Algorithms II 89-322-01, 89-322-02, FINAL EXAM
MOED B

<div align="center">

Instructor: *Prof. Amihood Amir*
Length of Exam: 2 hours
Time: September 14, 2008, 08:30
NO OUTSIDE MATERIAL ALLOWED!!!

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1. The algorithm we saw for finding the maximum agreement homeo-morphic subtree of two evolutionary trees, the dynamic programming considered subtrees. We saw that there are $O(n)$ subtrees for an n-leaf tree. Write the **exact** number of subtrees for a **binary** evolutionary tree with n leaves. Prove your claim.

   Answer:

   The number is $4n - 6$. Since every edge in the tree has precisely two subtrees (one on each side) and the number of edges is $2n - 3$, because this is an unrooted full binary tree. See the proof for that in *moed alef*.

   Errors:

2. Consider the following *integer program*:

   Objective function: $\max 4x_1 - 5x_2$.

   Constraints:
   quad $x_1 - x_2 \geq 0$
   quad $x_1 < 3$
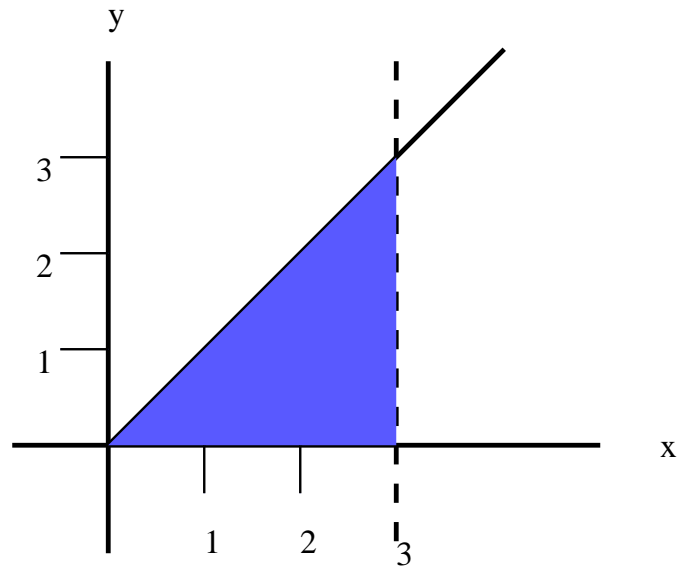   quad $x_1 \geq 0$
   quad $x_2 \geq 0$

   Does the above IP have a solution? What is it?

   Answer:

   For illustration purposes, take $x_1 = x$ and $x_2 = y$. The constraints give us the domain:

The function $4x - 5y$ in this domain gets the maximum value when x is largest and y is smallest. We are looking at integers which means $x = 2$ and $y = 0$.

Errors:

3. Let P be a pattern of length m. If you are given the KMP automaton, can you deduce from it the locations in the witness table that have a "$*$"? How?

Answer:

The fail link from the accepting state of the KMP automaton points to the first location where there is a "$*$" (assume it points to state k, then there is a "$*$" in location $m - k + 1$ of the witness table). Following the failure link from state k, say to state ℓ, means putting a "$*$" in location $m - \ell + 1$ of the witness table. We continue in this fashion until the failure link points to state 0.

Errors:

GOOD LUCK