Worst-case Analysis for Interactive Evaluation of Boolean Provenance

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Boolean Provenance

Acquisitions

Acquired	Acquiring	Date	
A2Bdone	Zazzer	7/11/2020	a_0
microBarg	Fiffer	1/5/2017	a_1
fPharm	Fiffer	1/2/2016	a_2
Optobest	microBarg	8/8/2015	a_3

Roles

Organization	Role	Member	
A2Bdone	Founder	Usha Koirala	r_0
A2Bdone	Founding member	Pavel Lebedev	r_1
A2Bdone	Founding member	Nana Alvi	r_2
microBarg	Co-founder	Nana Alvi	r_3
microBarg	Co-founder	Gao Yawen	r_4
microBarg	CTO	Amaal Kader	r_5

Education

Alumni	Institute	Year	
Usha Koirala	U. Melbourne	2017	e_0
Pavel Lebedev	U. Melbourne	2017	e_1
Nana Alvi	U. Sau Paolo	2010	e_2
Nana Alvi	U. Melbourne	2017	e_3
Gao Yawen	U. Sau Paolo	2010	e_4
Amaal Kader	U. Cape Town	2005	e_5

Input database

```
SELECT DISTINCT a.Acquired, e.Institute
FROM   Acquisitions AS a, Roles AS r, Education AS e
WHERE  a.Acquired = r.Organization AND
       r.Member = e.Alumni AND a.Date >= 2017.01.01 AND
       r.Role LIKE '%found%' AND e.YEAR <= year(a.Date)
```

Output relation

Acquired	Institute	
A2Bdone	U. Melbourne	$(a_0 \wedge r_0 \wedge e_0) \vee (a_0 \wedge r_1 \wedge e_1) \vee (a_0 \wedge r_2 \wedge e_3)$
A2Bdone	U. Sau Paolo	$(a_0 \wedge r_2 \wedge e_2)$
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```

$$a_0 \wedge r_0 \wedge e_0$$

Output relation

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       r.Role LIKE '%found%' AND e.YEAR <= year(a.Date)
```

$$(a_0 \wedge r_0 \wedge e_0) \vee (a_0 \wedge r_1 \wedge e_1)$$

relation

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Boolean Provenance: Possible Worlds

a_0= False, others=True

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Input database

For any truth valuation val:
an output tuple t evaluates to true iff it appears in the possible world of val

$(a_0 \wedge r_0 \wedge e_0) \vee (a_0 \wedge r_1 \wedge e_1) \vee (a_0 \wedge r_2 \wedge e_3)$=False

Output relation

Acquired	Institute	
~~A2Bdone~~	~~U. Melbourne~~	$(a_0 \wedge r_0 \wedge e_0) \vee (a_0 \wedge r_1 \wedge e_1) \vee (a_0 \wedge r_2 \wedge e_3)$
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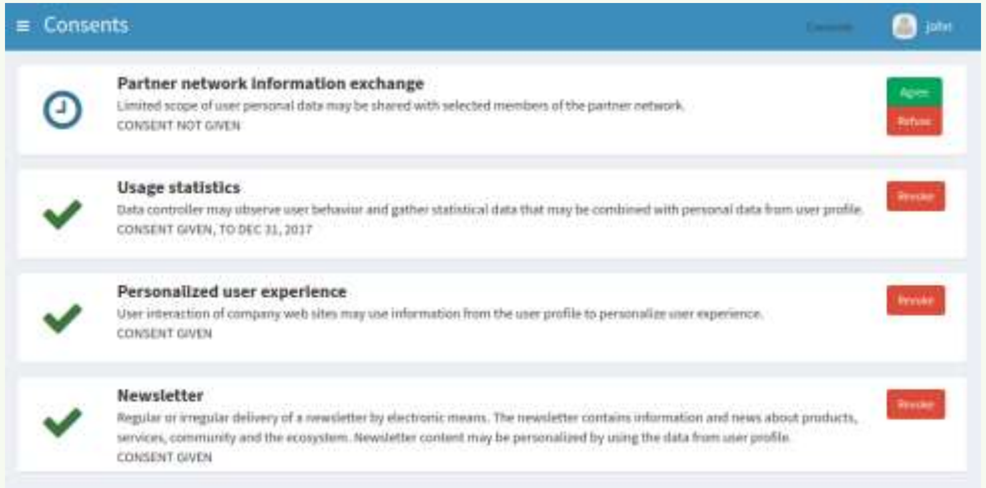
Boolean Provenance: Uses

- Deletion propagation
- Probabilistic databases
- Access control
- Consent Management

Consent Management*



Data owners are probed **on a need basis** for fine-grained consent – per tuple

*Managing Consent for Data Access in Shared Databases [ICDE 2021, Drien, Amarilli, A.]

