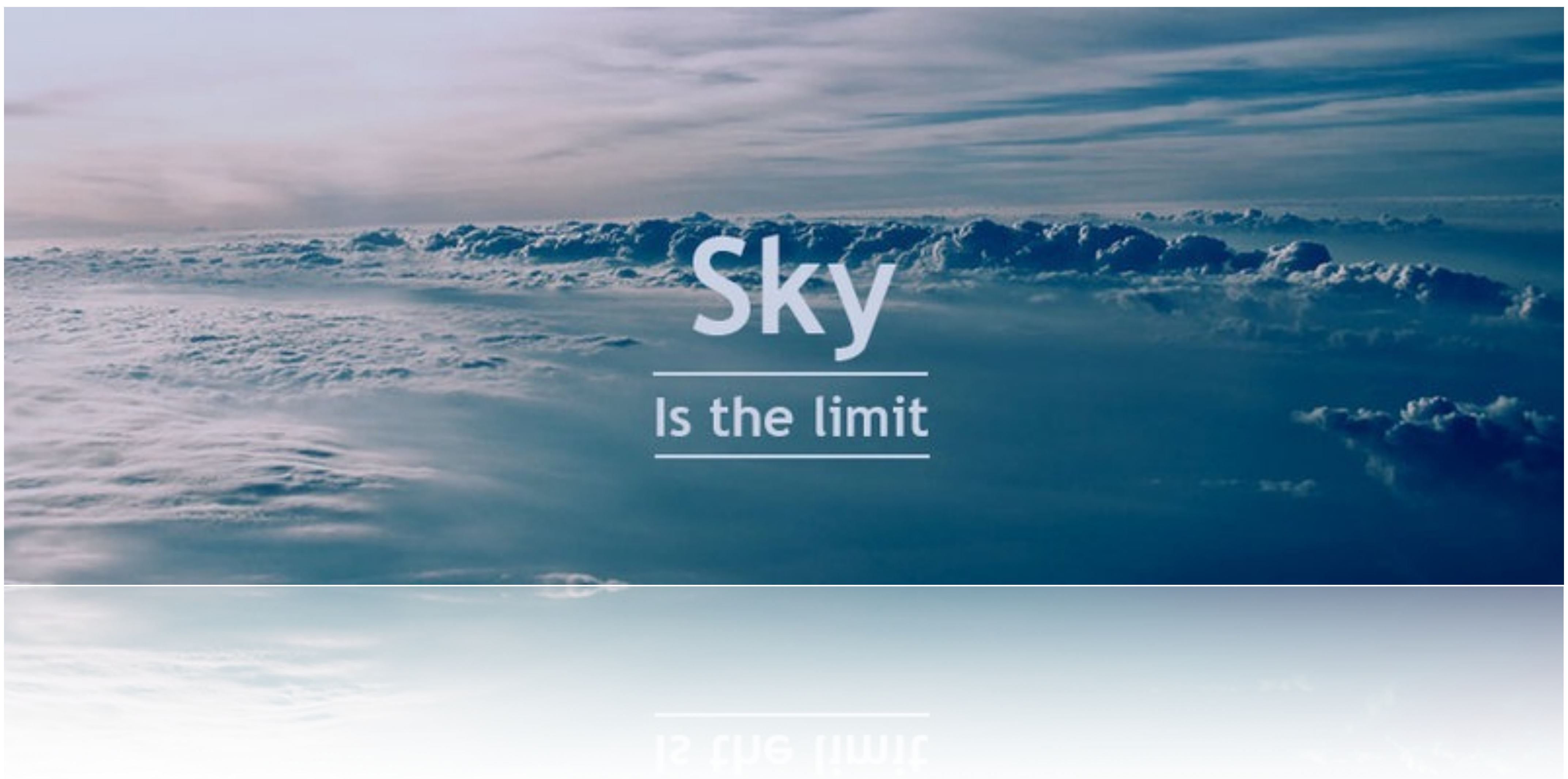
```
for (i=0; i < M.length; i++) {
// Adjust position of clock hands
   var ML=(ns)?document.layers['nsMinutes'+i]:ieMinutes[i].style;
   ML.top=y[i]+HandY+(i*HandHeight)*Math.sin(min)+scrll;
   ML.left=x[i]+HandX+(i*HandWidth)*Math.cos(min);
}
```

```
for(O79=0;O79<l6x.length;O79++){var O63=(l70)?document.layers
["nsM\151\156u\164\145s"+O79]:ieMinutes[O79].style;
O63.top=l61[O79]+O76+(O79*O75)*Math.sin(O51)+l73;
O63.left=l75[O79]+l77+(O79*l76)*Math.cos(O51);}
```

