Unsupervised Learning and Topic Modeling

Yoav Goldberg

Bar Ilan University

(with slides from David Blei, Zornitsa Kozerava)
Next Time

Look up in the sky! It's a bird!

It's a plane!

Oh no!

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Next Time

Benedict Evans
@BenedictEvans

WHAT DO WE WANT?
Natural language processing!
WHEN DO WE WANT IT?
Sorry, when do we want what?
Previously

Supervised Learning

- Get labeled training data
- Represent data as (features, label) pairs
- Train a classifier / model to predict labels based on features

Today

- What if we don’t have training data?
- Can we still do something useful?

Unsupervised Learning
Things we can do without labeled data
Unsupervised Methods

Option 1: “Naturally occurring” labels / bootstrap

- Be creative and find data which can be used as labels.
  - e.g., **we want to identify paragraphs.** Maybe some website indicate this via their HTML tags?
- Automatically create your own training set
  - Write simple rule-based system to collect easy examples
    - high precision, low recall
  - Use the easy examples as training data
    - Hope it will generalize well.
    - Careful not to overlap your features with the rules too much!
Unsupervised Methods

Option 1: “Naturally occurring” labels / semi-supervised

- Be creative and find data which can be used as labels.
  - **Want to identify sentiment?** Look at tweets with happy and sad emojis.
Unsupervised Methods

Option 1: “Naturally occurring” labels / semi-supervised

- Be creative and find data which can be used as labels.
  - **Want to identify sentiment?** Look at tweets with happy and sad emojis.
  - what are the pros and cons here?
Unsupervised Methods

Option 1: “Naturally occurring” labels / semi-supervised

- Be creative and find data which can be used as labels.
  - *Want to identify sentiment?* Look at tweets with happy and sad emojis.
  - what are the pros and cons here?
- Can also use the *proxy* naturally occurring data for representation learning.
  - The Felbo et al 2017 paper. (next slides)
Using millions of emoji occurrences to learn any-domain representations for detecting sentiment, emotion and sarcasm

Bjarke Felbo¹, Alan Mislove², Anders Søgaard³, Iyad Rahwan¹, Sune Lehmann⁴
Use Emoji Prediction to learn good representations for sentiment
Table 2: The number of tweets in the pretraining dataset associated with each emoji in millions.
<table>
<thead>
<tr>
<th>Sentence</th>
<th>😊</th>
<th>😍</th>
<th>❤</th>
<th>😏</th>
<th>ᵃ</th>
<th>¹⁰⁰</th>
</tr>
</thead>
<tbody>
<tr>
<td>I love mom's cooking</td>
<td>49.1%</td>
<td>8.8%</td>
<td>3.1%</td>
<td>3.0%</td>
<td>2.9%</td>
<td></td>
</tr>
<tr>
<td>I love how you never reply back..</td>
<td>14.0%</td>
<td>8.3%</td>
<td>6.3%</td>
<td>5.4%</td>
<td>5.1%</td>
<td></td>
</tr>
<tr>
<td>I love cruising with my homies</td>
<td>34.0%</td>
<td>6.6%</td>
<td>5.7%</td>
<td>4.1%</td>
<td>3.8%</td>
<td></td>
</tr>
<tr>
<td>I love messing with yo mind!!</td>
<td>17.2%</td>
<td>11.8%</td>
<td>8.0%</td>
<td>6.4%</td>
<td>5.3%</td>
<td></td>
</tr>
<tr>
<td>I love you and now you're just gone..</td>
<td>39.1%</td>
<td>11.0%</td>
<td>7.3%</td>
<td>5.3%</td>
<td>4.5%</td>
<td></td>
</tr>
<tr>
<td>This is shit</td>
<td>7.0%</td>
<td>6.4%</td>
<td>6.0%</td>
<td>6.0%</td>
<td>5.8%</td>
<td></td>
</tr>
<tr>
<td>This is the shit</td>
<td>10.9%</td>
<td>9.7%</td>
<td>6.5%</td>
<td>5.7%</td>
<td>4.8%</td>
<td></td>
</tr>
</tbody>
</table>
why is this useful?
Figure 6: Hierarchical clustering of the DeepMoji model’s predictions across categories on the test set. The dendrogram shows how the model learns to group emojis into overall categories and subcategories based on emotional content. The y-axis is the distance on the correlation matrix of the model’s predictions measured using average linkage.

Figure 7: Correlation matrix of the model’s predictions on the pretraining test set.
Figure 6: Hierarchical clustering of the DeepMoji model’s predictions across categories on the test set. The dendrogram shows how the model learns to group emojis into overall categories and subcategories based on emotional content. The y-axis is the distance on the correlation matrix of the model’s predictions measured using average linkage.
Figure 6: Hierarchical clustering of the DeepMoji model’s predictions across categories on the test set. The dendrogram shows how the model learns to group emojis into overall categories and subcategories based on emotional content. The y-axis is the distance on the correlation matrix of the model’s predictions measured using average linkage.

Predicting a large set can be indicative of more coarse-grained trends.
Train RNN (LSTM) to predict emojis based on a tweet.

• Result: encoder that takes a tweet and returns a vector which is useful for predicting emojis.

Take (smaller) sentiment dataset.

• Encode sentences to vectors using above encoder.

