Proof:

$\Rightarrow$ Obvious.

$\Leftarrow$ Assume all generated 4-leaf subtrees are homeomorphic.
Prove: 2 trees are homeomorphic.

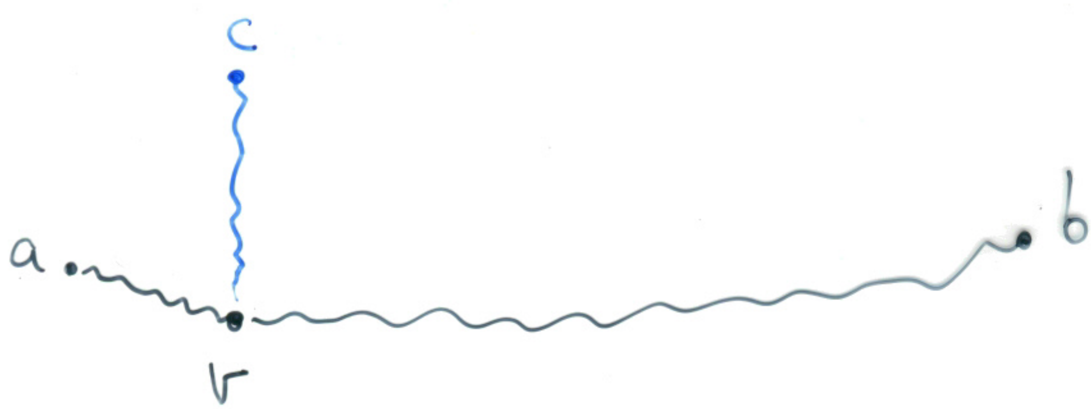
Def. Let v be a node of degree >2. We call v a super-2 node.
Two nodes a,b are twins if in the path from a to b are at most one internal node which is a super-2 node, and a,b are leaves.

Claim: A tree with at least 2 ~~nodes~~ leaves and a super 2 node has a pair of twins.

Proof: Let T be a tree with at least 2 ~~nodes.~~ leaves.

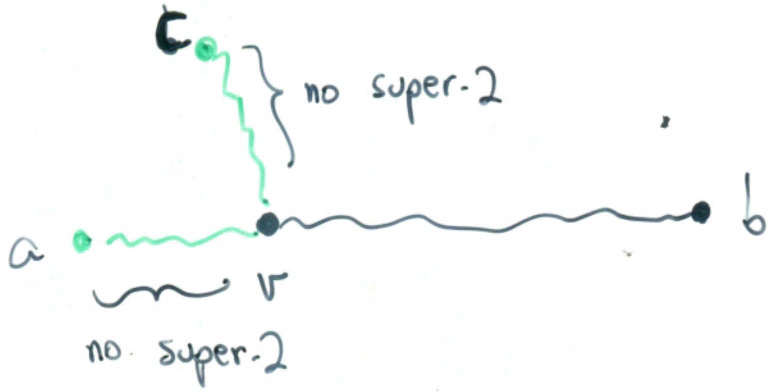
Consider a, b such that the path from a to b has the largest number of internal super-2 nodes.

Let v be the closest super-2 node to a.



v is super-2 so $\exists$ leaf c such that there is a path from v to c.

It is clear that there are no super-2 nodes on the path from v to c otherwise the path from c to b has more super-2 nodes than the path from a to b, contradicting its maximality.



So a and c are twins. ∎

Now we prove the theorem by induction on the number of leaves n.

