MLPs, Representations, Ngram Language Models

Yoav Goldberg

Linear Classifier

Binary:

$$f(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b$$

Multi class:

$$f(\mathbf{x}) = \mathbf{W} \cdot \mathbf{x} + \mathbf{b}$$

Linear Classifier

Binary:

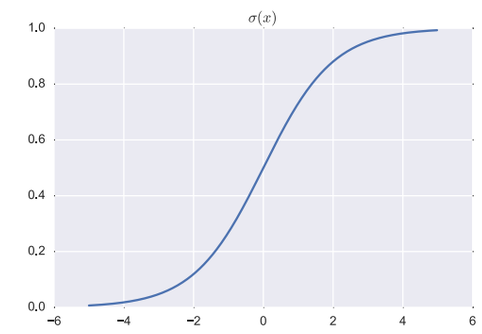
$$f(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b \qquad \text{sign}(\mathbf{w} \cdot \mathbf{x} + b)$$

Multi class:

$$f(\mathbf{x}) = \mathbf{W} \cdot \mathbf{x} + \mathbf{b}$$

Linear Classifier

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$



Binary:

$$f(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b \qquad \text{sign}(\mathbf{w} \cdot \mathbf{x} + b)$$

$$\sigma(\mathbf{w} \cdot \mathbf{x} + b)$$

Multi class:

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Linear Classifier

Binary:

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$$\text{sign}(\mathbf{w} \cdot \mathbf{x} + b)$$

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Multi class:

$$f(\mathbf{x}) = \mathbf{W} \cdot \mathbf{x} + \mathbf{b}$$

$$\arg\max_i (\mathbf{W} \cdot \mathbf{x} + \mathbf{b})_{[i]}$$

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$$\arg\max_i \text{softmax}(\mathbf{W} \cdot \mathbf{x} + \cdot b)_{[i]}$$

Linear Classifier

Binary:

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$$\arg\max_i (\mathbf{W} \cdot \mathbf{x} + \mathbf{b})_{[i]}$$

is this really needed?

$$\arg\max_i \text{softmax}(\mathbf{W} \cdot \mathbf{x} + \cdot b)_{[i]}$$

XOR

$$\text{xor}(0,0) = 0$$

$$\text{xor}(1,0) = 1$$

$$\text{xor}(0,1) = 1$$

$$\text{xor}(1,1) = 0$$

XOR

$$(0,0) \cdot \mathbf{w} + b < 0$$

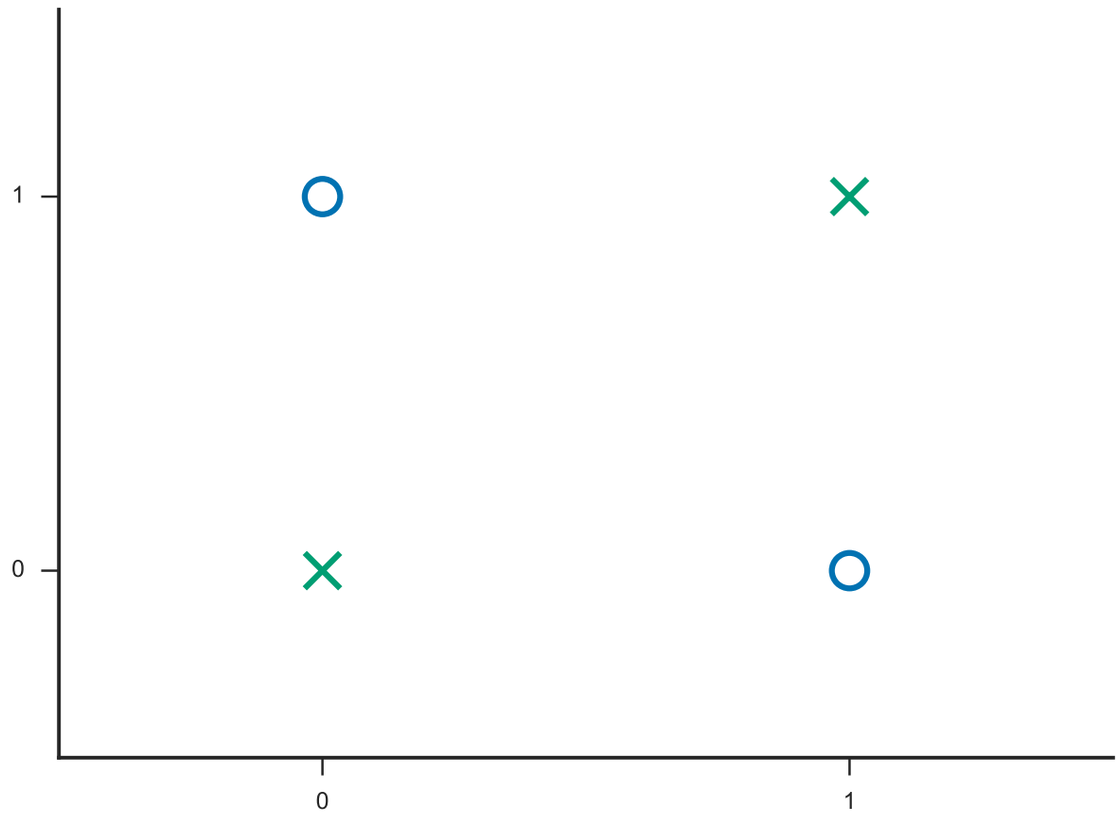
$$(0,1) \cdot \mathbf{w} + b \geq 0$$

$$(1,0) \cdot \mathbf{w} + b \geq 0$$

$$(1,1) \cdot \mathbf{w} + b < 0$$

$$\mathbf{w} = ?$$

XOR



XOR

$$\phi(x_1, x_2) = [x_1 \times x_2, x_1 + x_2],$$

XOR

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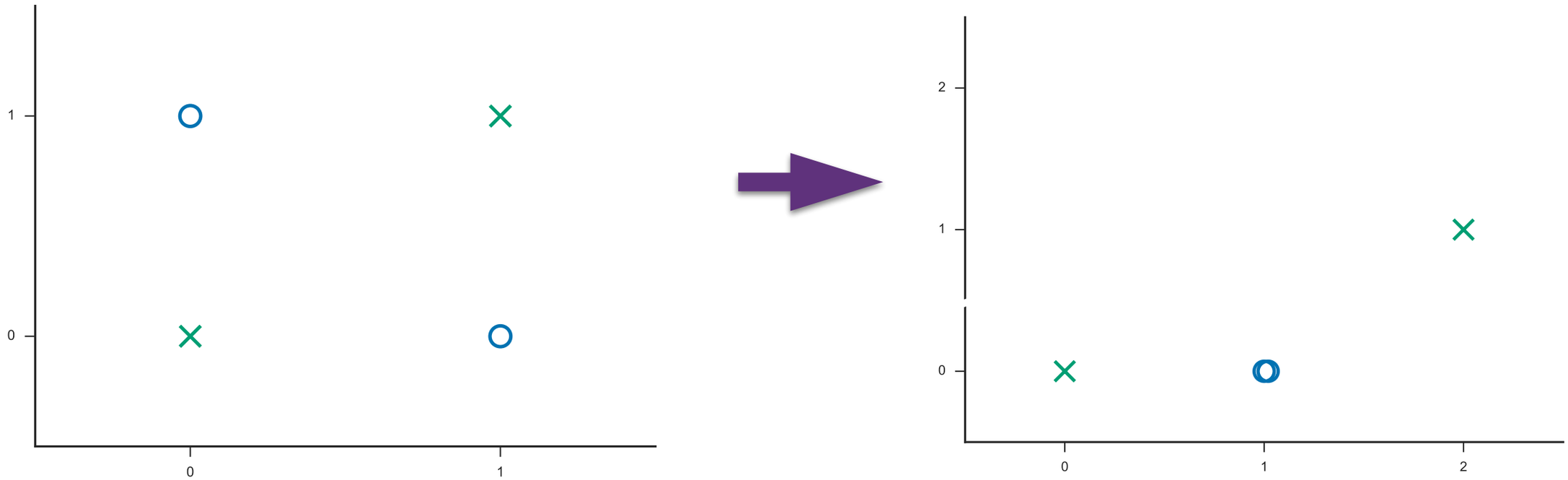
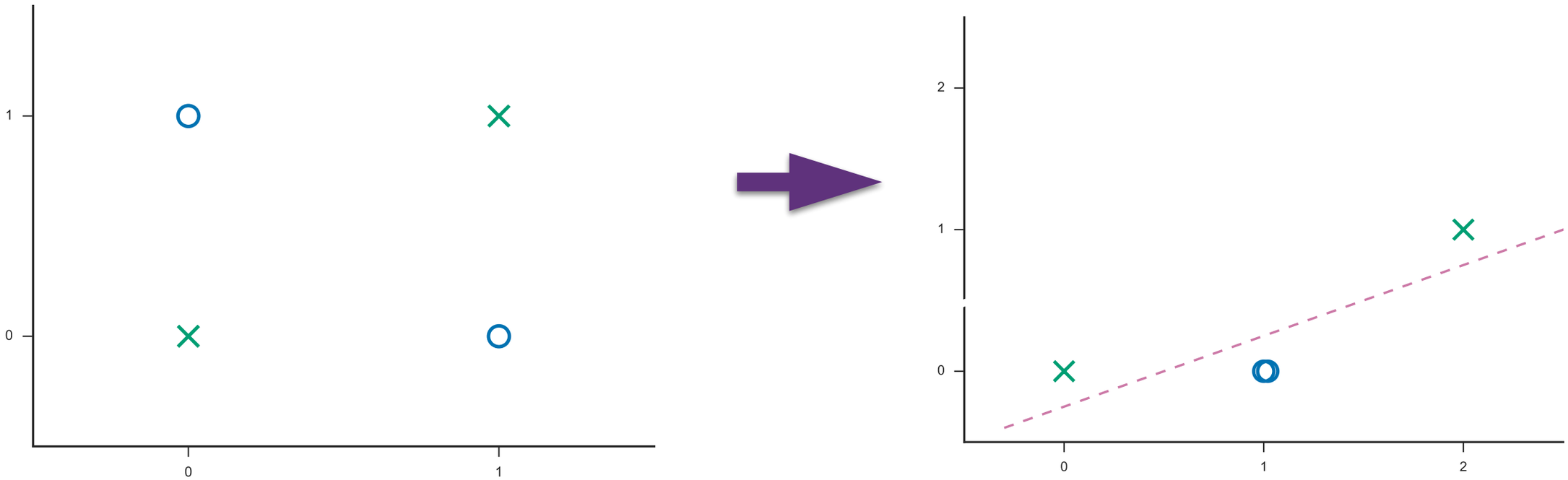