Consent Management

- We can use the output iff we can derive it from input tuples with consent
- We can choose **which** variables truth values to probe
- Effectiveness depends on the answer and Boolean expressions structure

e_5

Acquired	Institute	
A2Bdone	U. Melbourne	$(a_0 \wedge r_0 \wedge e_0) \vee (a_0 \wedge r_1 \wedge e_1) \vee (a_0 \wedge r_2 \wedge e_3)$
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Example Evaluation

$(a_0 \wedge r_0 \wedge e_0) \vee (a_0 \wedge r_1 \wedge e_1) \vee (a_0 \wedge r_2 \wedge e_3)$

$(a_0 \wedge r_2 \wedge e_2)$

$(a_1 \wedge r_3 \wedge e_3)$

$(a_1 \wedge r_3 \wedge e_2) \vee (a_1 \wedge r_4 \wedge e_4)$

a_0?

False

No need to ask about r_0, e_0, r_1, e_1

False

False

$(a_1 \wedge r_3 \wedge e_3)$

$(a_1 \wedge r_3 \wedge e_2) \vee (a_1 \wedge r_4 \wedge e_4)$

a_1?

True

False

False

$(r_3 \wedge e_3)$

$(r_3 \wedge e_2) \vee (r_4 \wedge e_4)$

r_3?

True

False

False

e_3

$e_2 \vee (r_4 \wedge e_4)$

e_3?

True

…

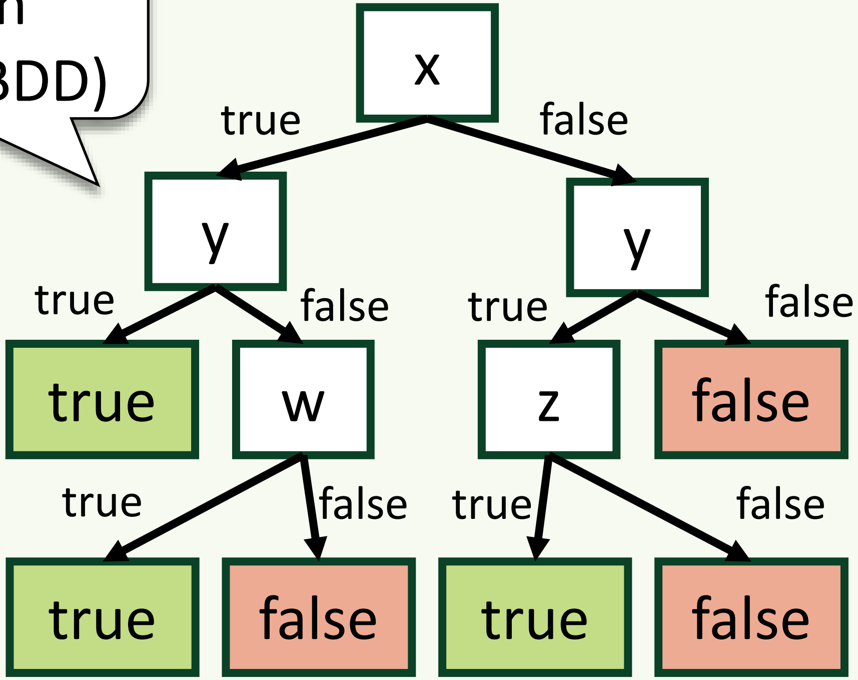
We can use an output tuple iff we can derive it from input tuples with consent

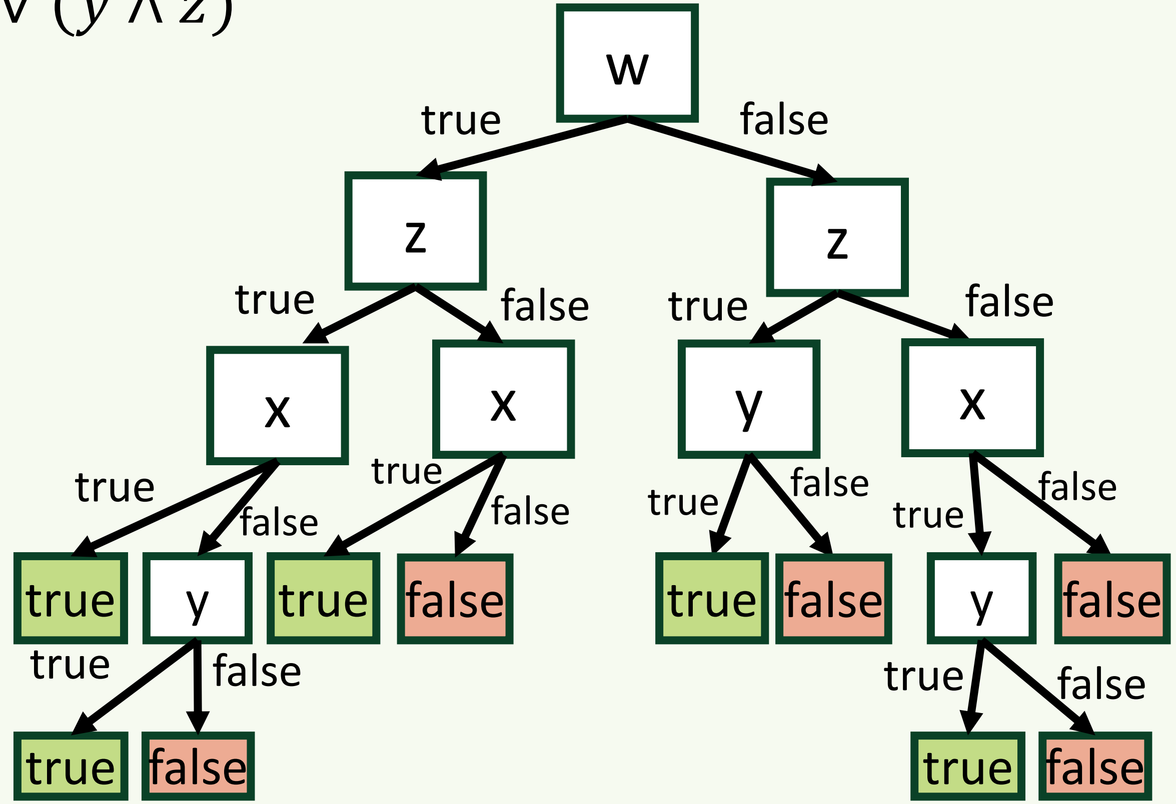
Optimizing the Worst-case Evaluation

- We are interested in a "cautious" probing strategy that minimizes the number of probed variables for any valuation

$$(w \land x) \lor (x \land y) \lor (y \land z)$$

Boolean Decision Diagram (BDD)



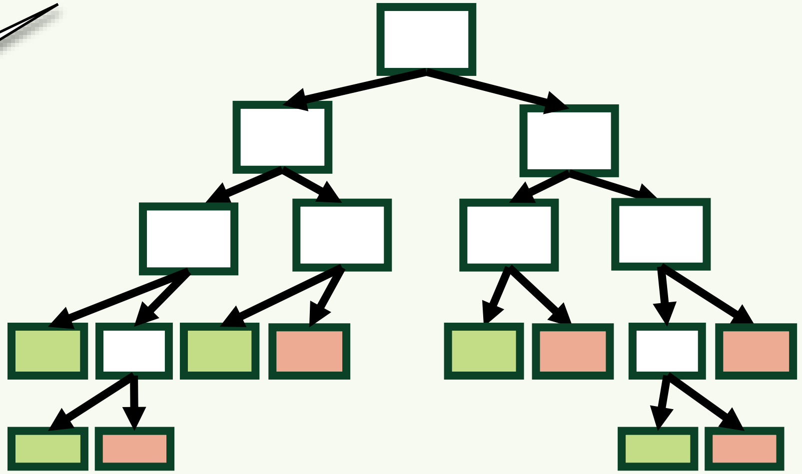


Three Problem Definitions (Intuitive)

Input: a set of Boolean provenance expressions

- **OPT-BDD-DEPTH:** minimize the worst-case number of probes