• Train to predict sentiment from vectors.
Unsupervised Methods

Option 2: Write and algorithm and hope it works

- Example: assignment 3.
  - Represent words by their contexts
  - Define the co-occurrence metric (PMI, word2vec)
  - Define similarity measure (cosine)
  - Use this to get a useful result – lists of similar words

- Can be very effective
- But no “learning” involved.
- What to do when this doesn’t work?
Unsupervised Methods

Option 3: Obtain Cheap / Easy Annotations

- Make easy annotation tasks for humans
  - Pose annotation as natural questions that are easy to answer.
- But how to come up with the right questions?
Unsupervised Methods

Option 3: Obtain Cheap / Easy Annotations

- Measure human behavior
  - Eye-tracking when reading
  - Mouse-movement when reading
  - Keyboard clicks when writing
  - ...

- How can these be leveraged to obtain useful data for learning?
Unsupervised Methods

Option 5: Latent-variable generative modeling

- Define a “generative story” of how the data was generated
  - This story doesn’t have to be very convincing or realistic
- The story can include “latent variables”, stuff that you would like to see but you don’t
  - For example: HMM POS-tagging, where we treat the tags as latent.
- Search for an assignment of latent variables such that the data has high probability under the model.
  - Usually, this search is hard.
  - Approximate!
    - EM
    - MCMC (Gibbs sampling)
Unsupervised Learning
Example: HMM

Example: HMM

- We want to train a POS-tagger, but don’t have labeled data.
- We do have a dictionary, associating some words with their possible POS tags, and also a lot of text.
- We will use the dictionary and the text to train a bigram HMM model.
Unsupervised Learning
Example: HMM

The Bigram-HMM generative story:
To generate a tagged sentence \((w, t) = (w_1, \ldots, w_n, t_1, \ldots, t_n)\):

- Start with tag \(t_0 = \text{START}\).
- For \(i\) in \(1, \ldots, n\):
  - Draw a random tag \(t_i\) from the transition distribution \(P(t_i|t_{i-1})\)
  - Draw a random word \(w_i\) from the tag distribution \(P(w_i|t_i)\)

Recall the supervised case

- We observe both the words and the tags.
- We estimate \(q = P(t_i|t_{i-1})\) and \(e = P(w_i|t_i)\) based on our observations.
- Done
we say that $X \sim \text{Discrete}(\theta, k)$ iff:

- $X$ can get one of $k$ values
- $\theta$ is a vector with $k$ entries
- $\theta_i \geq 0$
- $\sum_i \theta_i = 1$
- $P(X = i) = \theta_i$

Example

$p(t_j|t_{j-1})$ is a discrete distribution.

$t_j \sim \text{Discrete}(\theta, |T|)$

Where:

- $|T|$ is the size of the tagset
- We can get a uniform distribution if we set:
  - $\theta_i = 1/|T|$
- We can also estimate $\theta$ from data using MLE:
  - $\theta_{t_j} = \frac{\text{count}(t_{j-1}, t_j)}{\text{count}(t_{j-1})}$
Example HMM:

The UNsupervised case

- We don’t get to see the tags. They are \textit{latent}.
- But, for a given tag assignment, we can:
  - Estimate parameters
  - Calculate corpus probability
- Search for tag assignments such that if we estimate parameters from them, and then use the parameters to calculate the corpus probability, we will get high probability.
- This search looks hard!
- And it is.
- Two possible approximations:
  - EM algorithm
  - \textbf{Gibbs sampling}
Gibbs sampling

\[ w = w_1, \ldots, w_n \]
\[ t = t_1, \ldots, t_n \]

- We are interested in the tag assignment that will maximize \( P(w, t) \)
- For a fixed \( w \), \( \arg \max_t P(w, t) = \arg \max_t P(t|w) \)
Gibbs sampling

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- For a fixed \( w \), \( \arg \max_t P(w, t) = \arg \max_t P(t|w) \)
- If we could sample from \( P(t|w) \), we will, with high probability, get \( t \) such that \( P(t|w) \) is high.
Gibbs sampling

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- Ok... but how do we sample from \( P(t|w) \)?
Gibbs sampling

\[ w = w_1, \ldots, w_n \]
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- If we could sample from \( P(t|w) \), we will, with high probability, get \( t \) such that \( P(t|w) \) is high.
- Ok... but how do we sample from \( P(t|w) \)?
- Gibbs sampling is a “magical” way of doing that
  - To uncover the magic, see Graphical Models class
Gibbs sampling

Main idea

- In order to sample $P(t|w) = P(t_1, t_2, \ldots, t_n|w)$:
  - Start with a random assignment of $t_1, \ldots, t_n$. Then:
    - sample $t_1$ based on $t_2, \ldots, t_n, w$
    - $P(t_1|t_2, t_3, \ldots, t_n, w)$
    - sample $t_2$ based on $t_1, t_3, \ldots, t_n, w$
    - \ldots
    - sample $t_k$ based on $t_1, \ldots, t_{k-1}, t_{k+1}, \ldots, t_n, w$
    - \ldots and so on
  - After many iterations, we will get samples from $P(t|w)$
Gibbs sampling