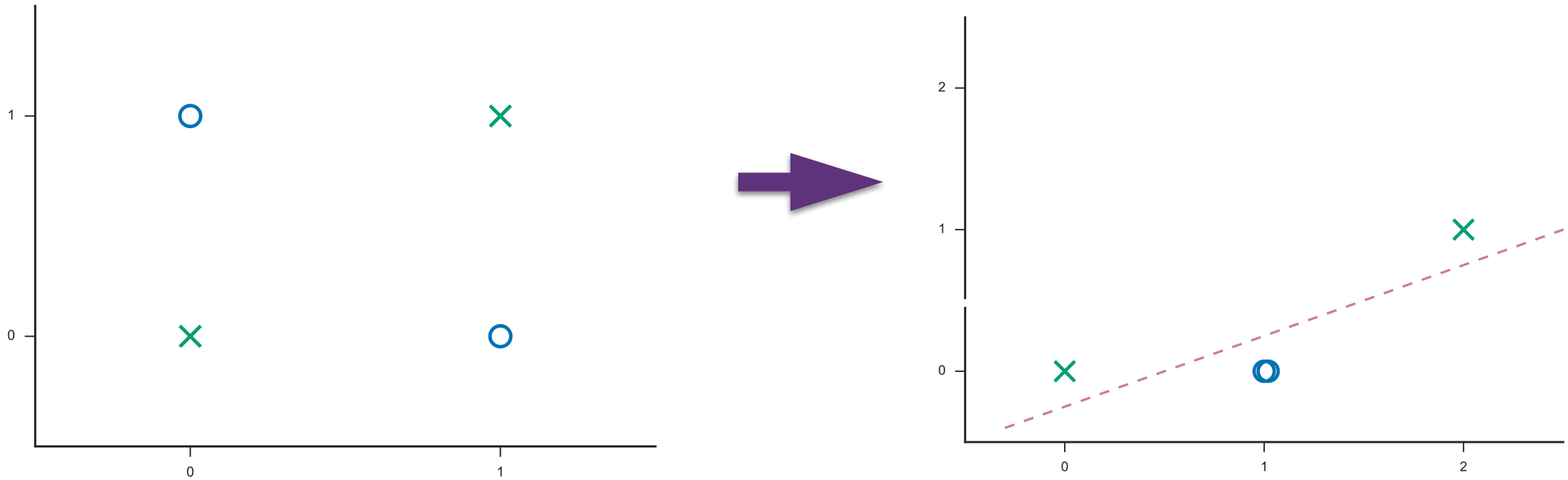
XOR

$$\phi(x_1, x_2) = [x_1 \times x_2, x_1 + x_2],$$



XOR

$$\phi(x_1, x_2) = [x_1 \times x_2, x_1 + x_2],$$



$$f(\mathbf{x}) = \mathbf{w} \cdot \phi(\mathbf{x}) + b$$

XOR

$$\phi(x_1, x_2) = [x_1 \times x_2, x_1 + x_2],$$

- Can make the transfer function **trainable**:

$$\phi(\mathbf{x}) = g(\mathbf{W}' \cdot \mathbf{x} + \mathbf{b}')$$

$$g(x) = \max(x, 0)$$

Multilayer Networks

Linear Classifier

Binary:

$$f(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b$$

$$\text{sign}(\mathbf{w} \cdot \mathbf{x} + b)$$

$$\sigma(\mathbf{w} \cdot \mathbf{x} + b)$$

Multi class:

$$f(\mathbf{x}) = \mathbf{W} \cdot \mathbf{x} + \mathbf{b}$$

$$\arg\max_i \text{softmax}(\mathbf{W} \cdot \mathbf{x} + \mathbf{b})_{[i]}$$

Non-Linear Classifier

$$f_\theta(\mathbf{x}) \ = \mathbf{w} g(\mathbf{W}' \cdot \mathbf{x} + \mathbf{b}') + b$$

Non-Linear Classifier

Multi-layer Perceptron (MLP):

$$f_\theta(\mathbf{x}) = \ \mathrm{NN}_{\mathrm{MLP2}}(\mathbf{x}) = \mathbf{y}$$

$$\mathbf{h^1} = g^1(\mathbf{x}\mathbf{W^1} + \mathbf{b^1})$$

$$\mathbf{h^2} = g^2(\mathbf{h^1}\mathbf{W^2} + \mathbf{b^2})$$

$$\mathbf{y} = \mathbf{h^2}\mathbf{W^3}$$

Non-Linear Classifier

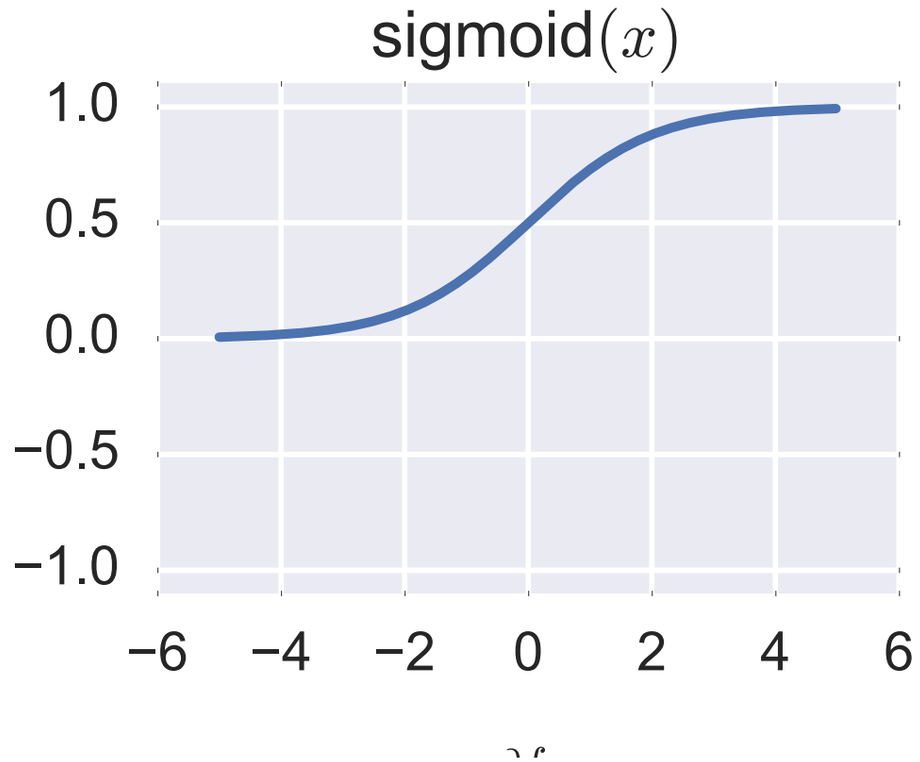
Multi-layer Perceptron (MLP):

$$
\begin{aligned}
f_\theta(\mathbf{x}) = \ & \mathrm{NN}_{\mathrm{MLP2}}(\mathbf{x}) = \mathbf{y} \\
\mathbf{h^1} = & g^1(\mathbf{x}\mathbf{W^1} + \mathbf{b^1}) \\
\mathbf{h^2} = & g^2(\mathbf{h^1}\mathbf{W^2} + \mathbf{b^2}) \\
\mathbf{y} = & \mathbf{h^2}\mathbf{W^3}
\end{aligned}
$$