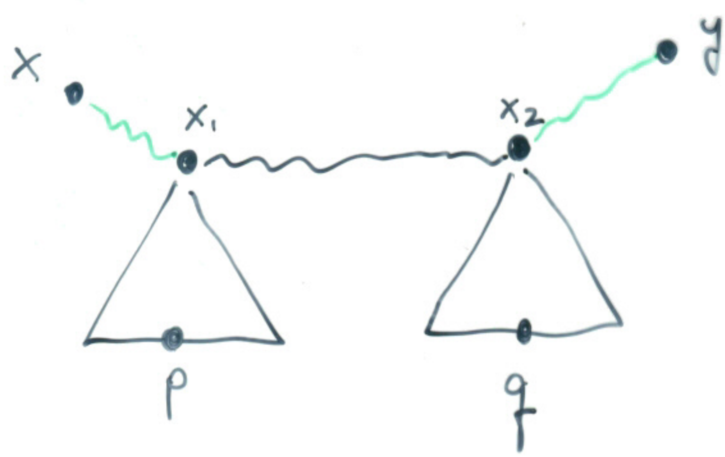
Base Case: $n \leq 4$. Thm true immediately.

Ind. hyp. Thm true for trees with $< n$ leaves. Prove for n. $(n > 4)$.

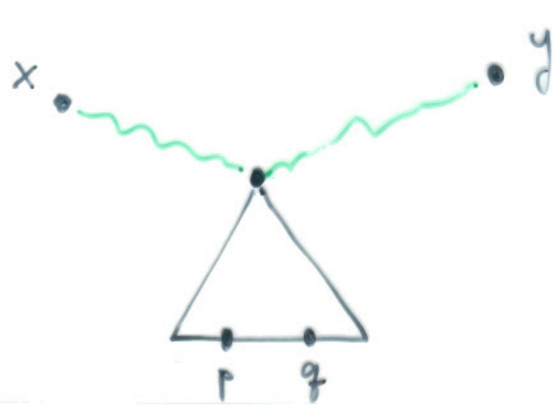
Let x, y be twins in tree T_1 and let v be super-2. node in path.

Claim: x & y are twins in T_2.

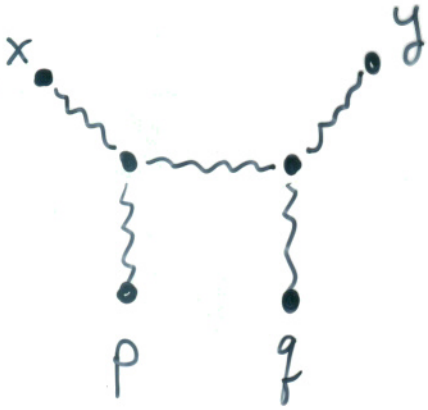
Proof: If x, y not twins in T_2 the situation in T_2 is:



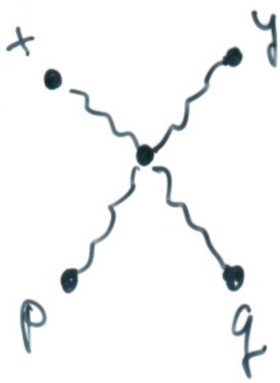
In T_1 the situation is:

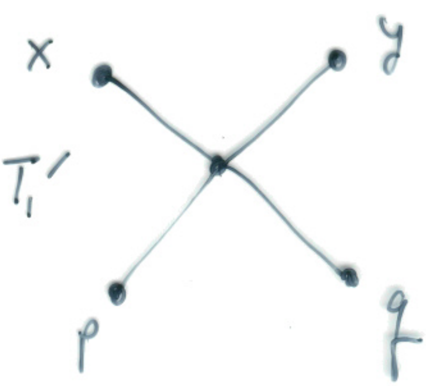


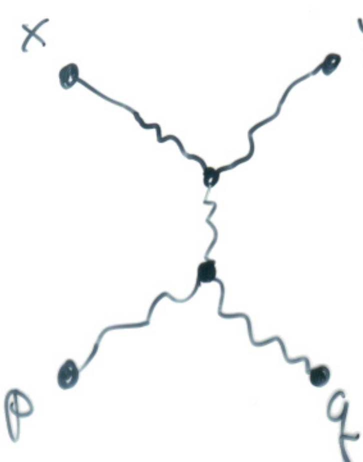
Consider the tree induced by x, y, p, q.