  - (there is always a trivial strategy that queries all variables in order)
- **DEC-BDD-DEPTH:** decide whether there exists a strategy making at most k probes
- **DEC-BDD-EVASIVE:** decide whether the expressions are **evasive** = no strategy is better than the trivial one
  (making less than n probes over n variables)

Used in Boolean Function Learning



Previous Work

- **Expected depth optimization by testing variables of Boolean formulas**
  - Interactive Boolean Evaluation, Sequential System Testing, Active Learning, Consent management
- **Worst-case BDD Analysis**
  - Graph/ String properties
  - Construction based on input-output pairs
  - Deciding among Boolean functions
- **Other metrics**

Today

1. Model
2. General Provenance Expressions
3. Read-Once Expressions
4. Monotone Expressions

BDDs for Expression Sets

$x \wedge \neg x$

false

φ_0: $x \wedge \neg x \wedge y$

φ_1: False

φ_2: $y \vee \neg y$

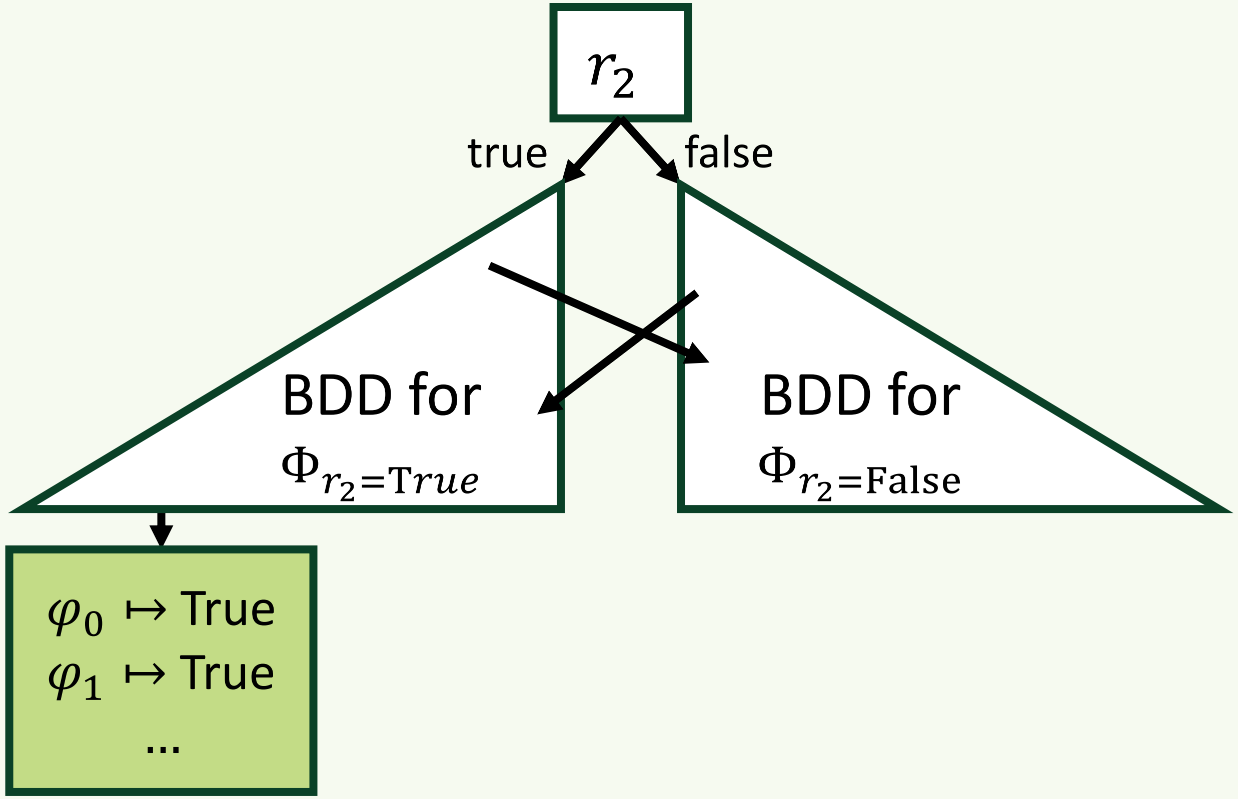
$\varphi_0 \mapsto$ False
$\varphi_1 \mapsto$ False
$\varphi_2 \mapsto$ True

Φ: $(a_0 \wedge r_0 \wedge e_0) \vee (a_0 \wedge r_1 \wedge e_1) \vee (a_0 \wedge r_2 \wedge e_3)$

$(a_0 \wedge r_2 \wedge e_2)$

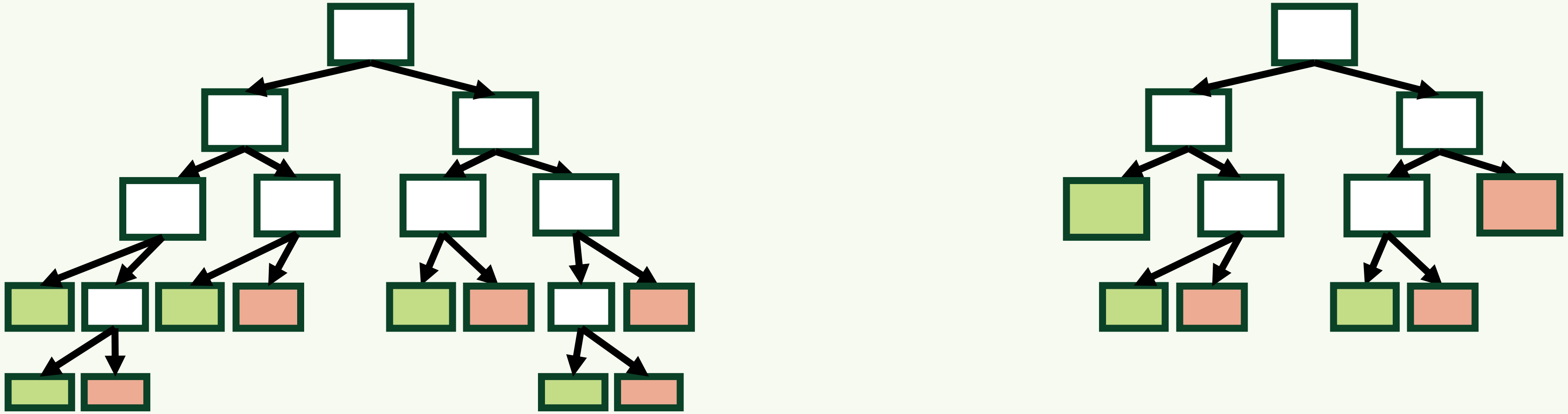
$(a_1 \wedge r_3 \wedge e_3)$

$(a_1 \wedge r_3 \wedge e_2) \vee (a_1 \wedge r_4 \wedge e_4)$



Depth

- **BDD Depth:** maximal path length from the root to a leaf
- **Expression Set Depth**: minimal BDD depth
  - Constant expression set ⇔ depth = 0



General Provenance Expressions

- **Proposition:** DEC-BDD-DEPTH is **coNP-hard**, even if the input Boolean expression is in DNF/CNF and the depth upper bound is $k = 0$.