Common Non-linearities

Sigmoid

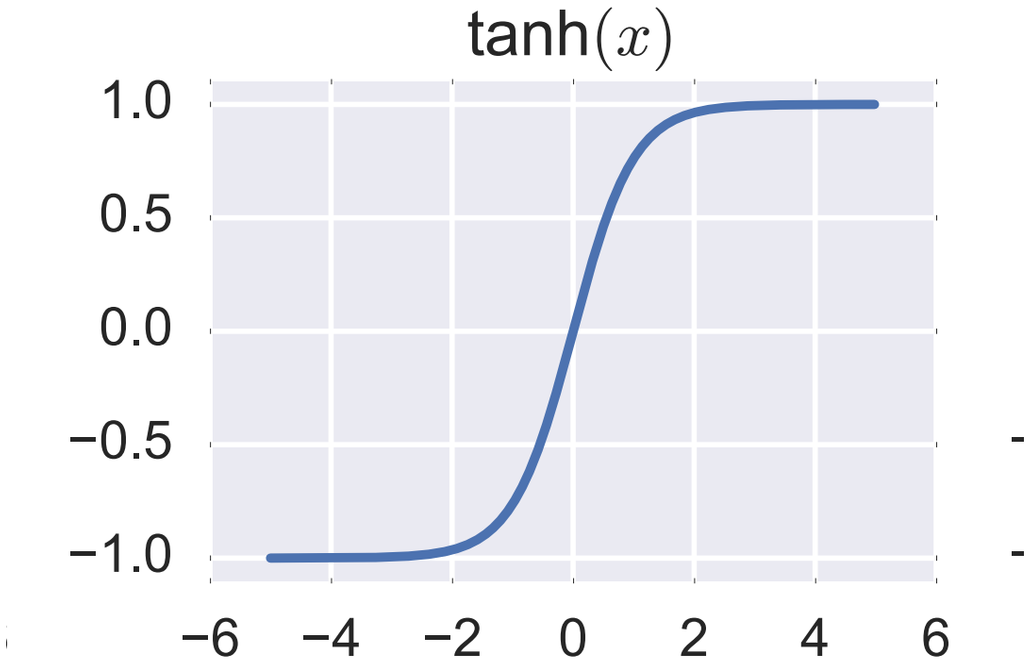
$$\sigma(x) = 1/(1 + e^{-x})$$



Common Non-linearities

tanh

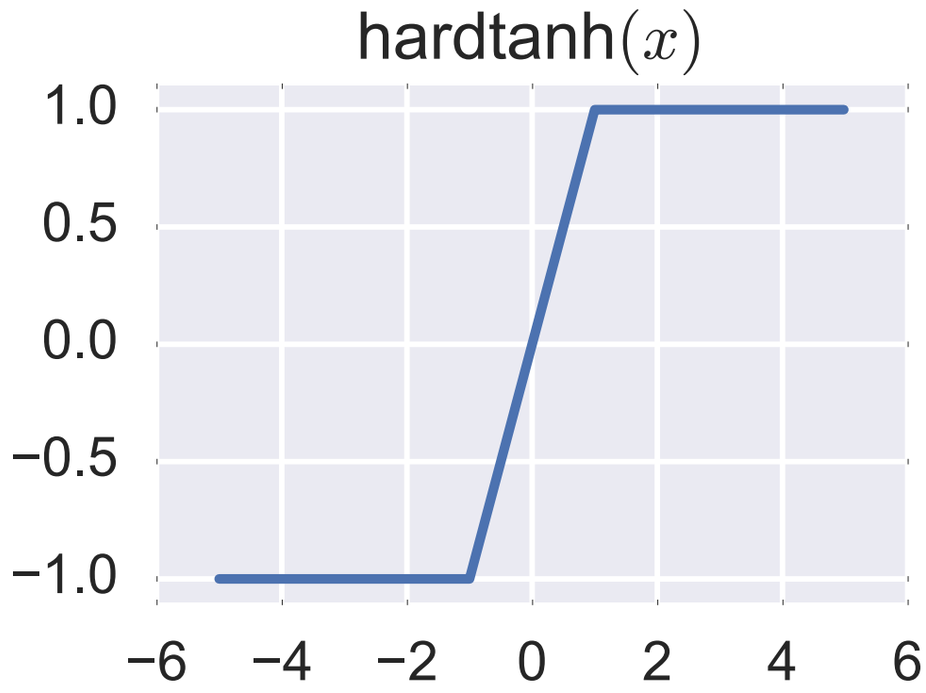
$$\tanh(x) = \frac{e^{2x}-1}{e^{2x}+1}$$



Common Non-linearities

hard-tanh

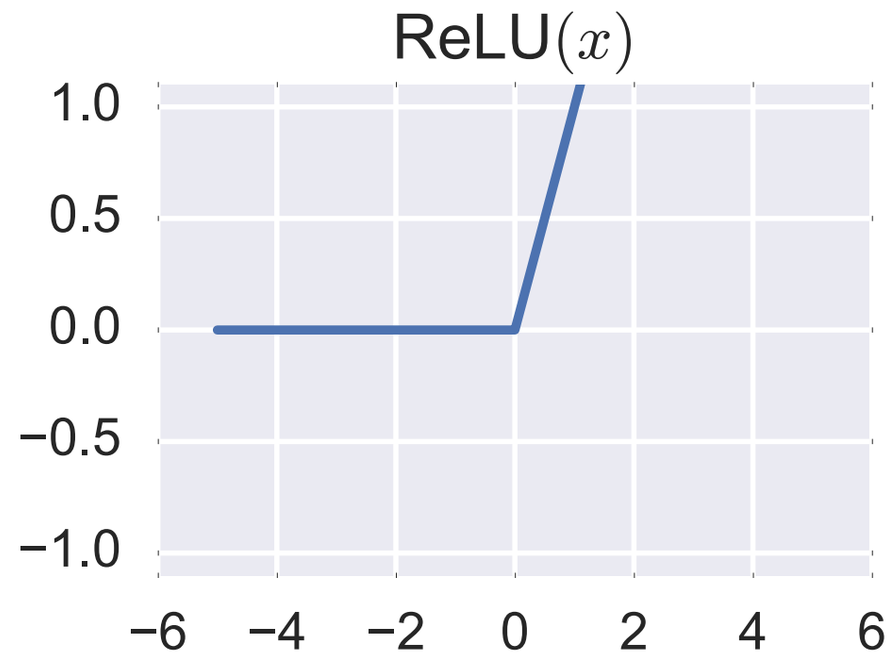
$$\text{hardtanh}(x) = \begin{cases} -1 & x < -1 \\ 1 & x > 1 \\ x & \text{otherwise} \end{cases}$$



Common Non-linearities

ReLU (rectifier, rectified linear unit)

$$\mathrm{ReLU}(x) = \max(0, x) = \begin{cases} 0 & x < 0 \\ x & \text{otherwise} \end{cases}$$



How many layers to use? And how wide should they be?

- No hard and fast rules.
- In vision, we see that "deeper is better".
  - Not always the case in text / sequences (though with transformers, we may be starting to see this).
- Can think of each layer as **transforming** the previous layer (remember the xor example).
- Narrower layers "compress" the information in the previous layer. Wider layers introduce redundancies.

Dropout

- **At each iteration, select a random subset of "neurons" and "drop" them.**

- Like training 2^n different networks.

- Prevents co-adaptation of neurons (prevents neurons from depending on each other).

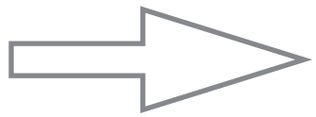
Dropout

$$\mathrm{NN}_{\mathrm{MLP2}}(\mathbf{x}) = \mathbf{y}$$

$$\mathbf{h^1} = g^1(\mathbf{x}\mathbf{W^1} + \mathbf{b^1})$$

$$\mathbf{h^2} = g^2(\mathbf{h^1}\mathbf{W^2} + \mathbf{b^2})$$

$$\mathbf{y} = \mathbf{h^2}\mathbf{W^3}$$



$$\mathrm{NN}_{\mathrm{MLP2}}(\mathbf{x}) = \mathbf{y}$$

$$\mathbf{h^1} = g^1(\mathbf{x}\mathbf{W^1} + \mathbf{b^1})$$

$$\mathbf{m^1} \sim \mathrm{Bernouli}(r^1)$$

$$\tilde{\mathbf{h}}^{\mathbf{1}} = \mathbf{m^1} \odot \mathbf{h^1}$$

$$\mathbf{h^2} = g^2(\tilde{\mathbf{h}}^{\mathbf{1}}\mathbf{W^2} + \mathbf{b^2})$$

$$\mathbf{m^2} \sim \mathrm{Bernouli}(r^2)$$

$$\tilde{\mathbf{h}}^{\mathbf{2}} = \mathbf{m^2} \odot \mathbf{h^2}$$

$$\mathbf{y} = \tilde{\mathbf{h}}^{\mathbf{2}}\mathbf{W^3}$$

Initialization

- With a (log) linear model, initialization doesn't matter much.

- With MLPs or more complex networks, initialization is **crucial** for achieving good performance.

Initialization

- With a (log) linear model, initialization doesn't matter much.
- With MLPs or more complex networks, initialization is **crucial** for achieving good performance.

$$\mathbb{R}^{n \times m} \sim \text{Uniform}[-\epsilon, +\epsilon]$$

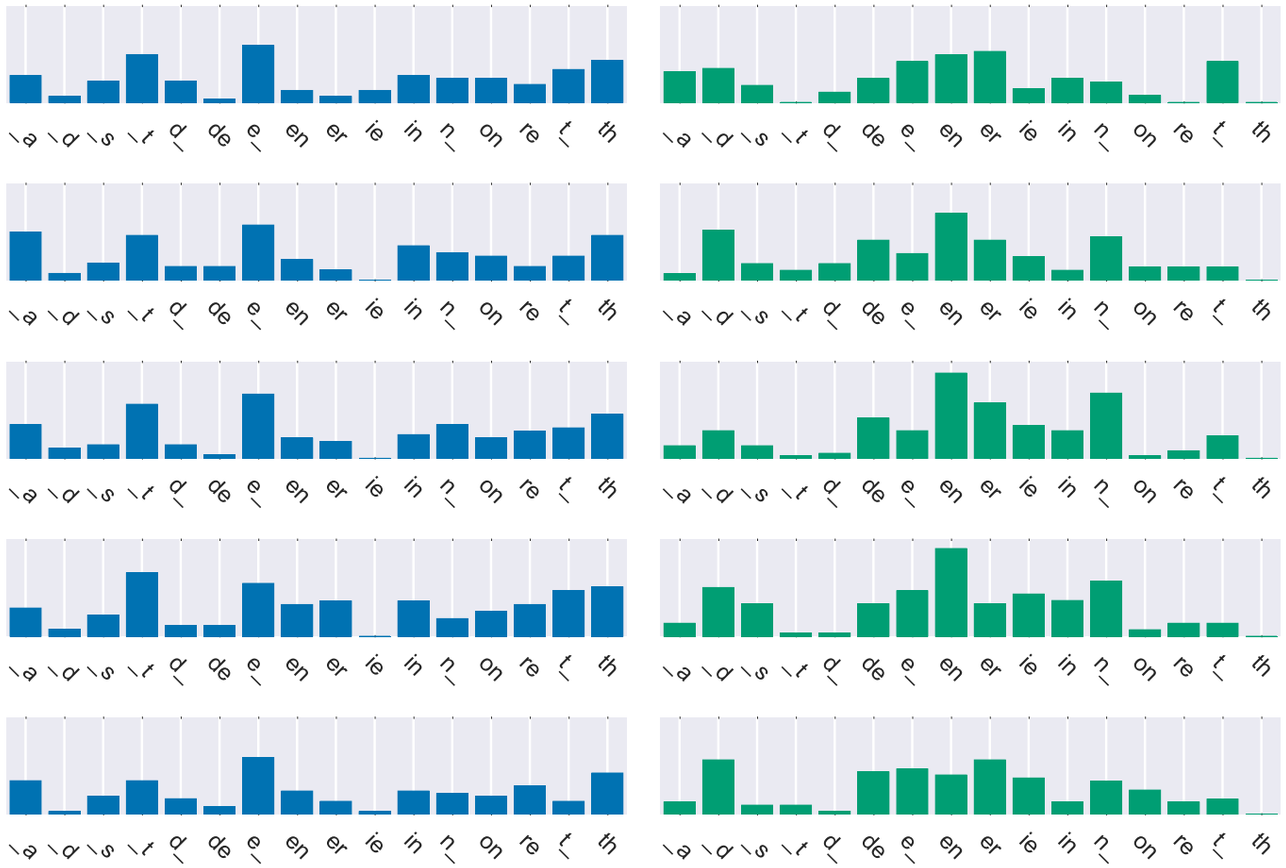
Xavier Glorot et al's suggestion:

$$\epsilon = \frac{\sqrt{(6)}}{\sqrt{m+n}}$$

Representations

Representations

Language Identification for 6 languages based on letter bigram counts.



$$\hat{\mathbf{y}} = f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{W} + \mathbf{b}$$

$$\hat{y} = \underset{i}{\operatorname{argmax}}\, \hat{\mathbf{y}}_{[i]}$$

Representations

assume 28 letters (including space).

the vector **x** is 784 dimensional vector.

each entry is the count for a particular letter pair.

$$\hat{\mathbf{y}} = f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{W} + \mathbf{b}$$

$$\hat{y} = \underset{i}{\operatorname{argmax}}\, \hat{\mathbf{y}}_{[i]}$$

Representations

consider the values $\hat{\mathbf{y}}$

$$\hat{\mathbf{y}} = f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{W} + \mathbf{b}$$

$$\hat{y} = \underset{i}{\operatorname{argmax}}\, \hat{\mathbf{y}}_{[i]}$$

Representations

consider the 6 columns of **W**

$$\hat{\mathbf{y}} = f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{W} + \mathbf{b}$$

$$\hat{y} = \underset{i}{\operatorname{argmax}}\, \hat{\mathbf{y}}_{[i]}$$

Representations

consider the 784 rows of **W**

$$\hat{\mathbf{y}} = f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{W} + \mathbf{b}$$

$$\hat{y} = \underset{i}{\operatorname{argmax}}\, \hat{\mathbf{y}}_{[i]}$$

Representations

think of **x** as a sum of one-hot vectors.

what is **xW** ?

$$\hat{\mathbf{y}} = f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{W} + \mathbf{b}$$

$$\hat{y} = \underset{i}{\operatorname{argmax}}\, \hat{\mathbf{y}}_{[i]}$$

Representations

what happens if we add layers?

$$\hat{\mathbf{y}} = g(\mathbf{x}\mathbf{W})\mathbf{U}$$

$$\mathbf{W} \in \mathbb{R}^{784\times 30} \quad \mathbf{U} \in \mathbb{R}^{30\times 6}$$

Language Modeling

Let's talk about sequences

- Predicting how a sequence will continue.

מה אתה _

Language Model

$$p(x_i|x_1, ..., x_{i-1})$$

Language Model: Markov Assumption

$$p(x_i|x_1, ..., x_{i-1}) \approx p(x_i|x_{i-4}, x_{i-3}, x_{i-2}, x_{i-1})$$

Language Model: Markov Assumption

$$p(x_i|x_1, ..., x_{i-1}) \approx p(x_i|x_{i-4}, x_{i-3}, x_{i-2}, x_{i-1})$$

(condition only on last n items)

this is called n-gram language model

Language Model

- LM can also be used to assign a probability to a sequence.

$$
\begin{aligned}
p(x_1, ..., x_n) = & p_{LM}(x_1 | \texttt{*S*}, \texttt{*S*}) \\
& \times p_{LM}(x_2 | \texttt{*S*}, x_1) \\
& \times p_{LM}(x_3 | x_1, x_2) \\
& \times p_{LM}(x_4 | x_2, x_3) \\
& \dots \\
& \times p_{LM}(x_n | x_{n-2}, x_{n-1})
\end{aligned}
$$

Language Model

- Very useful (used in Speech Recognition, Machine Translation.. and many others).
- Does not have to be over natural language.
- Huge research topic. We'll see a neural LM.

Neural LM

$$p(x_k|x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1}) = \text{softmax}(\text{MLP}(\mathbf{x}))$$

$$\mathbf{x} = encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

Neural LM

$$p(x_k|x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1}) = \text{softmax}(\text{MLP}(\mathbf{x}))$$

$$\mathbf{x} = \mathit{encode}(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

$$\text{softmax}(g(g(\mathbf{x}\mathbf{W^1} + \mathbf{b^1})\mathbf{W^2} + \mathbf{b^2})\mathbf{W^3} + \mathbf{b^3})$$

$$\text{softmax}(\square)$$

$$\uparrow$$

$$\square\mathbf{W^3} + \mathbf{b^3}$$

$$\uparrow$$

$$g(\square\mathbf{W^2} + \mathbf{b^2})$$

$$\uparrow$$

$$g(\square\mathbf{W^1} + \mathbf{b^1})$$

$$\uparrow$$

$$\mathbf{x}$$

$$\uparrow$$

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

$$\mathbb{R}^{d_{out}}$$

$$\text{softmax}(\square)$$

$$\mathbb{R}^{d_{out}}$$

$$\uparrow$$

$$\square\mathbf{W^3} + \mathbf{b^3}$$

$$\mathbb{R}^{d_2}$$

$$\uparrow$$

$$g(\square\mathbf{W^2} + \mathbf{b^2})$$

$$\mathbb{R}^{d_1}$$

$$\uparrow$$

$$g(\square\mathbf{W^1} + \mathbf{b^1})$$

$$\mathbb{R}^{d_{in}}$$

$$\uparrow$$

$$\mathbf{x}$$

$$\uparrow$$

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

$\mathbb{R}^{d_{out}}$

$\text{softmax}(\square)$

$\mathbb{R}^{d_{out}}$

$\uparrow$

$\square\mathbf{W^3} + \mathbf{b^3}$

$\mathbb{R}^{d_2}$

$\uparrow$

$g(\square\mathbf{W^2} + \mathbf{b^2})$

$\mathbb{R}^{d_1}$

$\uparrow$

$g(\square\mathbf{W^1} + \mathbf{b^1})$

$\mathbb{R}^{d_{in}}$

$\uparrow$

$\mathbf{x}$

$\uparrow$

$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$

Encoding k elements

$$\text{encode}(x_1, x_2, x_3, x_4)$$

We have k elements in a vocabulary of size |V|

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$$\text{encode}(x_1, x_2, x_3, x_4)$$

We have k elements in a vocabulary of size |V|

4

10

V={A,B,C,D,E,F,G,H,I,J}

Encoding k elements

$$\mathrm{encode}(D, A, G, C)$$

A= [1,0,0,0,0,0,0,0,0,0]
B= [0,1,0,0,0,0,0,0,0,0]
C= [0,0,1,0,0,0,0,0,0,0]
D= [0,0,0,1,0,0,0,0,0,0]
E= [0,0,0,0,1,0,0,0,0,0]
F= [0,0,0,0,0,1,0,0,0,0]
G= [0,0,0,0,0,0,1,0,0,0]
H= [0,0,0,0,0,0,0,1,0,0]
I= [0,0,0,0,0,0,0,0,1,0]
J= [0,0,0,0,0,0,0,0,0,1]

Encoding k elements

$$\text{encode}(D, A, G, C)$$

A= [1,0,0,0,0,0,0,0,0,0]
B= [0,1,0,0,0,0,0,0,0,0]
C= [0,0,1,0,0,0,0,0,0,0]
D= [0,0,0,1,0,0,0,0,0,0]
E= [0,0,0,0,1,0,0,0,0,0]
F= [0,0,0,0,0,1,0,0,0,0]
G= [0,0,0,0,0,0,1,0,0,0]
H= [0,0,0,0,0,0,0,1,0,0]
I= [0,0,0,0,0,0,0,0,1,0]
J= [0,0,0,0,0,0,0,0,0,1]

$$\mathbf{v}_D + \mathbf{v}_A + \mathbf{v}_G + \mathbf{v}_C$$

[0,0,0,1,0,0,0,0,0,0]
+
[1,0,0,0,0,0,0,0,0,0]
+
[0,0,0,0,0,0,1,0,0,0]
+
[0,0,1,0,0,0,0,0,0,0]
=
[1,0,0,1,0,0,1,0,0,0]

Encoding k elements

$$\text{encode}(D, A, G, C)$$

A= [1,0,0,0,0,0,0,0,0,0]
B= [0,1,0,0,0,0,0,0,0,0]
C= [0,0,1,0,0,0,0,0,0,0]
D= [0,0,0,1,0,0,0,0,0,0]
E= [0,0,0,0,1,0,0,0,0,0]
F= [0,0,0,0,0,1,0,0,0,0]
G= [0,0,0,0,0,0,1,0,0,0]
H= [0,0,0,0,0,0,0,1,0,0]
I= [0,0,0,0,0,0,0,0,1,0]
J= [0,0,0,0,0,0,0,0,0,1]

$$\mathbf{v}_D + \mathbf{v}_A + \mathbf{v}_G + \mathbf{v}_C$$

[0,0,0,1,0,0,0,0,0,0]
+
[1,0,0,0,0,0,0,0,0,0]
+
[0,0,0,0,0,0,1,0,0,0]
+
[0,0,1,0,0,0,0,0,0,0]
=
[1,0,1,1,0,0,1,0,0,0]

what does this miss?

