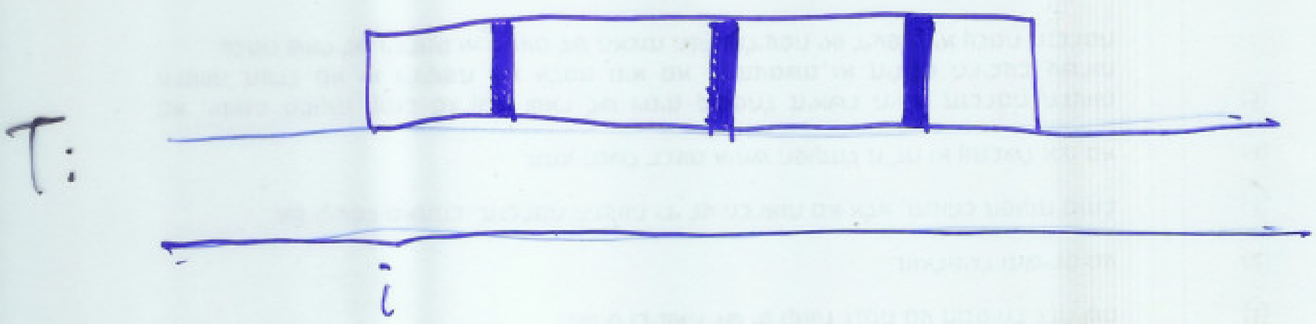
DETERMINISTIC SAMPLING

IDEA:

Find a small sample of P

such that $\forall i$



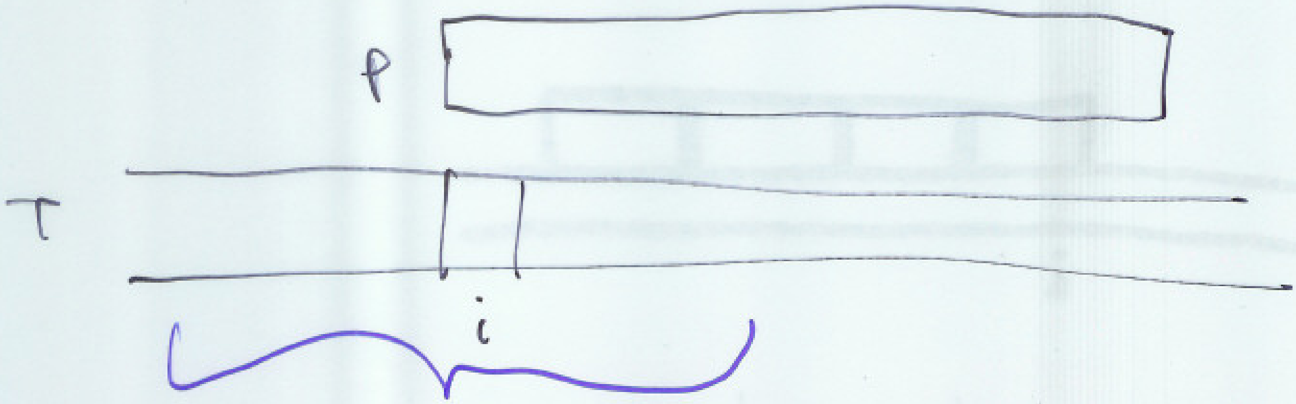
If the sample positions do not match then no occurrence of P in T.

(Easy to satisfy – any position can do that)

BUT

If the sample positions match

Then



Guaranteed a large area

where pattern can not start-

dead zone

large $= \frac{m}{c}$ where c is

some constant.

PATTERN MATCHING ALGORITHM

For $i=1$ to $n-m$ do

check sample

If positive then set location i as candidate

end For

For every candidate, kill all candidates in its dead zone.

For remaining candidates - verify in naive way whether pattern occurs.

Time: $O(ns)$: "For" loop, where

s = sample size.

$$O\left(m \cdot \frac{n}{\left(\frac{m}{c}\right)}\right) = O(nc) \overset{c \text{ is a constant}}{=} O(n)$$

for verification

VISHKIN (1990):

It is possible to construct a sample of size $\log m$ for non-periodic patterns.