- Proof: by reduction from CNF satisfiability / DNF falsifiability. A non satisfiable CNF ⇒ constant False ⇒ depth 0

$x \land \neg x$

false

Read-Once Provenance

Φ: $(\underline{a_0} \wedge r_0 \wedge e_0) \vee (\underline{a_0} \wedge r_1 \wedge e_1) \vee (\underline{a_0} \wedge r_2 \wedge e_3)$

$(\underline{a_0} \wedge r_2 \wedge e_2)$

$(a_1 \wedge r_3 \wedge e_3)$

$(a_1 \wedge r_3 \wedge e_2) \vee (a_1 \wedge r_4 \wedge e_4)$

Not read-once: variables repeat within/across expressions

Previous work: query classes yielding read-once provenance or compiling provenance to read-once form.
E.g., SP queries

Φ: $(a_0 \wedge r_0 \wedge e_0) \vee (a_0 \wedge r_1 \wedge e_1) \vee (a_0 \wedge r_2 \wedge e_3)$

$(a_1 \wedge r_3 \wedge e_2) \vee (a_1 \wedge r_4 \wedge e_4)$

Read once: no variable repetitions (in equivalent)

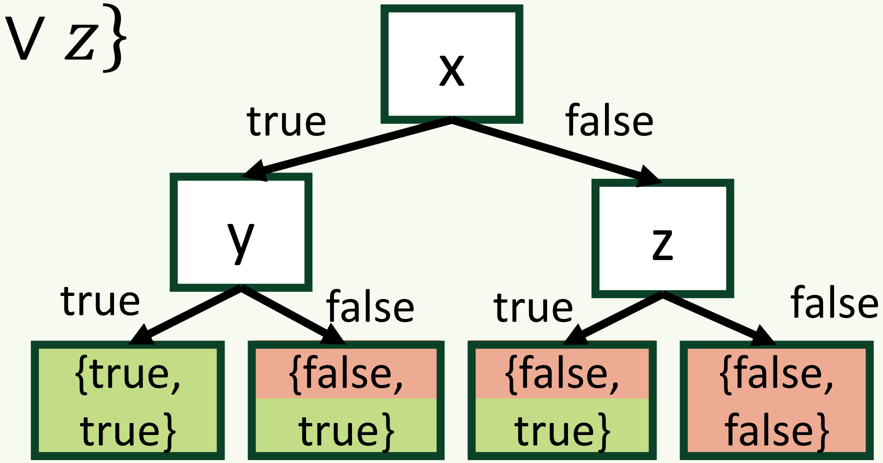
Φ: $\underline{a_0 \wedge} \big((r_0 \wedge e_0) \vee (r_1 \wedge e_1) \vee (r_2 \wedge e_3)\big)$

$\underline{a_1 \wedge} \big((r_3 \wedge e_2) \vee (r_4 \wedge e_4)\big)$

Analysis for Read-Once

- **Proposition:** Sets of read-once of Boolean expressions (without constants), and their equivalents, are **evasive**.
- Proof: by induction
- This result does not hold if variables repeat **across** expressions

$$\Phi = \{x \land y, x \lor z\}$$



Monotone Provenance

- Monotone k-DNF expressions: no negation, every term (conjunction) contains up to k unique variables
- **In the paper:** we show a **2-way correspondence** between k-DNF expressions and SPJU queries
- **Question:** monotone expressions are satisfiable and falsifiable. What is the minimal depth for monotone Boolean expressions?