Encoding k elements

$$\text{encode}(D, A, G, C)$$

A= [1,0,0,0,0,0,0,0,0,0]
B= [0,1,0,0,0,0,0,0,0,0]
C= [0,0,1,0,0,0,0,0,0,0]
D= [0,0,0,1,0,0,0,0,0,0]
E= [0,0,0,0,1,0,0,0,0,0]
F= [0,0,0,0,0,1,0,0,0,0]
G= [0,0,0,0,0,0,1,0,0,0]
H= [0,0,0,0,0,0,0,1,0,0]
I= [0,0,0,0,0,0,0,0,1,0]
J= [0,0,0,0,0,0,0,0,0,1]

$$\mathbf{v}_D \circ \mathbf{v}_A \circ \mathbf{v}_G \circ \mathbf{v}_C$$

Encoding k elements

$$\text{encode}(D, A, G, C)$$

$$\mathbf{v}_D \circ \mathbf{v}_A \circ \mathbf{v}_G \circ \mathbf{v}_C$$

[0,0,0,1,0,0,0,0,0,0]
o
[1,0,0,0,0,0,0,0,0,0]
o
[0,0,0,0,0,0,1,0,0,0]
o
[0,0,1,0,0,0,0,0,0,0]
=
[0,0,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0]

$\mathbb{R}^{d_{out}}$

$\text{softmax}(\square)$

$\mathbb{R}^{d_{out}}$

$\uparrow$

$\square\mathbf{W^3} + \mathbf{b^3}$

$\mathbb{R}^{d_2}$

$\uparrow$

$g(\square\mathbf{W^2} + \mathbf{b^2})$

$\mathbb{R}^{d_1}$

$\uparrow$

$g(\square\mathbf{W^1} + \mathbf{b^1})$

$\mathbb{R}^{d_{in}}$

$\uparrow$

$\mathbf{x}$

$\uparrow$

$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$

$$(\mathbf{v}_D + \mathbf{v}_A + \mathbf{v}_G + \mathbf{v}_C)\mathbf{W}$$

[0,0,0,1,0,0,0,0,0,0]
+
[1,0,0,0,0,0,0,0,0,0]
+
[0,0,0,0,0,0,1,0,0,0]
+
[0,0,1,0,0,0,0,0,0,0]
=
[1,0,0,1,0,0,1,0,0,0]

W

A= [-0.32, 0.09, 0.33,-0.44]
B= [0.29, 0.02,-0.46,-0.39]
C= [-0.46, 0.24,-0.16, 0.08]
D= [-0.15,-0.31, 0.34, 0.00]
E= [-0.10,-0.37, 0.01, 0.40]
F= [-0.28,-0.26,-0.24, 0.31]
G= [-0.32,-0.42,-0.21, 0.18]
H= [-0.09,-0.01, 0.06, 0.14]
I= [0.28,-0.02,-0.39, 0.12]
J= [0.23,-0.22,-0.14, 0.28]

$$(\mathbf{v}_D + \mathbf{v}_A + \mathbf{v}_G + \mathbf{v}_C)\mathbf{W}$$

$$= \mathbf{v}_D \cdot \mathbf{W} + \mathbf{v}_A \cdot \mathbf{W} + \mathbf{v}_G \cdot \mathbf{W} + \mathbf{v}_C \cdot \mathbf{W}$$

[0,0,0,1,0,0,0,0,0,0]
+
[1,0,0,0,0,0,0,0,0,0]
+
[0,0,0,0,0,0,1,0,0,0]
+
[0,0,1,0,0,0,0,0,0,0]
=
[1,0,0,1,0,0,1,0,0,0]

W

A= [-0.32, 0.09, 0.33,-0.44]
B= [0.29, 0.02,-0.46,-0.39]
C= [-0.46, 0.24,-0.16, 0.08]
D= [-0.15,-0.31, 0.34, 0.00]
E= [-0.10,-0.37, 0.01, 0.40]
F= [-0.28,-0.26,-0.24, 0.31]
G= [-0.32,-0.42,-0.21, 0.18]
H= [-0.09,-0.01, 0.06, 0.14]
I= [0.28,-0.02,-0.39, 0.12]
J= [0.23,-0.22,-0.14, 0.28]

$$(\mathbf{v}_D + \mathbf{v}_A + \mathbf{v}_G + \mathbf{v}_C)\mathbf{W}$$

$$= \mathbf{v}_D \cdot \mathbf{W} + \mathbf{v}_A \cdot \mathbf{W} + \mathbf{v}_G \cdot \mathbf{W} + \mathbf{v}_C \cdot \mathbf{W}$$

sum of rows in **W**

each row corresponds to a certain vocabulary item.

$$(\mathbf{v}_D \circ \mathbf{v}_A \circ \mathbf{v}_G \circ \mathbf{v}_C)\mathbf{W}$$

$$(\mathbf{v}_D \circ \mathbf{v}_A \circ \mathbf{v}_G \circ \mathbf{v}_C)\mathbf{W}$$

still sum of rows in **W**
but **W** has 4x many rows.

[0,0,0,1,0,0,0,0,0,0]

o

[1,0,0,0,0,0,0,0,0,0]

o

[0,0,0,0,0,0,1,0,0,0]

o

[0,0,1,0,0,0,0,0,0,0]

=

[0,0,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0]

A(-3)= [0.42,-0.15, 0.12, 0.02]
B(-3)= [0.28,-0.15,-0.11, 0.32]
C(-3)= [0.15,-0.24, 0.23, 0.41]
D(-3)= [-0.12,-0.24, 0.12,-0.34]
E(-3)= [-0.42,-0.21, 0.08, 0.40]
F(-3)= [0.20, 0.11,-0.31, 0.33]
G(-3)= [0.07,-0.05, 0.16, 0.23]
H(-3)= [0.28, 0.03, 0.22,-0.49]
I(-3)= [0.08, 0.39,-0.25, 0.27]
J(-3)= [0.10,-0.42,-0.37, 0.35]
A(-2)= [-0.00, 0.41, 0.19, 0.49]
B(-2)= [0.24, 0.48, 0.34,-0.42]
C(-2)= [-0.46, 0.22, 0.24,-0.21]
D(-2)= [-0.11,-0.48, 0.18,-0.22]
E(-2)= [-0.32, 0.10,-0.41,-0.43]
F(-2)= [0.32, 0.02,-0.22, 0.06]
G(-2)= [-0.31,-0.36, 0.09, 0.39]
H(-2)= [0.01,-0.22,-0.09,-0.15]
I(-2)= [0.01, 0.10,-0.16,-0.21]
J(-2)= [-0.24, 0.40,-0.34,-0.13]
A(-1)= [-0.23,-0.38, 0.02, 0.32]
B(-1)= [-0.34, 0.04,-0.18,-0.00]
C(-1)= [0.40,-0.02, 0.10,-0.16]
D(-1)= [0.13,-0.07,-0.19,-0.01]
E(-1)= [0.40, 0.27,-0.33, 0.36]
F(-1)= [0.04,-0.13,-0.43, 0.39]
G(-1)= [0.44, 0.38, 0.03,-0.39]
H(-1)= [0.41,-0.23, 0.33,-0.08]
I(-1)= [-0.50,-0.16,-0.42,-0.27]
J(-1)= [-0.15, 0.41, 0.46,-0.16]
A(+0)= [-0.11, 0.03, 0.20, 0.50]
B(+0)= [0.16,-0.34, 0.20,-0.21]
C(+0)= [0.05,-0.13,-0.23,-0.31]
D(+0)= [0.13,-0.02, 0.38,-0.09]
E(+0)= [0.30, 0.39, 0.10, 0.38]
F(+0)= [-0.16,-0.31,-0.02,-0.34]
G(+0)= [0.06,-0.04, 0.02,-0.32]
H(+0)= [0.25, 0.30, 0.29, 0.24]
I(+0)= [0.40,-0.17,-0.18,-0.19]
J(+0)= [0.27, 0.33,-0.42,-0.07]

$$(\mathbf{v}_D \circ \mathbf{v}_A \circ \mathbf{v}_G \circ \mathbf{v}_C)\mathbf{W}$$

still sum of rows in **W**
but **W** has 4x many rows.

alternatively:

$$= \mathbf{v}_D \cdot \mathbf{W}' + \mathbf{v}_A \cdot \mathbf{W}'' + \mathbf{v}_G \cdot \mathbf{W}''' + \mathbf{v}_C \cdot \mathbf{W}''''$$

$$(\mathbf{v}_D \circ \mathbf{v}_A \circ \mathbf{v}_G \circ \mathbf{v}_C)\mathbf{W}$$

still sum of rows in **W**
but **W** has 4x many rows.

alternatively:

$$= \mathbf{v}_D \cdot \mathbf{W}' + \mathbf{v}_A \cdot \mathbf{W}'' + \mathbf{v}_G \cdot \mathbf{W}''' + \mathbf{v}_C \cdot \mathbf{W}''''$$

$$\mathbf{W} = \mathbf{W}' \circ \mathbf{W}'' \circ \mathbf{W}''' \circ \mathbf{W}''''$$

[0,0,0,1,0,0,0,0,0,0]

o

[1,0,0,0,0,0,0,0,0,0]

o

[0,0,0,0,0,0,1,0,0,0]

o

[0,0,1,0,0,0,0,0,0,0]

A(-3)= [0.42,-0.15, 0.12, 0.02]
B(-3)= [0.28,-0.15,-0.11, 0.32]
C(-3)= [0.15,-0.24, 0.23, 0.41]
D(-3)= [-0.12,-0.24, 0.12,-0.34]
E(-3)= [-0.42,-0.21, 0.08, 0.40]
F(-3)= [0.20, 0.11,-0.31, 0.33]
G(-3)= [0.07,-0.05, 0.16, 0.23]
H(-3)= [0.28, 0.03, 0.22,-0.49]
I(-3)= [0.08, 0.39,-0.25, 0.27]
J(-3)= [0.10,-0.42,-0.37, 0.35]