DEFINITION: P is periodic if

$$P = U^{v} U'$$

where $v > 1$ and U' is a prefix of U.

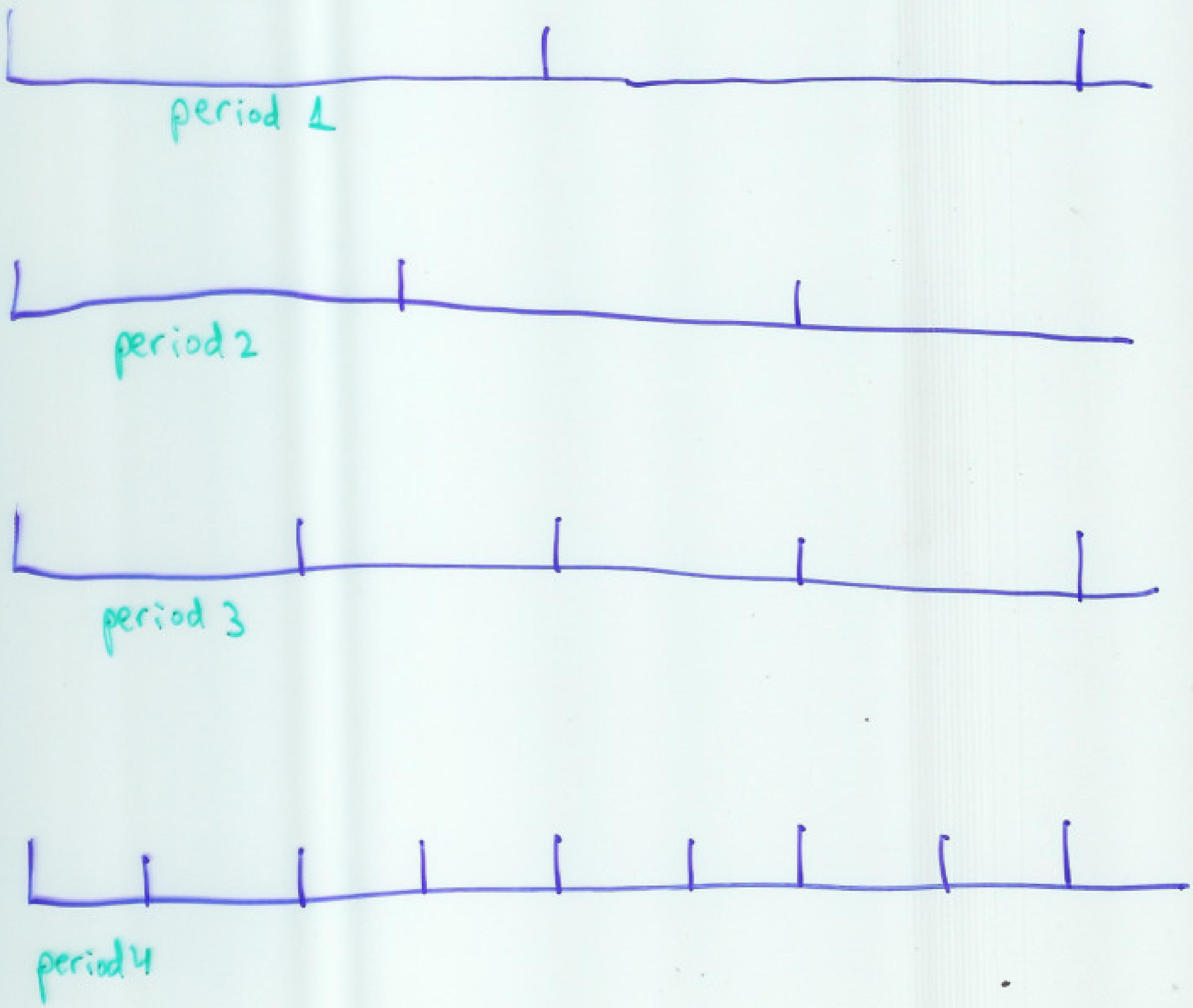
U is a period of P.