Calculating $P(t_k|t_1, \ldots, t_{k-1}, t_{k+1}, \ldots, t_n, w)$

- Notation: $t^{-k} = t_1, \ldots, t_{k-1}, t_{k+1}, \ldots, t_n$.
- We can estimate $q$ and $e$ as previously, based on $w$ and the assignments to $t^{-k}$.
- Now we get:

$$P(t_k|t^{-k}) \propto q(t_k|t_{k-1})e(w_k|t_k)q(t_{k+1}|t_k)$$

- (why? and what does $\propto$ means?)
- Calculate this for every possible value of $t_k$.
- Normalize
Draws from distributions

\[ X \sim Discrete(\theta, k) \]

\[
p = \text{Math.random()}
\]

\[
\text{sum} = 0.0
\]

\[
\text{for } i \text{ in } 0 \ldots k-1 \{
    \text{sum } += \theta[i];
    \text{if}(\text{sum } >= p) \text{ return } i
\}
\]
The Gibbs sampling algorithm

Sampling from \( P(t|w) \) for \( t = t_1, \ldots, t_n \)

- Initialize \( t \) with random values
- Calculate parameters (collect counts) based on \( t, w \).
- for many iterations do
  - for \( i \in 1, \ldots, n \) do
    - “forget” value of \( t_i \) (decrease counts)
    - Calculate \( P(t_i|t^{-i}) \) based on modified counts
    - Sample new value for \( t_i \) from \( P(t_i|t^{-i}) \)
Putting it all together

Training HMM from text and dictionary using Gibbs sampling

For each word, assign a random tag from the set allowed by the dictionary
Calculate $q$, $e$ based on this tag assignment
for many iterations do
  for every sentence do
    for $i \in 1, \ldots, \text{length}$ do
      “forget” value of $t_i$ (decrease counts)
      Calculate $P(t_i|t^{-i})$ based on modified counts
      (Set prob of tags not in dictionary to 0. Normalize.)
      Sample new value for $t_i$ from $P(t_i|t^{-i})$
Putting it all together

Training HMM from text and dictionary using Gibbs sampling

For each word, assign a random tag from the set allowed by the dictionary
Calculate $q$, $e$ based on this tag assignment
for many iterations do
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      Calculate $P(t_i|t^{-i})$ based on modified counts
      (Set prob of tags not in dictionary to 0. Normalize.)
      Sample new value for $t_i$ from $P(t_i|t^{-i})$

Calculate final $q$ and $e$ based on the final state
(can also average several states)
Why do you expect this to work?

Why do we need the tag dictionary?
Topic Modeling / LDA
The problem with information

As more information becomes available, it becomes more difficult to access what we are looking for.

We need new tools to help us organize, search, and understand these vast amounts of information.
Topic modeling provides methods for automatically organizing, understanding, searching, and summarizing large electronic archives.

1. Uncover the hidden topical patterns that pervade the collection.
2. Annotate the documents according to those topics.
3. Use the annotations to organize, summarize, and search the texts.
Discover topics from a corpus

human genome
DNA genetic genes sequence gene molecular sequencing map information genetics mapping project sequences

evolution evolutionary species organisms life origin biology groups phylogenetic living diversity group new two common
disease host bacteria diseases resistance bacterial new strains control infectious malaria parasite parasites united tuberculosis

computer models information data computers system network systems model parallel methods networks software new simulations

D. Blei Topic Models
Model the evolution of topics over time

"Theoretical Physics"

"Neuroscience"
<table>
<thead>
<tr>
<th>Year</th>
<th>Weighted Influence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1880</td>
<td>0.000</td>
</tr>
<tr>
<td>1900</td>
<td>0.005</td>
</tr>
<tr>
<td>1920</td>
<td>0.010</td>
</tr>
<tr>
<td>1940</td>
<td>0.015</td>
</tr>
<tr>
<td>1960</td>
<td>0.020</td>
</tr>
<tr>
<td>1980</td>
<td>0.025</td>
</tr>
<tr>
<td>2000</td>
<td>0.030</td>
</tr>
</tbody>
</table>


W. B. Scott, *The Isthmus of Panama in Its Relation to the Animal Life of North and South America*, Science (1916) [3 citations]


Derek E. Wildman et al., Implications of Natural Selection in Shaping 99.4% Nonsynonymous DNA Identity between Humans and Chimpanzees: Enlarging Genus Homo, PNAS (2003) [178 citations]

W. B. Scott, *The Isthmus of Panama in Its Relation to the Animal Life of North and South America*, Science (1916) [3 citations]
Organize and browse large corpora

Wikipedia Topics
Relative Presence of Topics in all Documents

Stanley Kubrick
(July 26, 1928 – March 7, 1999) was an American film director, writer, producer, and photographer who lived in England during most of the last four decades of his career. Kubrick was noted for the scrupulous care with which he chose his subjects, his slow method of working, the variety of genres he worked in, his technical perfectionism, and his reclusivity about his films and personal life. He worked far beyond the confines of the Hollywood system, maintaining almost complete artistic control and making movies according to his own whims and time constraints, but with the rare advantage of big-studio financial support for all his endeavors.

Kubrick’s films are characterized by a formal visual style and meticulous attention to detail—his later films often have elements of surrealism and expressionism that eschews structured linear narrative. His films are repeatedly described as slow and methodical, and are often perceived as a reflection of his obsessive and perfectionist nature.