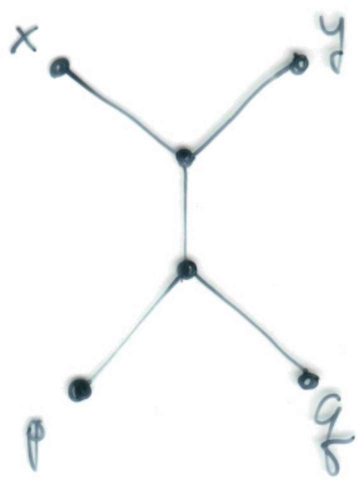
In T_2: 

Homeomorphic image: T_2' 

In T_1: Either 

$\approx T_1'$ 

or: 

$\approx T_1'$ 

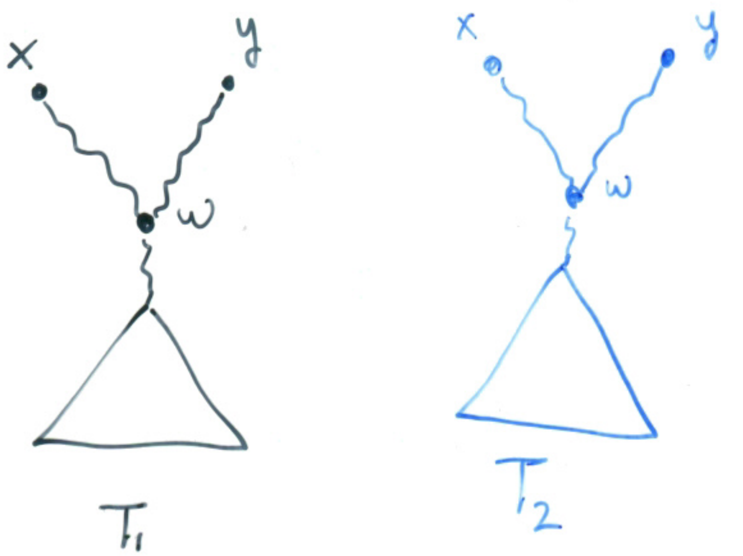
$T_1' \neq T_2'$. Contradiction. ∎

Return to proof of thm.

Let x, y be twins in T_1 and T_2.

For reasons similar to previous claim, either the super-2 node w on the path from x to y has degree 3 in both T_1, T_2 or degree > 3 in both T_1, T_2.

Cases: 1)

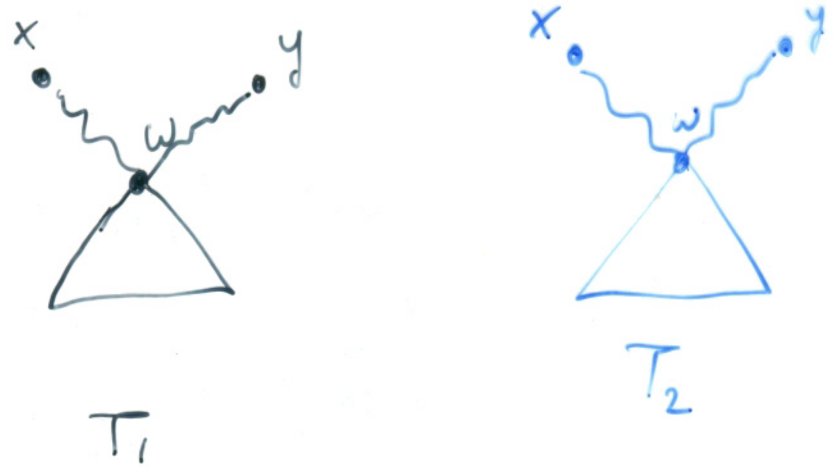


Delete x, y & the path from them to w.

By ind. remaining trees homeomorphic.

Adding x, y to both does not change homeomorphism.

2)



Delete y and its path to w.

By induction remaining trees homeomorphic.

Now add y and its path. ∎