EXAMPLE:

ABCAB is not periodic

ABCABCA is periodic

ABC ABC ABC ABC ABC ABC ABC ABC A



PERIODICITY LEMMA: (Fine & Wilf)

Let U_1, U_2 be periods of P

Let $|U_1| = x_1$, $|U_2| = x_2$.

Then P has a period U_3, where

$$|U_3| = \gcd(x_1, x_2)$$

Proof: Generalize the concept of period to allow $v \geq 1$

($P = UU'$ where U' is a non-empty prefix of U, makes U a period).

Alternate view of periodicity:

Let $|P| = m$ $\quad |U| = x$

$\forall i \quad 1 \leq i \leq m - x$

$$P[i] = P[i+x]$$

Claim: If b and c are co-prime then $b-c$ and c are co-prime (for $b > c$).

Proof: O/w
$$b-c = x \cdot w$$
$$c = x \cdot z \qquad x \neq 1$$
$$b = x \cdot w + x \cdot z = x \cdot (w+z)$$
contradicting co-primality of b, c. ∎

Conclude: $\gcd(p,q) = \gcd(p-q, q)$ (for $p > q$).

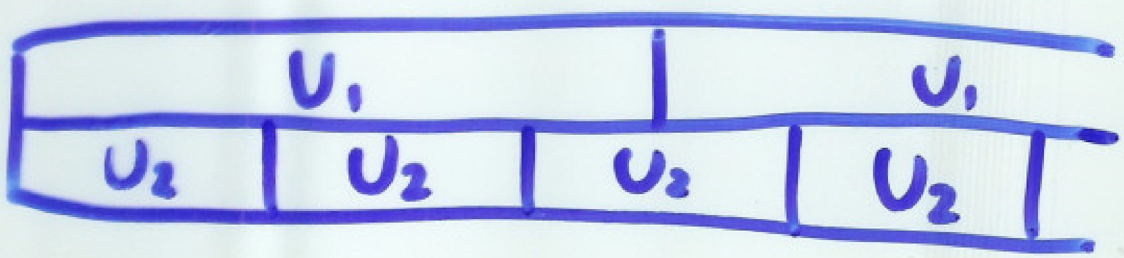
Proof:
$$p = ab$$
$$q = ac \quad \text{where } b, c \text{ co-prime.}$$
$$p - q = a(b-c)$$
By claim $(b-c)$ and c are co-prime. ∎

Return to Proof of Periodicity Lemma:

By induction on $|P|$.

$|P| = 1$ is obvious.

Assume lemma true $\forall$ strings of length less than n. Prove for $|P| = n$.



Consider U_1.

$$U_1[i] = U_1[i+x_2] \qquad \forall\ 1 \le i \le x_1 - x_2$$

But

because P has period U_2 of length x_2

$$P[i] = P[i+x_1] = P[i+x_1-x_2]$$

$$\forall\ 1 \le i \le m - x_1$$

Conclude: U_1 has period $(x_1 - x_2)$

We have: U_1 (of length $< |P|$)

has period of length x_2

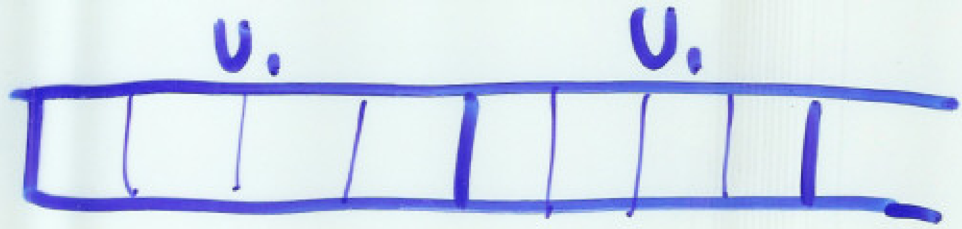
and period of length $(x_1 - x_2)$

By ind hyp it has period of

length $\gcd(x_1 - x_2, x_2)$.