W'

A(-2)= [-0.00, 0.41, 0.19, 0.49]
B(-2)= [0.24, 0.48, 0.34,-0.42]
C(-2)= [-0.46, 0.22, 0.24,-0.21]
D(-2)= [-0.11,-0.48, 0.18,-0.22]
E(-2)= [-0.32, 0.10,-0.41,-0.43]
F(-2)= [0.32, 0.02,-0.22, 0.06]
G(-2)= [-0.31,-0.36, 0.09, 0.39]
H(-2)= [0.01,-0.22,-0.09,-0.15]
I(-2)= [0.01, 0.10,-0.16,-0.21]
J(-2)= [-0.24, 0.40,-0.34,-0.13]

W''

A(-1)= [-0.23,-0.38, 0.02, 0.32]
B(-1)= [-0.34, 0.04,-0.18,-0.00]
C(-1)= [0.40,-0.02, 0.10,-0.16]
D(-1)= [0.13,-0.07,-0.19,-0.01]
E(-1)= [0.40, 0.27,-0.33, 0.36]
F(-1)= [0.04,-0.13,-0.43, 0.39]
G(-1)= [0.44, 0.38, 0.03,-0.39]
H(-1)= [0.41,-0.23, 0.33,-0.08]
I(-1)= [-0.50,-0.16,-0.42,-0.27]
J(-1)= [-0.15, 0.41, 0.46,-0.16]

W'''

A(+0)= [-0.11, 0.03, 0.20, 0.50]
B(+0)= [0.16,-0.34, 0.20,-0.21]
C(+0)= [0.05,-0.13,-0.23,-0.31]
D(+0)= [0.13,-0.02, 0.38,-0.09]
E(+0)= [0.30, 0.39, 0.10, 0.38]
F(+0)= [-0.16,-0.31,-0.02,-0.34]
G(+0)= [0.06,-0.04, 0.02,-0.32]
H(+0)= [0.25, 0.30, 0.29, 0.24]
I(+0)= [0.40,-0.17,-0.18,-0.19]
J(+0)= [0.27, 0.33,-0.42,-0.07]

W''''

- 1-hot times matrix: row selection
- sum of 1-hot times matrix: row selection + sum
- concat of 1-hot: like using 1-hot from larger vocab

"Embedding Layer"

- Very common in neural network land:
  - associate each vocabulary item with a row in matrix **E** of dense vectors (dim of row << |V|)
  - concat or sum rows of **E** for input.

"Embedding Layer"

$$\text{encode}(D, A, G, C)$$

$$= \mathbf{E}_{[D]} \circ \mathbf{E}_{[A]} \circ \mathbf{E}_{[G]} \circ \mathbf{E}_{[C]}$$

E

A= [-0.32, 0.09, 0.33,-0.44]
B= [0.29, 0.02,-0.46,-0.39]
C= [-0.46, 0.24,-0.16, 0.08]
D= [-0.15,-0.31, 0.34, 0.00]
E= [-0.10,-0.37, 0.01, 0.40]
F= [-0.28,-0.26,-0.24, 0.31]
G= [-0.32,-0.42,-0.21, 0.18]
H= [-0.09,-0.01, 0.06, 0.14]
I= [0.28,-0.02,-0.39, 0.12]
J= [0.23,-0.22,-0.14, 0.28]

[-0.15,-0.31, 0.34, 0.00,-0.32, 0.09, 0.33,-0.44,-0.32,-0.42,-0.21, 0.18,-0.46, 0.24,-0.16, 0.08]

$$\mathbb{R}^{d_{out}}$$

$$\text{softmax}(\square)$$

$$\mathbb{R}^{d_{out}}$$

$$\uparrow$$

$$\square\mathbf{W^3} + \mathbf{b^3}$$

$$\mathbb{R}^{d_2}$$

$$\uparrow$$

$$g(\square\mathbf{W^2} + \mathbf{b^2})$$

$$\uparrow \qquad \mathbb{R}^{d_1}$$

$$g(\square\mathbf{W^1} + \mathbf{b^1})$$

$$\uparrow \qquad \mathbb{R}^{d_{in}}$$

$$\mathbf{E}_{[x_{k-4}]} \circ \mathbf{E}_{[x_{k-3}]} \circ \mathbf{E}_{[x_{k-2}]} \circ \mathbf{E}_{[x_{k-1}]}$$

$$\uparrow$$

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

"Embedding Layer"

$$\text{encode}(D, A, G, C)$$

$$= \mathbf{E}_{[D]} \circ \mathbf{E}_{[A]} \circ \mathbf{E}_{[G]} \circ \mathbf{E}_{[C]}$$

E

A= [-0.32, 0.09, 0.33,-0.44]
B= [0.29, 0.02,-0.46,-0.39]
C= [-0.46, 0.24,-0.16, 0.08]
D= [-0.15,-0.31, 0.34, 0.00]
E= [-0.10,-0.37, 0.01, 0.40]
F= [-0.28,-0.26,-0.24, 0.31]
G= [-0.32,-0.42,-0.21, 0.18]
H= [-0.09,-0.01, 0.06, 0.14]
I= [0.28,-0.02,-0.39, 0.12]
J= [0.23,-0.22,-0.14, 0.28]

[-0.15,-0.31, 0.34, 0.00,-0.32, 0.09, 0.33,-0.44,-0.32,-0.42,-0.21, 0.18,-0.46, 0.24,-0.16, 0.08]

how does this relate to what we had before?

what is

$$(\mathbf{E}_{[D]} \circ \mathbf{E}_{[A]} \circ \mathbf{E}_{[G]} \circ \mathbf{E}_{[C]})\mathbf{W}$$

?

"Embedding Layer"

$$(\mathbf{E}_{[D]} \circ \mathbf{E}_{[A]} \circ \mathbf{E}_{[G]} \circ \mathbf{E}_{[C]})\mathbf{W}$$

same row in **E** regardless of position in the sequence.

but **W** will transform this row differently for each position.

$$\mathbb{R}^{d_{out}}$$

$$\text{softmax}(\square)$$

$$\mathbb{R}^{d_{out}}$$

$$\uparrow$$

$$\square\mathbf{W^3} + \mathbf{b^3}$$

$$\mathbb{R}^{d_2}$$

$$\uparrow$$

$$g(\square\mathbf{W^2} + \mathbf{b^2})$$

$$\uparrow \qquad \mathbb{R}^{d_1}$$

$$g(\square\mathbf{W^1} + \mathbf{b^1})$$

$$\uparrow \qquad \mathbb{R}^{d_{in}}$$

$$\mathbf{E}_{[x_{k-4}]} \circ \mathbf{E}_{[x_{k-3}]} \circ \mathbf{E}_{[x_{k-2}]} \circ \mathbf{E}_{[x_{k-1}]}$$

$$\uparrow$$

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

what's in **W1**?
what's in **W2**?
what's in **W3**?

$$\mathbb{R}^{d_{out}}$$

$$\text{softmax}(\square)$$

$$\mathbb{R}^{d_{out}}$$

$$\uparrow$$

$$\square\mathbf{W^3} + \mathbf{b^3}$$

$$\mathbb{R}^{d_2}$$

$$\uparrow$$

$$g(\square\mathbf{W^2} + \mathbf{b^2})$$

$$\uparrow \qquad \mathbb{R}^{d_1}$$

$$g(\square\mathbf{W^1} + \mathbf{b^1})$$

$$\uparrow \qquad \mathbb{R}^{d_{in}}$$

$$\mathbf{E}_{[x_{k-4}]} \circ \mathbf{E}_{[x_{k-3}]} \circ \mathbf{E}_{[x_{k-2}]} \circ \mathbf{E}_{[x_{k-1}]}$$

$$\uparrow$$

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

Neural LM

$$\text{softmax}(\square)$$

$$\uparrow$$

$$\square\mathbf{W^3} + \mathbf{b^3}$$

$$\uparrow$$

$$g(\square\mathbf{W^2} + \mathbf{b^2})$$

$$\uparrow$$

$$g(\square\mathbf{W^1} + \mathbf{b^1})$$

$$\uparrow$$

$$\mathbf{E}_{[x_{k-4}]} \circ \mathbf{E}_{[x_{k-3}]} \circ \mathbf{E}_{[x_{k-2}]} \circ \mathbf{E}_{[x_{k-1}]}$$