Obfuscation

- [BarakGoldreichImpagliazzoRudichSahaiVadhanYang01] :
  - **Virtual black-box obfuscation (VBB)**
    Obfuscated program reveals no more than a black box implementing the program
    impossible
  - **Indistinguishability obfuscation (iO)**
    Obfuscations of any two functionally-equivalent programs be computationally indistinguishable
    may be **possible**
- [GargGentryHaleviRaykovaSahaiWaters12] :
  A candidate **indistinguishability obfuscator** (iO)

The Power of Indistinguishability Obfuscation



The Power of Indistinguishability Obfuscation

- Public-key encryption, short "hash-and-sign" signatures, CCA-secure public-key encryption, non-interactive zero-knowledge proofs, Injective trapdoor functions, oblivious transfer [SW14]
- Deniable encryption scheme [SW14]
- One-way functions [KMN+14]
- Trapdoor permutations [BPW15]
- Multiparty key exchange [BZ14]
- Efficient traitor tracing [BZ14]
- Full-domain hash without random oracles [HSW14]
- Multi-input functional encryption [GGG+14, AJ15]
- Functional encryption for randomized functionalities [GJK+15]
- Adaptively-secure multiparty computation [GGH+14a, CGP15, DKR15, GP15]
- Communication-efficient secure computation [HW15]
- Adaptively-secure functional encryption [Wat14]
- Polynomially-many hardcore bits for any one-way function [BST14]
- ZAPs and non-interactive witness-indistinguishable proofs [BP15]
- Constant-round zero-knowledge proofs [CLP14]
- Fully-homomorphic encryption [CLT+15]
- Cryptographic hardness for the complexity class PPAD [BPR14]

(Last update: April 2015)

Is there a natural task that **cannot** be solved using indistinguishability obfuscation?

Black-Box Seperations

- The main technique for proving lower bound in cryptography: **Black Box Separations**

- The vast majority of constructions in cryptography are “black box”

“Building a primitive X from

any implementation *of a primitive Y”*

  - The construction and security proof rely only on the input-output behavior of **Y** and of **X**'s adversary
  - The construction ignores the internal structure of **Y**

- **Examples**:
  - PRF from PRG [GGM86], PRG from OWFs [HILL93,99]

Black-Box Separations

- Typically, show impossibility of "X ⇒Y" by:

"There exists an oracle relative to which Y exists but X does not exist"

- **Examples**:
  - No key agreement from OWFs [IR89]
  - No CRHF from OWFs [Sim98]

Our Challenge: Non-Black-Box Constructions

- **Constructions that are based on *iO* or *FE*, almost always have some *non-black-box* ingredient**
- **Typical example**
  From private-key to public-key encryption [SW14] **(simplified)**
  - Private-key scheme: $Enc(K,m) = (r, \mathrm{PRF}(K,r) \oplus m)$
  - Public-key scheme: $SK = K,\ PK = iO(Enc(K,\cdot))$

Non-black-box ingredient:

Need the specific evaluation circuit of the PRF

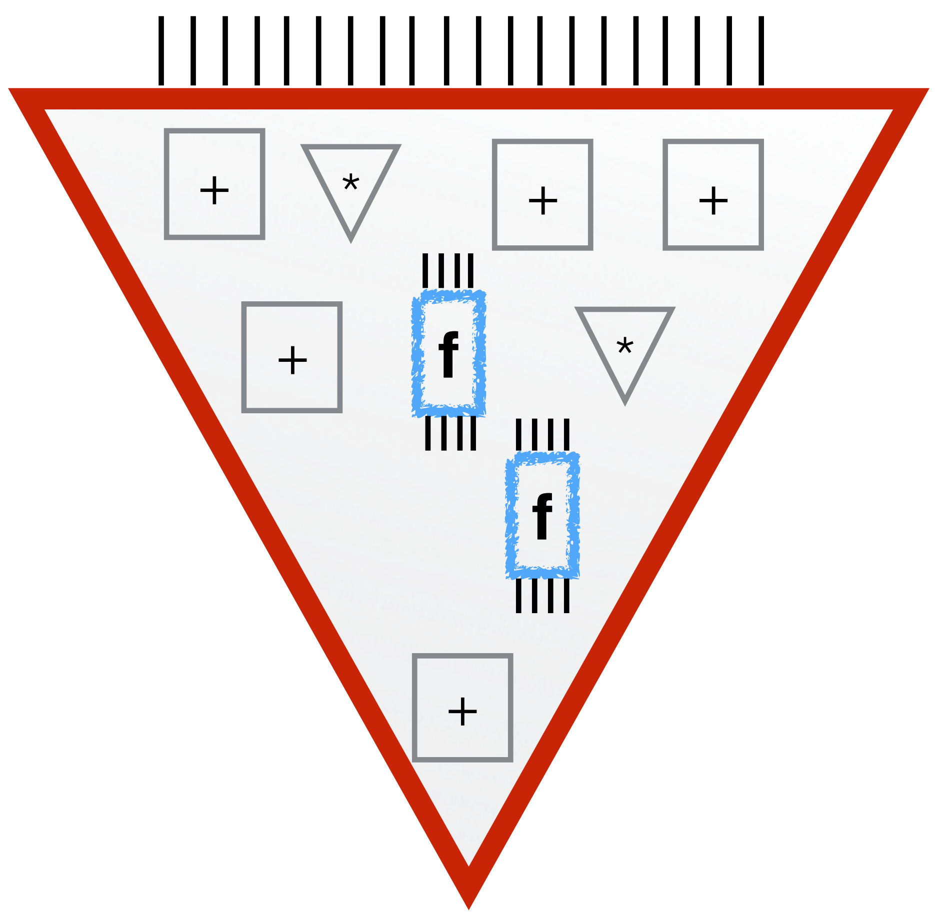
- **How can one reason about such non-black-box techniques?**

Our Solution

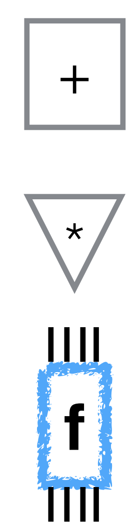
- Overcome this challenge by considering $i\mathcal{O}$ for a richer class of circuits:

oracle-aided circuits

(circuits with oracle gates)



Possible gates:



Our Solution

- Transform **almost all** iO-based constructions from non-black-box to black-box

$$iO(r, \mathrm{PRF}(K, r) \oplus m))$$

$$\Downarrow$$

$$iO(r, C^{OWF}(K, r) \oplus m)$$

(possible due to [GGM86]+[HILL89])

- Constructing iO for **oracle-aided** circuits
  is clearly **harder than**
  constructing iO for standard circuits

- Limits on the power of iO for **oracle-aided** circuits
  clearly **implies**
  limits on the power of iO for standard circuits

$$\text{iO} + \text{TDP} \not\Rrightarrow \text{CRHF}$$

iO+TDP $\not\Rightarrow$ CRHF

- **Theorem:**
  There is no black-box construction of
  a collision-resistant hash function family from
  - a trapdoor permutation **f** and
  - an **indistinguishability obfuscator** for all oracle-aided circuits $\mathbf{C^f}$
- Unless with an **exponential** security loss
  (rules out sub-exponential hardness as well!)
- Also rules out: homomorphic encryption, homomorphic commitment, two-message PIR [IKO05]