But by claim, this equals

$\gcd(x_1, x_2)$

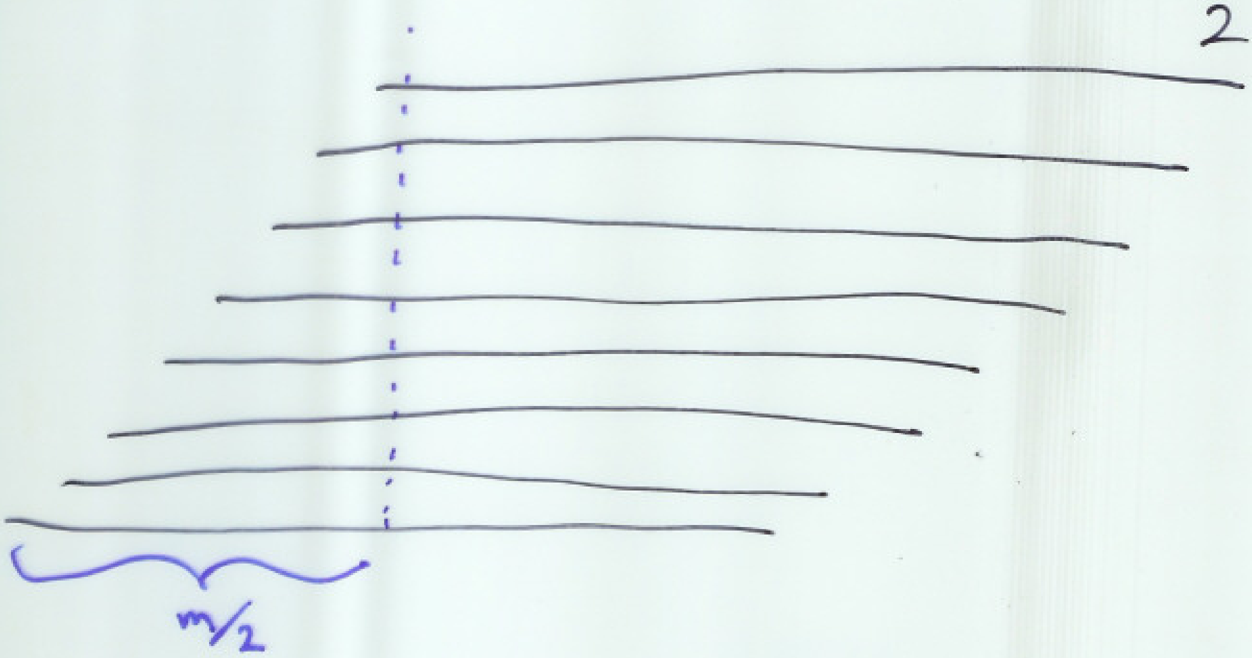
So: $P =$ 

also has period of length

$\gcd(x_1, x_2)$. ∎

HOW IS SAMPLE CONSTRUCTED?

Consider P shifted and stacked $\frac{m}{2}$ times.



Because non-periodic, exists a column that has at least 2 different symbols

Choose column j where symbol a occurs < $\frac{1}{2}$ times.

Discard all rows where not a in column j.

Repeat until only one row left.

EXAMPLE:

A : B A B B A A B A B

A B : A B B A A B A B

A B A : B B A A B A B

A B A B : B A A B A B

A B A B B : A A B A B

A B A B B A A B A B

A B A B B A A B A B

1 2 3 4 5 6 7 8 9 10

SAMPLE: 7, A

8, B

Cancels from $i-1$ to $i+3$ except i.

SAMPLE SIZE: $O(\log m)$

Why? At every iteration, at least half of the rows are eliminated.

WHAT ABOUT PERIODIC PATTERNS?

1. Find smallest period.

2. Look for consecutive repetitions of period.