$$\uparrow$$

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

Neural LM

A Neural Probabilistic Language Model

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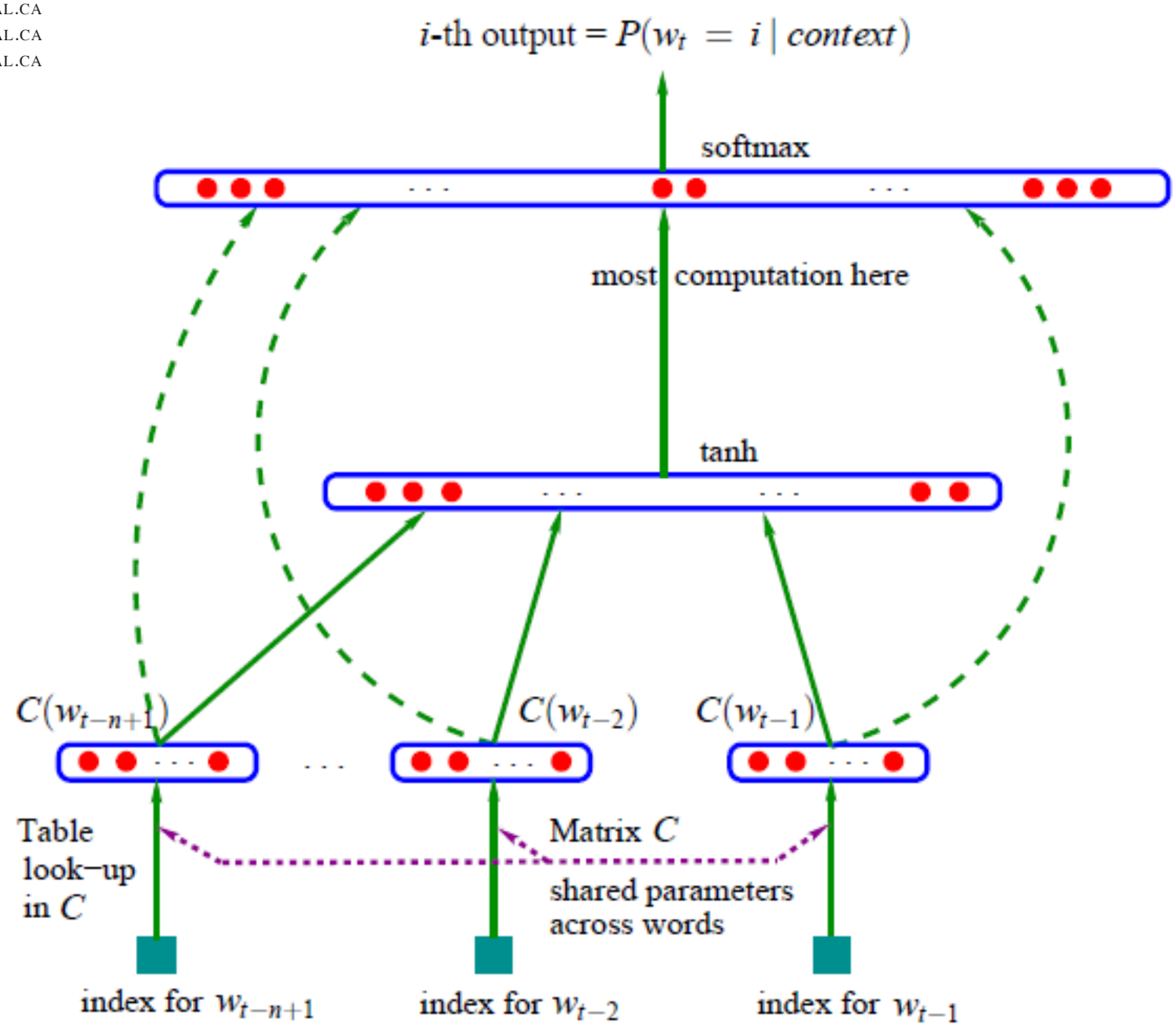
Université de Montréal, Montréal, Québec, Canada

Bengio et al 2003

Pretty much same model,
But fewer layers.



Neural LM

- What is the cost of increasing the history length?

Neural LM

- We can use a trained LM for scoring a given sentence. (how?)

Neural LM

- We can use a trained LM for comparing two given sentences. (how?)

Neural LM

- We can use a trained LM for **generating new sentences**. (how?)

Neural LM

- We can use K trained LMs for **k-class classification**. (how?)

Neural LM

- We can use K trained LMs for **k-class classification**. (how?)

$$p(w_1, w_1, ..., w_n|LM_1)$$
$$p(w_1, w_1, ..., w_n|LM_2)$$
$$p(w_1, w_1, ..., w_n|LM_3)$$

$$\hat{y} = \arg\max_k p(w_1, w_1, ..., w_n|LM_k)$$

sequence prediction tasks

- In LM:
  - predict **word i** based on **k previous words**.
- But we could also predict **label** based on **k items**.
  - which tasks can this be used for?

Neural LM

$$\text{softmax}(\square)$$

softmax over words

$$\uparrow$$

$$\square\mathbf{W^3} + \mathbf{b^3}$$

$$\uparrow$$

$$g(\square\mathbf{W^2} + \mathbf{b^2})$$

$$\uparrow$$

$$g(\square\mathbf{W^1} + \mathbf{b^1})$$

$$\uparrow$$

$$\mathbf{E}_{[x_{k-4}]} \circ \mathbf{E}_{[x_{k-3}]} \circ \mathbf{E}_{[x_{k-2}]} \circ \mathbf{E}_{[x_{k-1}]}$$

$$\uparrow$$

window of prev words

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

Sequence Prediction

$$\text{softmax}(\square)$$

softmax over k classes

$$\uparrow$$

$$\square\mathbf{W^3} + \mathbf{b^3}$$

$$\uparrow$$

$$g(\square\mathbf{W^2} + \mathbf{b^2})$$

$$\uparrow$$

$$g(\square\mathbf{W^1} + \mathbf{b^1})$$

$$\uparrow$$

$$\mathbf{E}_{[x_{k-4}]} \circ \mathbf{E}_{[x_{k-3}]} \circ \mathbf{E}_{[x_{k-2}]} \circ \mathbf{E}_{[x_{k-1}]}$$

$$\uparrow$$

sequence items

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$

classification

ACGCGCTCGATCG	V
GTGCGCTCGAACG	V
ACGCGTTCTACCG	X

tagging

DET ADJ NOUN VERB PREP DET ADJ NOUN

The brown fox jumped over the lazy dog

tagging

Window-based approach

NOUN

The brown **fox** jumped over

VERB

brown fox **jumped** over the

PREP

fox jumped **over** the lazy

back to LM

Training a language model

- Set dimensions of **E, W3**, according to vocab size.
- Initial random values for **E, W1, W2, W3, b1, b2, b3**
- For every n-tuple in some text:
    - try to predict last item based on prev n-1
    - use cross-entropy loss.

What happens after training?

- Consider the columns of **W3**.

- Consider the rows of **E**.

what's in **W3**?

columns of W3 correspond to vocab items!

$$\text{softmax}(\square) \qquad \mathbb{R}^{d_{out}}$$

$$\uparrow$$

$$\square\mathbf{W^3} + \mathbf{b^3} \qquad \mathbb{R}^{d_{out}}$$

$$\uparrow$$

$$g(\square\mathbf{W^2} + \mathbf{b^2}) \qquad \mathbb{R}^{d_2}$$

$$\uparrow$$

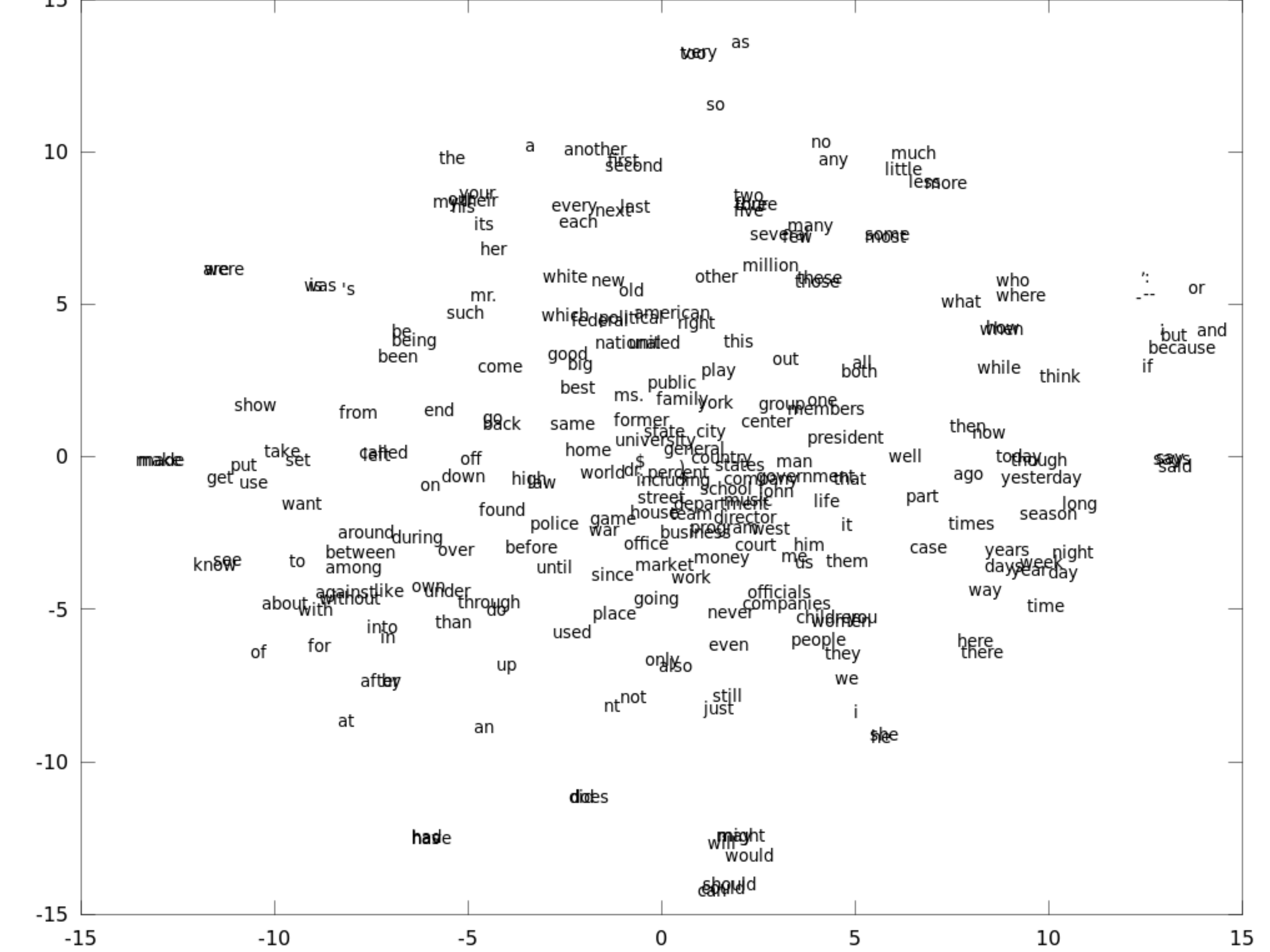
$$g(\square\mathbf{W^1} + \mathbf{b^1}) \qquad \mathbb{R}^{d_1}$$