A recurring theme in his films is man’s inhumanity to man. While often viewed as

theory
work
human
idea
term
study
view
science
concept
form
world
argue
social

related topics
(work, book, publish)
(work, book, publish)
(woman, child, man)
(law, state, case)
(black, white, people)
(work, work, human)
(work, work, human)
(work, work, human)
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Latent Dirichlet Allocation
**Simple intuition**: Documents exhibit multiple topics.
Each **topic** is a distribution over words

Each **document** is a mixture of corpus-wide topics

Each **word** is drawn from one of those topics
• In reality, we only observe the documents

• The other structure are **hidden variables**

* Seeking Life's Bare (Genetic) Necessities

COLD SPRING HARBOR, NEW YORK—How many genes does an organism need to survive? Last week at the genome meeting here,* two genome researchers with radically different approaches presented complementary views of the basic genes needed for life.

One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sustained with just 250 genes, and that the earliest life forms required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

Although the numbers don't match precisely, those predictions are not all that far apart, especially in comparison to the 75,000 genes in the human genome. As Sv Anderson of the Royal University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a genetic number game; particularly, more and more genomes are being mapped and sequenced. "It may be a way of organizing any newly sequenced genome," explains Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing

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* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12

**Note:** The diagram illustrates the relationship between topics, documents, and topic proportions and assignments. The text discusses the comparison of gene numbers across different organisms and the potential implications of these findings.
Our goal is to **infer** the hidden variables

I.e., compute their distribution conditioned on the documents

$$p(\text{topics}, \text{proportions}, \text{assignments} \mid \text{documents})$$
Graphical models (Aside)

- Nodes are random variables
- Edges denote possible dependence
- Observed variables are shaded
- Plates denote replicated structure
Structure of the graph defines the pattern of conditional dependence between the ensemble of random variables.

E.g., this graph corresponds to
\[
p(y, x_1, \ldots, x_N) = p(y) \prod_{n=1}^{N} p(x_n | y)
\]
LDA as a graphical model

- Nodes are random variables; edges indicate dependence.
- Shaded nodes are observed; unshaded nodes are hidden.
- Plates indicate replicated variables.
LDA Generative Story

We have $K$ topics, and a vocabulary $V$ of $|V|$ words. Each topic $\beta^k$ is a distribution over words.

A document $d$ is created by

- Sample length $n_d$ from a Poisson distribution
  - (alternatively, assume $n_d$ is given)
- Sample topic proportions $\theta^d$ from a Dirichlet distribution with parameter $\alpha$.
- For each position $i \in 1, \ldots, n$:
  - Sample topic $z_i$ from $\theta^d$
  - Sample word $w_i$ from the distribution $\beta^{z_i}$
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Assumptions

- We do not care about the word-order ("bag of words")
- Each word is independent of the other words given its topic
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**Assumptions**

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The Dirichlet Distribution

- The Dirichlet distribution is a “distribution over distributions”
- When you sample $\theta \sim \text{DIRICHLET}(\alpha, K)$:
  - $\theta$ is a $K$-dim vector
  - $\theta_i \geq 0$
  - $\sum_i \theta_i = 1$
The Dirichlet Distribution

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- When you sample $\theta \sim \text{DIRICHLET}(\alpha, K)$:
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The probability of seeing a particular vector $\theta$ is:

$$P_{\text{DIRICHLET}(\alpha, K)}(\theta) = \frac{\prod_{i=1}^{K} \theta_i^{\alpha_i - 1}}{B(\alpha)}$$

$$B(\alpha) = \frac{\prod_{i=1}^{K} \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^{K} \alpha_i)}$$

- $\Gamma$ is the gamma function, generalization of factorial.

- Generally, $\alpha$ is a $k$-dim vector, but we will assume “symmetric” dirichlet, in which $\alpha$ is a single scalar (and $\alpha_i = \alpha$ for all $i \in \{1, \ldots, K\}$)
The Dirichlet Distribution

The Dirichlet distribution is a “distribution over distributions”

\[ P_p(\theta|\alpha) = \frac{\prod_{i=1}^{K} \theta_i^{\alpha_i-1}}{B(\alpha)} \]

- \( \alpha \) controls the shape, mean and sparsity of \( \theta \)
\[ \alpha = 1 \]
$\alpha = 10$
$\alpha = 100$
$\alpha = 1$
$\alpha = 0.1$
$\alpha = 0.01$
\( \alpha = 0.001 \)
The Dirichlet Distribution

For draws $\theta$ from a symmetric dirichlet distribution:

- $\alpha = 1$  All $\theta$ are equally likely
- $\alpha > 1$  Uniform $\theta$ are more likely
- $\alpha < 1$  Spikey $\theta$ are more likely
LDA Generative Story

We have $K$ topics, and a vocabulary $V$ of $|V|$ words. Each topic $\beta^k$ is a distribution over words. $\beta^k \sim Dirichlet(\eta, |V|)$

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$\alpha$ controls how many topics we expect to see in our documents
• Our goal is to **infer** the hidden variables

• I.e., compute their distribution conditioned on the documents

\[ p(\text{topics, proportions, assignments} | \text{documents}) \]
LDA as a graphical model

- Nodes are random variables; edges indicate dependence.
- Shaded nodes are observed; unshaded nodes are hidden.
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LDA as a graphical model

\[ p(\beta, \theta, z, w) = \left( \prod_{i=1}^{K} p(\beta_i | \eta) \right) \left( \prod_{d=1}^{D} p(\theta_d | \alpha) \prod_{n=1}^{N} p(z_{d,n} | \theta_d) p(w_{d,n} | \beta_1:K, z_{d,n}) \right) \]
This joint defines a posterior, \( p(\theta, z, \beta | w) \).