Bounds for Monotone Provenance

- **Lower bound on depth:** maximal term in DNF/clause in CNF
  - Each can be a minimal 0/1 certificate

- **Theorem:** for arbitrarily large n there exists a monotone Boolean expression with a BDD of depth **linear** in this bound
  - Term/clause size is $O(\log n)$ - exponentially smaller than "trivial" solution.
  - The BDD is optimal in this case

$$(\psi_{i-1} \wedge u_i) \vee (u_i \wedge v_i) \vee (v_i \wedge \psi'_{i-1})$$

4 Proof Sketch

Monotone

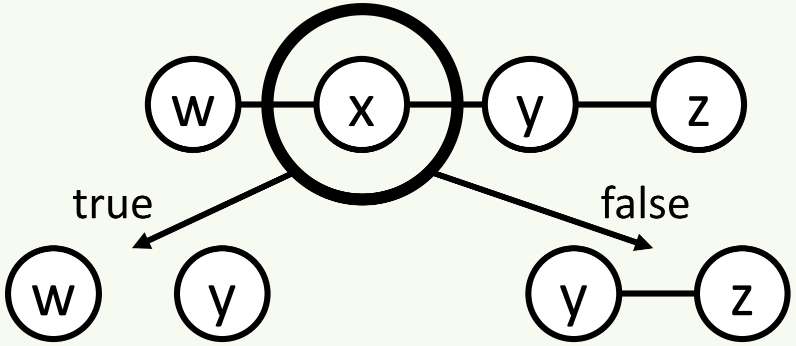
- Recursively define: $\psi_i = (\psi_{i-1} \wedge u_i) \vee (u_i \wedge v_i) \vee (v_i \wedge \psi'_{i-1})$ where u_i, v_i are fresh variables and ψ'_{i-1} is a copy of ψ_{i-1} using fresh variables.
  Let $\psi_0 = (w_0 \wedge x_0) \vee (x_0 \wedge y_0) \vee (x_0 \wedge y_0)$
- **Observation:** ψ_i cannot be evaluated without probing at least one of u_i, v_i
  - If $u_i = v_i$ we're done by probing both
  - Otherwise, we need to evaluate either ψ_{i-1} or ψ'_{i-1} but not both
- **Observation:** ψ_i includes 2^i copies of ψ_0 and $n = \Theta(2^i)$ variables
- "Bad" algorithm: evaluate all copies of ψ_0 first. Each copy requires 2-4 probes.
- "Good" algorithm: evaluate u_i, v_i first, then if needed proceed to one of the ψ_{i-1} and continue recursively. We query at most $2i + 3 = O(\log n)$

Monotone Acyclic Graph DNF

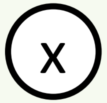
- When each term is of size 2, terms can be viewed as edges
- When the resulting graph is acyclic, we have the following

- **Theorem:** Given a monotone acyclic graph DNF, DEC-BDD-EVASIVE is in PTIME.