$$\uparrow$$

$$\mathbf{E}_{[x_{k-4}]} \circ \mathbf{E}_{[x_{k-3}]} \circ \mathbf{E}_{[x_{k-2}]} \circ \mathbf{E}_{[x_{k-1}]} \qquad \mathbb{R}^{d_{in}}$$

$$\uparrow$$

$$encode(x_{k-4}, x_{k-3}, x_{k-2}, x_{k-1})$$



15
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some
most
million
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while
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play
public
best
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family
york
group
one
members
show
from
end
back
same
former
state
city
center
president
then
now
university
general
take
called
left
home
country
states
man
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made
put
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world
percent
government
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school
john
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street
department
life
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found
game
team
director
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house
war
business
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office
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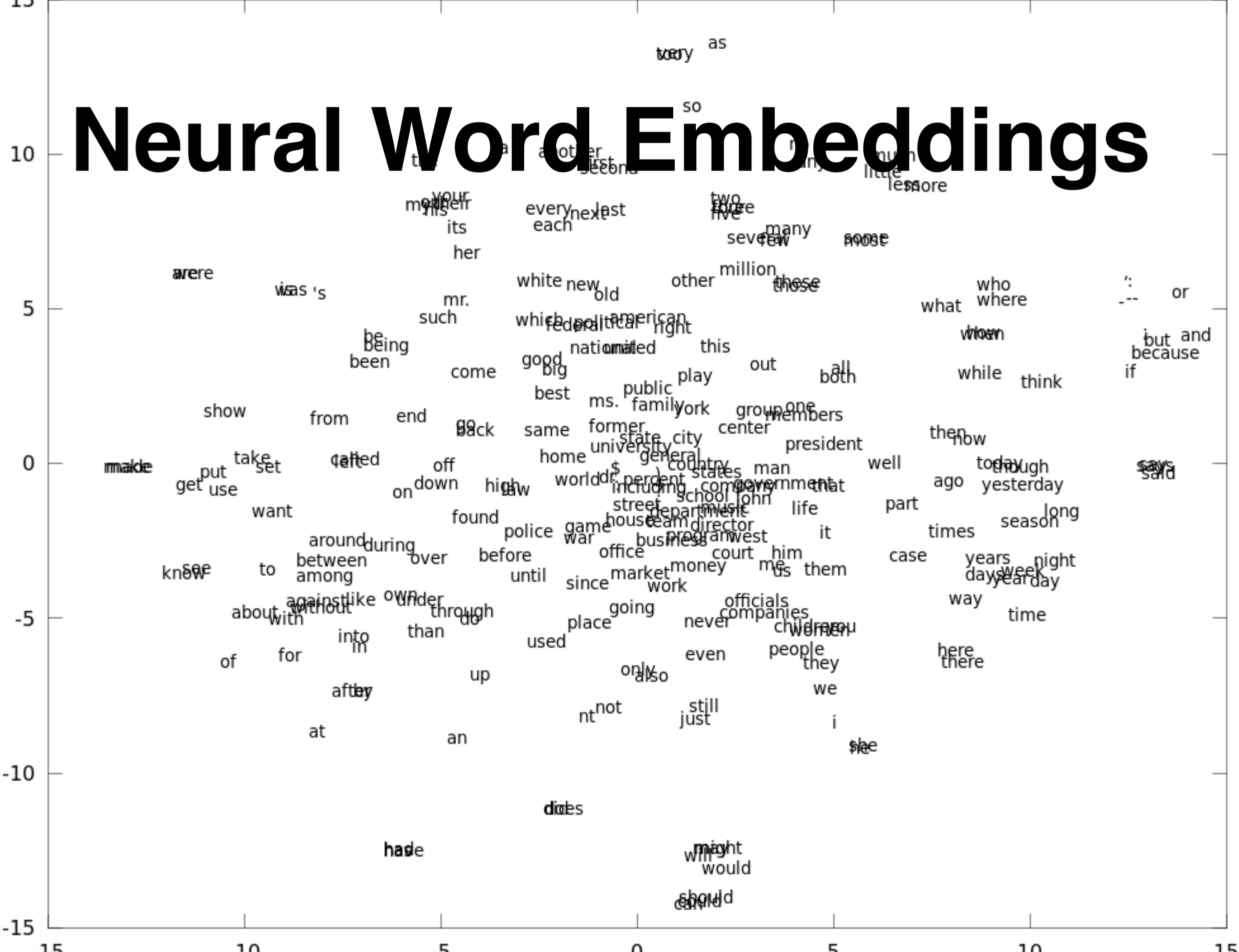
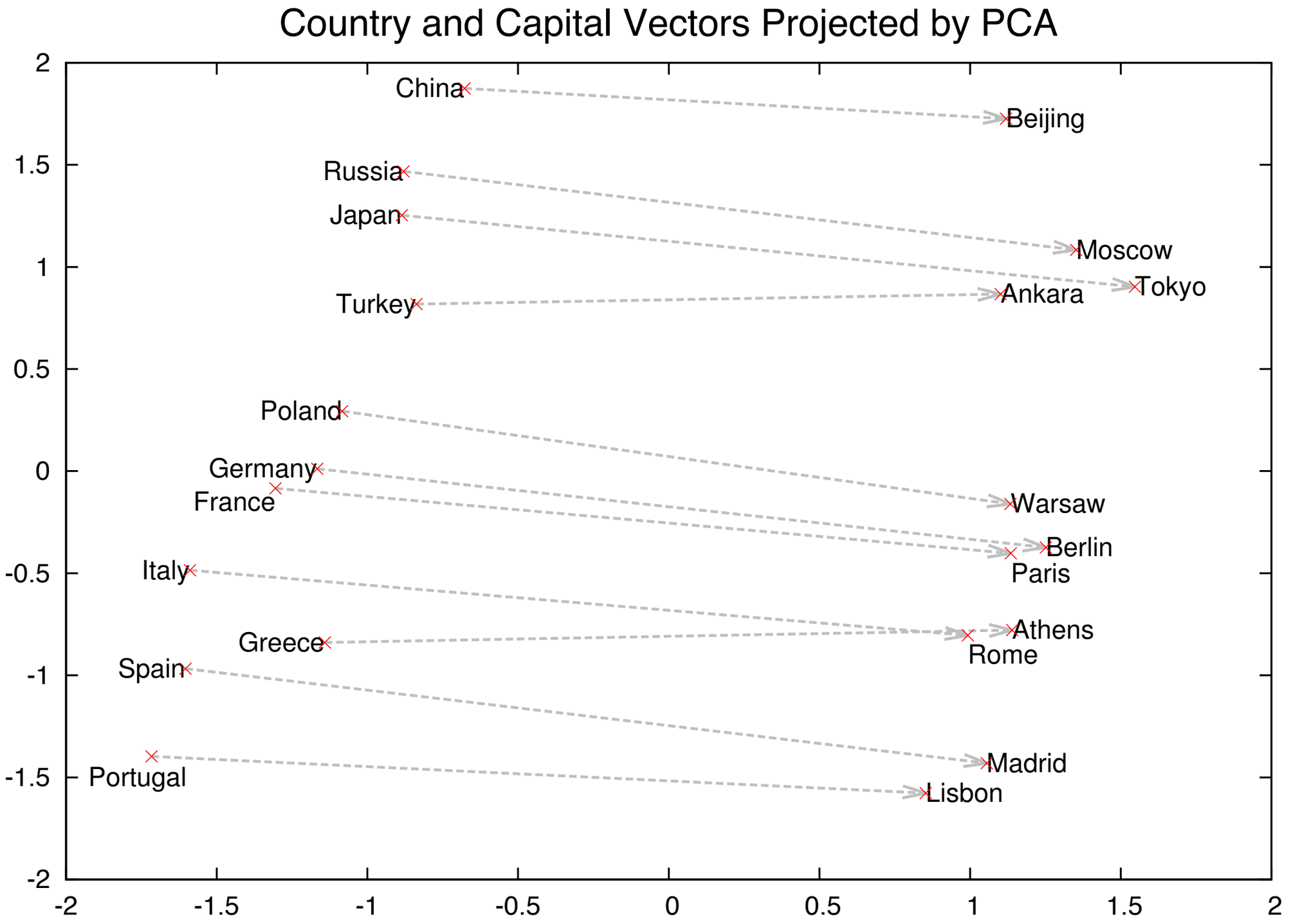
can

Neural Word Embeddings



Neural Word Embeddings

Neural Word Embeddings



Neural Word Embeddings

Target Word	BoW5
batman	nightwing aquaman catwoman superman manhunter
hogwarts	dumbledore hallows half-blood malfoy snape
turing	nondeterministic non-deterministic computability deterministic finite-state
florida	gainesville fla jacksonville tampa lauderdale
object-oriented	aspect-oriented smalltalk event-driven prolog domain-specific
dancing	singing dance dances dancers tap-dancing

Neural Word Embeddings

- We trained a language model.
- We ended up with **vector representations of words**.
- These representations are useful -- they encode various aspects of word similarity.

tagging + pre-training

we can use the **E** we got from LM training to initialize **E** for the POS tagging task.

(why is that helpful?)

NOUN

The brown **fox** jumped over

VERB

brown fox **jumped** over the

PREP

fox jumped **over** the lazy

pre-training

- This is a sort of **semi-supervised** learning or **multi-task** learning.

- We learn from "unannotated" data.

- We then use the representations on tasks with annotated data.

Neural Word Embeddings

- We trained a language model.
- We ended up with **vector representations of words**.
- These representations are useful -- they encode various aspects of word similarity.
- **A form of semi-supervised learning.**
- More next week.