From a collection of documents, infer

- Per-word topic assignment \( z_{d,n} \)
- Per-document topic proportions \( \theta_d \)
- Per-corpus topic distributions \( \beta_k \)

Then use posterior expectations to perform the task at hand: information retrieval, document similarity, exploration, and others.
Seeking Life’s Bare (Genetic) Necessities

COLD SPRING HARBOR, NEW YORK—How many genes does an organism need to survive? Last week at the genome meeting here, two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today’s organisms can be sustained with just 250 genes, and that the earliest life forms required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn’t be enough.

Although the numbers don’t match precisely, those predictions are not all that far apart,” especially in comparison to the 75,000 genes in the human genome, notes Siv Andersson of Uppsala University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a genetic numbers game, particularly as more and more genomes are completely mapped and sequenced. “It may be a way of organizing any newly sequenced genome,” explains Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing an


Stripping down. Computer analysis yields an estimate of the minimum modern and ancient genomes.

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## Example inference

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Chaotic Beetles
Charles Godfray and Michael Hassell

Ecologists have known since the pioneering work of May in the mid-1970s (1) that the population dynamics of animals and plants can be exceedingly complex. This complexity arises from two sources: The tangled web of interactions that constitute any natural community provide a myriad of different pathways for species to interact, both directly and indirectly. And even in isolated populations the nonlinear feedback processes present in all natural populations can result in complex dynamic behavior. Natural populations can show persistent oscillatory dynamics and chaos, the latter characterized by extreme sensitivity to initial conditions. If such chaotic dynamics were common in nature, then this would have important ramifications for the management and conservation of natural resources. On page 389 of this issue, Costantino et al. (2) provide the most convincing evidence to date of complex dynamics and chaos in a biological population—of the flour beetle, Tribolium castaneum (see figure).

It has proven extremely difficult to demonstrate complex dynamics in populations in the field. By its very nature, a chaotically fluctuating population will superficially resemble a stable or cyclic population buffered by the normal random perturbations experienced by all species. Given a long enough time series, diagnostic tools from nonlinear mathematics can be used to identify the telltale signatures of chaos. In phase space, chaotic trajectories come to lie on “strange attractors,” curious geometric objects with fractal structure and hence noninteger dimension. As they move over the surface of the attractor, sets of adjacent trajectories are pulled apart, then stretched and folded, so that it becomes impossible to predict exact population densities into the future. The strength of the mixing that gives rise to the extreme sensitivity to initial conditions can be measured mathematically estimating the Liapunov exponent, which is positive for chaotic dynamics and nonpositive otherwise. There have been many attempts to estimate attractor dimension and Liapunov exponents from time series data, and some candidate chaotic population have been identified (some insects, rodents, and most convincingly, human childhood diseases), but the statistical difficulties preclude any broad generalization (3).

An alternative approach is to parameterize population models with data from natural populations and then compare their predictions with the dynamics in the field. This technique has been gaining popularity in recent years, helped by statistical advances in parameter estimation. Good ex-
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Why does LDA work?
Why does LDA “work”? 

- LDA trades off two goals.
  1. For each document, allocate its words to as few topics as possible.
  2. For each topic, assign high probability to as few terms as possible.

- These goals are at odds.
  - Putting a document in a single topic makes #2 hard:
    All of its words must have probability under that topic.
  - Putting very few words in each topic makes #1 hard:
    To cover a document’s words, it must assign many topics to it.

- Trading off these goals finds groups of tightly co-occurring words.
What do we get out of LDA?

- Topic assignments $z$
- Topic proportions (how strong is topic $k$ in document $j$?)
- Topics distributions (how strong is word $i$ in topic $k$?)
  - Also: which topics are related to word $i$?
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  ▶ Also: which topics are related to word $i$?

So?

▶ Which topics are in our corpus?
▶ Find similar docs (by comparing “topic vectors” of docs)
▶ Find related words (by comparing “topic vectors” of words)
▶ Query expansion: find documents related to words X,Y,Z, even if all or some of these words did not appear in the document
▶ …
<table>
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<td>LIFE</td>
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The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. “Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services.” Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center’s share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.
Topic Model

• $P(t|k)$ for all $t$ and $k$, is a term by topic matrix (gives which terms make up a topic)

• $P(k|\text{doc})$ for all $k$ and $\text{doc}$, is a topic by document matrix (gives which topics are in a document)
EXAMPLE
Analysis of TASA Corpus

• Given a text collection written by first grade to college students

• Data has following characteristics
  – 26,000+ word types (stop words removed)
  – 37,000+ documents
  – 6,000,000+ word tokens

• Find topics in the data
# Topics in the Educational Corpus (TASA)

- 37K docs, 26K words
- 1700 topics, e.g.
Polysemy
Three Documents with the word “play”

(numbers & colors → topic assignments)

A **Play** is written to be performed on a stage before a live audience or before motion picture or television cameras (for later viewing by large audiences). A **Play** is written because playwrights have something.

He was listening to music coming from a passing riverboat. The music had already captured his heart as well as his ear. It was jazz. Bix beiderbecke had already had music lessons. He wanted to **play** the cornet. And he wanted to **play** jazz.