- Proof: We define an **non-evasiveness pattern**, which exists iff the provenance is not evasive

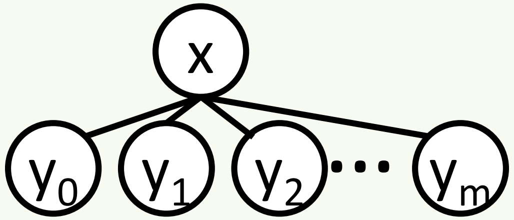
$(w \wedge x) \vee (x \wedge y) \vee (y \wedge z)$



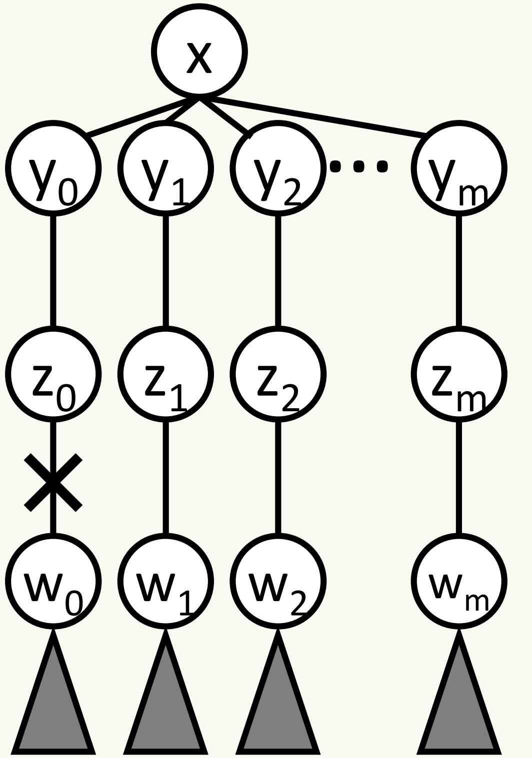
Proof Sketch



Isolated vertex
= non-evasive



Evasive (e.g., if all are true)



Each child has grand-child with non-evasiveness pattern
= non-evasive

Probe every y_{i}.
If all are false – no need to probe x.
Assume w.l.o.g y_0 is true.

$z_0 \wedge \mathrm{True} = z_0$ absorbs $z_0 \wedge w_0$

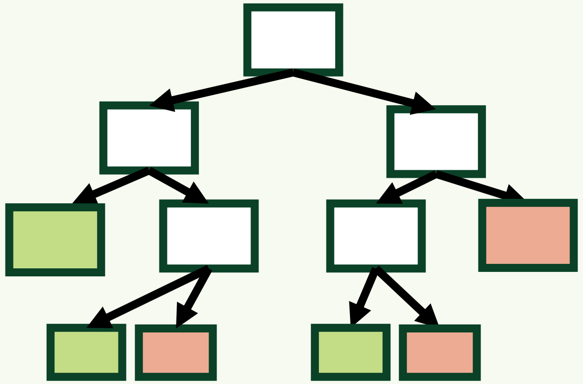
w_0 is the new root.
By recursive argument – it is non-evasive!

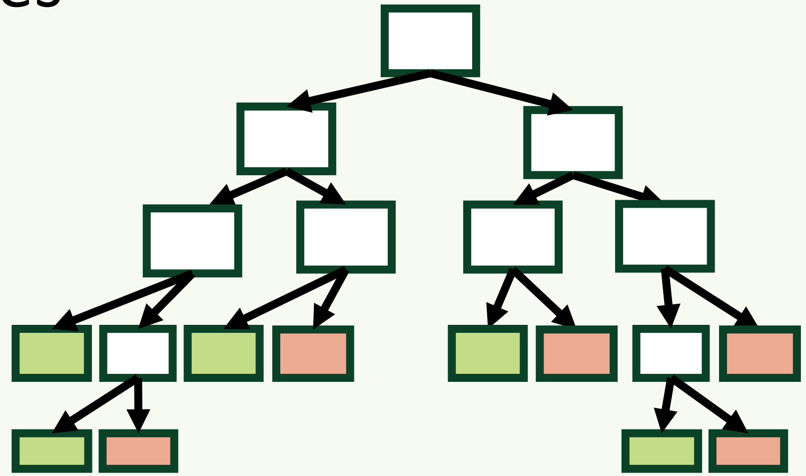
The other direction is by induction on the tree structure, showing having no pattern entails that any probe and any answer yields remaining sub-graphs without our pattern

Conclusion and Future Work

- Overview
  - Optimizing the BDD depth for deciding the truth value of Boolean provenance expressions
  - Results for different classes of queries and provenance shapes
  - Many open questions

- Further application domains, further query classes





Thank you!

Questions/
remarks?