Techniques We Don't Capture

- Constructions that use NIZK proofs for languages that are defined relative to a computational primitive

- **NIZK proof** $L = \{(d,r) \mid \exists r \text{ s.t. } d = Enc(i;r)\}$
  - Uses Cook-Levin reduction to SAT
  - Makes use of the circuit for deciding L by representing its computation state as boolean formula - ***non-black-box***

- [BKSY11] seems as a promising approach for extending our framework to capture such constructions

- Other (less common) techniques (so far not used with iO)

Proof Sketch

- Builds upon and generalizes [Sim98,HHRS07]
- We define an oracle Γ such that relative to it:
  1. There exists a **one-way permutation f**
     (for this talk - OWP and not TDP…)
  2. There exists an **indistinguishability obfuscator** for all oracle-aided circuits $\mathbf{C^f}$
  3. There does not exist a **collision-resistant hash function**

The Oracle Γ

The one-way permutation f

$f = \{f_n\}_n$, where each f_n is a uniformly chosen permutation over $\{0,1\}^n$

O and Eval

$O = \{O_n\}_{n \in \mathbb{N}}$, where each O_n is a uniformly chosen permutation over $\{0,1\}^{2n}$

$Eval(\tilde{C}, a)$ with $|\tilde{C}| = |a| = n$

Looks for the unique pair $(C, r) \in \{0,1\}^{2n}$ such that $O_n(C, r) = \tilde{C}$

Returns $C^f(a)$

ColFinder

1) On input C, ColFinder chooses a uniform w, evaluates C(w)
2) Samples a uniform w' such that C(w')=C(w)
3) Returns (w,w')

- **We implement iO as follows:** $\hat{C}(\cdot) = iO(C)$
  - On input oracle-aided circuit **C** (with |C|=n), choose a random **r**
  - Outputs $\tilde{C} = O_n(C, r)$

We Need to Prove

1. f is a **one-way permutation** relative to Γ
2. $i\mathcal{O}$ is an **indistinguishability obfuscator** relative to Γ
3. There is no CRHF relative to Γ (easy)

- **Main difficulty**:
  Both **Eval** and **ColFinder** may carry out an exponential amount of "work"
  - Need to show that it does not help the adversary in inverting f or in breaking $i\mathcal{O}$
  - In [Sim98, HHRS07] there was only **ColFinder**; here we also have **Eval** - we have to deal with two "exp-time" oracles and their interaction
- Details: see the paper

Follow-up Work

- **A**, Gil Segev, "*On Constructing One-Way Permutations from Indistinguishability Obfuscation*". In TCC-2016-A, ePrint 2015/752
- **Theorem:** There are no fully black-box constructions of **a domain-invariant one-way permutation family** (the domain is independent of the underlying primitives - f and iO) from
  - a one-way function **f** and
  - an **indistinguishability obfuscator** for all oracle-aided circuits $\mathbf{C}^{f}$
- Matching positive result: **There exists** a construction of a non-domain-invariant TDP from iO+OWF (Bitansky-Paneth-Wichs, TCC-2016-A)

This Talk

A framework for proving impossibility results for commonly-used non-black-box techniques

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- **Limits on the Power of Functional Encryption**

Private-Key FE $\not\Rightarrow$ Public-Key Crypto

- **Theorem:**
  There is no black-box construction of
  a key-agreement protocol
  with perfect completeness from
  - a one-way permutation f and
  - a **private-key functional encryption** for the class of oracle-aided circuits $\mathcal{C}=\{C^f\}$

- Captures the known constructions [BS15,KSY15,BKS15]

Conclusions

- **Limits on the Power of Indistinguishability Obfuscation**
  - iO $\not\Rightarrow$ CRHF
- **Limits on the Power of Private-Key Functional Encryption**
  - Private-Key FE $\not\Rightarrow$ Key Agreement

Thank You!