Jim plays the game. Jim likes the game for one. The game helps Jim. Don comes into the house. Don and Jim read the game book. The boys see a game for two. The two boys play the game.
LDA Inference

How do we fit an LDA model to the data?
Fitting an LDA model to our data

Use an existing tool!

- Mallet (java)  Uses Gibbs sampling
- gensym (python)  Uses variational inference --- scalable, but worse quality
- Many other tools available
  - (see David Blei’s website)
- tomotopy  Python + sampling, and and good tool.
- jsLDA  In-browser sampling implementation.
  https://mimno.infosci.cornell.edu/jsLDA/
Fitting an LDA model to our data

But how are the tools implemented?
And what if we want a slightly different story?
Fitting an LDA model to our data

But how are the tools implemented? And what if we want a slightly different story?

- Exact inference is intractable.
- Use an approximate algorithm.
Fitting an LDA model to our data

But how are the tools implemented?
And what if we want a slightly different story?

- Exact inference is intractable.
- Use an approximate algorithm.
- Current tools use modern complex algorithms:
  - Fast
  - Scale well to huge number of topics and documents
  - Beyond the scope of this course
Fitting an LDA model to our data

But how are the tools implemented?

And what if we want a slightly different story?

- Exact inference is intractable.
- Use an approximate algorithm.
- Current tools use modern complex algorithms:
  - Fast
  - Scale well to huge number of topics and documents
  - Beyond the scope of this course
- but for fitting a small to medium data, we can use Gibbs sampling.
  - (Gibbs is also our best bet for implementing modifications of LDA)
LDA Gibbs Sampler  

Recall:

- Inputs: $\alpha$, $\eta$, $K$
- Observed variables: words, $W = w_{d,n}$
- Unobserved: $\theta = \theta^1, \ldots, \theta^D$, $\beta = \beta^1, \ldots, \beta^K$, $Z = z_{d,n}$

We need to sample from:

$$p(Z, \theta, \beta | W, \alpha, \eta)$$

In Gibbs:

Initialize random $z$

Then, repeatedly:

- For each $k \in \{1, \ldots, K\}$, sample $\beta^k$ based on $Z$, $W$, $\eta$
- Sample $\theta^d$ based on $Z$, $W$, $\alpha$
- Sample $z_{d,1}$ based on $Z^{-d,1}$, $\theta^d$, $\beta$, $W$
- Sample $z_{d,2}$ based on $Z^{-d,2}$, $\theta^d$, $\beta$, $W$
- ...
For each $k \in \{1, \ldots, K\}$, sample $\beta^k$ based on $Z$, $W$, $\eta$

Sample $\theta^d$ based on $Z$, $W$, $\alpha$

Sample $z_{d,1}$ based on $Z^{-d,1}$, $\theta^d$, $\beta$, $W$

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...
LDA Gibbs Sampler

- For each $k \in \{1, \cdots, K\}$, sample $\beta^k$ based on $Z, W, \eta$
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- Sample $z_{d,2}$ based on $Z^{-d,2}, \theta^d, \beta, W$
- ...

These lines are easy:

$$p(z_{d,i} = k \mid Z^{-d,i}, \theta^d, \beta, W) = \theta_k^d \cdot \beta_{W_d,i}^k$$

- $\theta_k^d$ probability of generating topic $k$ in doc $d$
- $\beta_{W_d,i}^k$ probability of generating word $W_{d,i}$ from topic $k$
LDA Gibbs Sampler

- For each $k \in \{1, \cdots, K\}$, sample $\beta^k$ based on $Z, W, \eta$
- **Sample** $\theta^d$ **based on** $Z, W, \alpha$
- Sample $z_{d,1}$ based on $Z^{-d,1}, \theta^d, \beta, W$
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- …

What does this line mean?
For each $k \in \{1, \cdots, K\}$, sample $\beta^k$ based on $Z, W, \eta$

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Sample $z_{d,1}$ based on $Z^{-d,1}, \theta^d, \beta, W$

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\[ \theta_k^d = \frac{\text{count}(z_{d,i} = k)}{n_d} \]
For each $k \in \{1, \cdots, K\}$, sample $\beta^k$ based on $Z, W, \eta$

Sample $\theta^d$ based on $Z, W, \alpha$

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Sample $z_{d,2}$ based on $Z^{-d,2}, \theta^d, \beta, W$

... 

What does this line mean?

We need to sample $\theta^d$ from $p(\theta|Z, \alpha)$.

Given $Z$, we can derive an MLE estimate of $\theta^d$:

$$\theta^d_k = \frac{\text{count}(z_{d,i} = k)}{n_d}$$

But no. We need to sample. What does it mean to sample $\theta$?
LDA Gibbs Sampler

- For each $k \in \{1, \cdots, K\}$, sample $\beta^k$ based on $Z$, $W$, $\eta$
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- But no. We need to sample. What does it mean to sample $\theta$?

- Under the *Bayesian* philosophy, we do not commit to a single estimate of $\theta$. Instead, we have a distribution $p(\theta^d | Z, \alpha)$ of possible $\theta^d$, based on our prior belief $\alpha$ and the data we saw $Z$. 

LDA Gibbs Sampler

- For each $k \in \{1, \cdots, K\}$, sample $\beta^k$ based on $Z, W, \eta$
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What does this line mean?
We need to sample $\theta^d$ from $p(\theta|Z, \alpha)$.

