Algorithms II 89-322-01, 89-322-02, FINAL EXAM
MOED A

Instructor: *Prof. Amihood Amir*
Length of Exam: 2 hours
Time: July 9th, 08:30
NO OUTSIDE MATERIAL ALLOWED!!!

1. (33 points) Describe how the Abrahamson-Kosaraju algorithm can be modified in order to achieve a time complexity of $O(n\sqrt{m \log m})$. Prove that indeed this is the time reached.

   Answer:

   The algorithm had two part. Alphabet reduction and then "fixing". The alphabet reduction was to an alphabet of size $\sqrt{m}$ and this led to a size of "fixing set" $O(\sqrt{m})$. However, the fixing is faster than the convolution.

   In general if we reduce the alphabet to size a, it means that the convolutions part takes time $O(a \cdot m \log m)$. However, the size of a block, and the minimum number of times that a frequent element occurs is the $\frac{n}{a}$, thus the fixing time is $O(\frac{m}{a} m)$. The optimum is reached when both numbers are equal, i.e.

$$a \cdot m \log m = \frac{m^2}{a}$$

$$a^2 \log m = m$$

$$a^2 = \frac{m}{\log m}$$

$$a = \sqrt{\frac{m}{\log m}}$$

   Thus the best time achieved by the Abrahamson method is $O(m\sqrt{m \log m})$.

2. (33 points)

   (a) Construct an Integer Programming system for finding the smallest set of labels in two evolutionary trees, whose deletion from the trees leaves an agreement homeomorphic tree.

   (b) Which relaxation can be done in order to get an approximation to the answer?

   (c) What approximation factor is achieved by your relaxation?

   Answer:

   (a) Recall that by the Bandelt and Dress theorem, the result is the minimum number of leaves that occur in all quadruples of leaves that are not homeomorphic. We therefore assign a variable x_i to every symbol x_i, These variables get integer value from $\{0, 1\}$. Assume there are n symbols.

The objective function is min $\sum_{i=1}^{n} x_i$.
Subject to the constraints:
$x_i \in \{0, 1\}$, for $i = 1, ..., n$.
For every quadruple $< x_{i_1}, x_{i_2}, x_{i_3}, x_{i_4} >$ that is not homeomorphic to all trees, write:

$$\sum_{j=1}^{4} x_{i_j} \geq 1.$$

(b) The relaxation is choosing all symbols whose value is at least $\frac{1}{4}$.

(c) The approximation ratio is 4. Clearly, every quadruple will have at least one element we choose, so our answer is correct, albeit not minimal. However, suppose the set of symbols we choose is C. Clearly,

$$|C| \leq 4 \sum_{i \in C} x_i \leq 4 \sum_{i=1}^{n} x_i \leq OPT$$

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3. (34 points) Consider the following strategy for the three server problem: Use the servers in a round-robin fashion cyclically, i.e. 1,2,3,1,2,3,1,2,3,... Is this strategy competitive? Prove your claim.

Answer:

The strategy is not competitive.Consider an equilateral triangle, where every corner has two point of distance 1 to each other, and the edge length is d. Assume that initially, the servers are sitting each on one of the points in a different corner. Assume that the inputs come at the unoccupied point in the triangle corners, also in a round-robin fashion. If we serve the interrupts in the cyclic strategy described, then the travel distance is kd for k inputs.However, the optimum is to leave the servers in the initial corners and just serve that corner, so the optimal cost is d. Clearly there is no constant c for which $kd \leq ck, \quad \forall d$, thus the strategy is not competitive.

GOOD LUCK