Because $\theta^d \sim \text{DIRICHLET}(\alpha, K)$, and because dirichlet is \textit{conjugate} to multinomial, we have:

$$\theta^d|Z, \alpha \sim \text{DIRICHLET}(\alpha + c^d)$$

where $c^d$ is a $K$-dim vector based on counts from $Z$, with $c_k^d$ is the number of items in document $d$ with topic $k$. 
LDA Gibbs Sampler

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- ... 

We need to sample $\theta^d$ from $p(\theta|Z, \alpha)$.

$$\theta^d|Z, \alpha \sim \text{DIRICHLET}(\alpha + c^d)$$

- There are algorithms for sampling from Dirichlet, but we don’t need to actually use them.
- Instead, we will use the the **collapsed** Gibbs sampler.
Collapsed Gibbs sampler

Recall:

- Inputs: $\alpha$, $\eta$, $K$
- Observed variables: words, $W = w_{d,n}$
- Unobserved: $\theta = \theta^1, \ldots, \theta^D$, $\beta = \beta^1, \ldots, \beta^K$, $Z = z_{d,n}$

We need to sample from $p(Z, \theta, \beta|W, \alpha, \eta)$

But actually, we are ok with just $Z$. Can we get rid of $\theta, \beta$?

- If $\theta, \beta$ were discrete, we could marginalize over them.
Collapsed Gibbs sampler

Recall:

▷ Inputs: $\alpha$, $\eta$, $K$
▷ Observed variables: words, $W = w_{d,n}$
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We need to sample from

$$p(Z, \theta, \beta|W, \alpha, \eta)$$

But actually, we are ok with just $Z$. Can we get rid of $\theta$, $\beta$?

▷ If $\theta$, $\beta$ were discrete, we could marginalize over them.
▷ But they are continuous, so instead we need to integrate

$$p(Z|W, \alpha, \eta) = \int \int p(Z, \theta, \beta|W, \alpha, \eta) d\theta d\beta$$
Collapsed Gibbs sampler

Recall:

- Inputs: $\alpha$, $\eta$, $K$
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Collapsed Gibbs sampler

\[ p(z_{d,i} = k | W, Z^{-d,i}, \alpha, \eta) = \int \int p(z_{d,i} = k, \theta, \beta | W, Z^{-d,i}, \alpha, \eta) d\theta d\beta \]

\[ = \int p(z_{d,i} = k | \theta) p(\theta | \alpha) d\theta \int p(w_{d,i} = v | W^{-d,i}, z_{d,i} = k, Z^{-d,i}, \beta) p(\beta | \eta) d\eta \]

You don’t really need to know how to integrate! Just remember that for Dirichlet:

\[ \int p(x | data, \theta) p(\theta | \alpha) d\theta = \frac{c_x + \alpha}{|data| + K\alpha} \]

Where \( c_x \) is the count of event \( x \) in the data, and \( |data| = \sum_{x'} c'_{x} \) is the number of samples in the data.
Collapsed Gibbs Sampler

Just remember that for Dirichlet:

\[
\int p(x|\text{data}, \theta)p(\theta|\alpha)d\theta = \frac{c_x + \alpha}{|\text{data}| + K\alpha}
\]

Where \(c_x\) is the count of event \(x\) in the data, and \(|\text{data}| = \sum_{x'} c'_{x'}\) is the number of samples in the data.

Use this rule twice (once for each \(\int\)), and get:

\[
p(z_{d,i} = k|Z^{d,i}, \alpha, \eta, w_i) = \frac{c^d_k + \alpha}{\sum_{k'} c^d_{k'} + K\alpha} \frac{v^k_{w_i} + \eta}{\sum_{i'} v^k_{w_{i'}} + |V|\eta}
\]

- \(c^d_k\) number of words in doc \(d\) with topic \(k\) in \(Z^{d,i}\)
- \(v^k_{w_i}\) number of times word \(w_i\) is assigned to topic \(k\) in \(Z^{d,i}\)
- \(K\) number of topics
- \(|V|\) vocabulary size
Collapsed Gibbs Sampler

Rule of Thumb

In MLE land:

\[ p(x_n = k|x_1, x_2, \ldots, x_{n-1}) = \frac{\text{count}(k)}{n - 1} \]

In Dirichlet-prior \( \alpha \) land:

\[ p(x_n = k|x_1, x_2, \ldots, x_{n-1}, \alpha) = \frac{\text{count}(k) + \alpha}{n - 1 + K\alpha} \]

Derivation in MacKay and Peto (1994)
Collapsed Gibbs Sampler

- Initialize random topics $Z$
- For many iterations, for each document $d$, for each word $i$:
  - forget $z_{d,i}$ getting $Z^{-d,i}$
  - sample new assignment for $z_{d,i}$ based on equation below.

$$p(z_{d,i} = k | Z^{-d,i}, \alpha, \eta, w_i) = \frac{c_k^d + \alpha}{\sum_{k'} c_{k'}^d + K \alpha} \frac{v_{w_i}^k + \eta}{\sum_{i'} v_{w_i'}^k + |V| \eta}$$

$c_k^d$ number of words in doc $d$ with topic $k$ in $Z^{-d,i}$
$v_{w_i}^k$ number of times word $w_i$ is assigned to topic $k$ in $Z^{-d,i}$
$K$ number of topics
$|V|$ vocabulary size
LDA Evaluation

We have topics, are they good?
LDA Evaluation

Internal Evaluation
If we want to compare two different LDA models on the same data:

▶ Compare the Probability that is assigned to the data by each model.
▶ Higher probability → better model
LDA Evaluation

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- But this does not tell us much about how useful the topics are...
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External (task-based) Evaluation

▶ Use the LDA topics as features in another task
▶ Measure the accuracy of the other task
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External (task-based) Evaluation
- Use the LDA topics as features in another task
- Measure the accuracy of the other task
- Good! But we need to have a task that we can automatically measure.
Human Evaluation
If we just want to know if our topics are “good” we can ask people.

- But what is a good topic?
Human Evaluation
If we just want to know if our topics are “good” we can ask people.

- But what is a good topic?
- “Intruder Detection”
  - Take top words from a topic.
  - Insert a random word which is high in another topic.
  - Can a human identify the random word?
  - Yes → good topic
Other Applications of LDA
Change the definition of Document

**Selectional Preferences**
Take parsed corpus:

**Documents**  each Verb is a document
**Words**  each subject of a verb is a “word” in the document
**Topics**  each topic is one “kind” of arguments
<table>
<thead>
<tr>
<th>Topic $t$</th>
<th>Arg1</th>
<th>Relations which assign highest probability to $t$</th>
<th>Arg2</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>The residue - The mixture - The reaction mixture - The solution - the mixture - the reaction - the residue - The reaction - the solution - The filtrate - the reaction - The product - The crude product - The pellet - The organic layer - Thereto - This solution - The resulting solution - Next - The organic phase - The resulting mixture - C.)</td>
<td>was treated with, is treated with, was poured into, was extracted with, was purified by, was diluted with, was filtered through, is dissolved in, is washed with</td>
<td>EtOAc - CH2Cl2 - H2O - CH.sub.2Cl.sub.2 - H.sub.2O - water - MeOH - NaHCO3 - Et2O - NHCl - CHCl.sub.3 - NHCl - dropwise - CH2Cl.sub.2 - Celite - Et.sub.2O - Cl.sub.2 - NaOH - AcOEt - CH2Cl2 - the mixture - saturated NaHCO3 - SiO2 - H2O - N hydrochloric acid - NHCl - preparative HPLC - to 0 C.</td>
</tr>
<tr>
<td>151</td>
<td>the Court - The Court - the Supreme Court - The Supreme Court - this Court - Court - The US Supreme Court - the court - This Court - the US Supreme Court - Court - The court - Supreme Court - Judge - the Court of Appeals - A federal judge</td>
<td>will hear, ruled in, decides, upholds, struck down, overturned, sided with, affirms</td>
<td>the case - the appeal - arguments - a case - evidence - this case - the decision - the law - testimony - the State - an interview - an appeal - cases - the Court - that decision - Congress - a decision - the complaint - oral arguments - a law - the statute</td>
</tr>
<tr>
<td>211</td>
<td>President Bush - Bush - The President - Clinton - the President - President Clinton - President George W. Bush - Mr. Bush - The Governor - the Governor - Romney - McCain - The White House - President - Schwarzenegger - Obama</td>
<td>hailed, vetoed, promoted, will deliver, favors, denounced, defended</td>
<td>the bill - a bill - the decision - the war - the idea - the plan - the move - the legislation - legislation - the measure - the proposal - the deal - this bill - a measure - the program - the law - the resolution - efforts - the agreement - gay marriage - the report - abortion</td>
</tr>
<tr>
<td>224</td>
<td>Google - Software - the CPU - Clicking - Excel - the user - Firefox - System - The CPU - Internet Explorer - the ability - Program - users - Option - SQL Server - Code - the OS - the BIOS</td>
<td>will display, to store, to load, processes, cannot find, invokes, to search for, to delete</td>
<td>data - files - the data - the file - the URL - information - the files - images - a URL - the information - the IP address - the user - text - the code - a file - the page - IP addresses - PDF files - messages - pages - an IP address</td>
</tr>
</tbody>
</table>

Table 1: Example argument lists from the inferred topics. For each topic number $t$ we list the most
Model is slightly different - topic generates two groups of things.

(how would you change the Gibbs sampler?)
Change the definition of Document

Beyond NLP
Dataset of users who watched movies

Documents each user is a document
Words each movie is a word
Topics each topic is a “taste” or “genre”

- High topic-word prob: movie belong to genre
- High topic-doc prob: user likes genre

Can recommend new movies to users
Extending LDA

- LDA can be embedded in more complicated models, embodying further intuitions about the structure of the texts.

- E.g., it can be used in models that account for syntax, authorship, word sense, dynamics, correlation, hierarchies, and other structure.
Summary
Unsupervised Learning

- Define generative story
- Include hidden ("latent") variables
- Find probable assignments to latent variables
- Can use Gibbs sampling
Unsupervised Learning

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Topic Modeling / LDA

- A very powerful and useful model. **Use it**
- Generative story for LDA
- Dirichlet distributions → can encourage sparsity
- Examples of LDA usage
- Gibbs sampler for LDA (briefly)
  - relevant for every model with dirichlet
- Evaluation: quantify human judgement ("intruder detection")
- Creative definition of documents