Do Scripture and Mathematics Agree on the Number π?

Professor Isaac Elishakoff and Elliot M. Pines, PhD

Presented at the Fifth Miami International Conference on Torah and Science, 16-18 December 2003

The five parts of this paper discuss the seeming contradiction between scripture and mathematics concerning the value of π (pi), and offer possible resolutions. Alongside a review of the widely accepted opinions and some recent investigations, we humbly offer our own suggestions. In Part One, we introduce the apparent conflict and its significance. In Part Two, Professor Elishakoff takes a direct approach, investigating some pertinent issues of Jewish law and offering an analysis in terms of engineering practice. In Part Three, Professor Elishakoff and Dr. Pines discuss evidence that the Sages of the talmudic era had knowledge of π to greater accuracy than that implied by a surface reading of Scripture that defines the Jewish legal standard. A hint of knowledge of π of still greater accuracy is found in the Bible itself. In Part Four, Dr. Pines continues this train of thought into the esoteric, commencing with a supporting information-theory-based analysis. Pines follows up his discussion with an exploration of possible kabbalistic meaning. An appendix with a physics-based speculation further develops Part Four. Finally, in Part Five, the authors conclude that the contradiction implicit in a superficial understanding may be masking an underlying harmony on several levels that makes itself known only through careful examination, which scientific and popular texts should be providing.

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Part One: Introduction

Isaac Elishakoff and Elliot Pines

Lost among the often abstract debates between Torah and science is the down-to-earth issue of π, the ratio between the circumference and diameter of a circle. *Tanakh* (Bible) and mathematics appear at odds over this simple constant.

Although we all learned in school that π equals 3.141592…, it appears that the Tanakh claims 3 as an exact or at least approximate value of π. "The Bible is very clear on where it stands regarding π," writes David Blanter, in his 1997 book *The Joy of* π. Blanter quotes from the description in I Kings 7:23 of the basin that King Solomon placed in the Temple: "Also he made a molten sea of ten cubits from brim to brim, round in compass five cubits the height thereof; and a line of thirty cubits did compass it round about." Blanter says that this passage and the nearly identical one in II Chronicles 4:2 indicate an approximation "so far from truth" that either "the Bible is false" or "scientists are lying to us."[1] In his review of Blanter's popular book, Roz Kaveny takes special note of "…the Biblical 3 (which patently left a lot to be desired)."[2]

Jörg Arndt and Christoph Haenel call biblical π "pretty pathetic, not only when considered in absolute terms, but also for the time 550 BCE."[3] Jonathan Borwein, Peter Borwein, and David H. Bailey proclaim, "Not all ancient societies were so accurate however—nearly 1500 years later the Hebrews were perhaps still content to use the value of 3.…"[4] Similarly, Petr Beckmann brings I Kings to task for using the number 3 for π.[5] Gerd Almkvist and Bruce Berndt[6] attack Cecil Read[7] for suggesting that the molten sea was elliptical and accuse him of being a person who "perhaps believes that G-d makes no mistakes…."

The *Encyclopedia Judaica* questions the talmudic use of the biblical value when "in the third century BCE Archimedes had already given a more exact value."[8]

The *Universal Jewish Encyclopedia* is harsher: "…the Mishnah and Gemara erroneously suggested the value of the Greek letter π as being equal to three (I Kings 7:23). This deduction was fallaciously based upon the

Roman school of logic."[9]Shlomo Edward Belaga[10] attempts to capture the psychology here, referring to "those who mention this verse, who either cannot or do not want to, hide (or even are happy, for ideological reasons, to emphasize) their surprise by such low accuracy of the Biblical approximation of $\pi_0 = 3$."

Why does the Bible seem so inaccurate? Let's examine this question from direct to esoteric points of view.

Part Two: The Direct Approach

Isaac Elishakoff

2.1 "Torah Speaks in Human Language"

The talmudic principle that "the Torah speaks in human language"[11] leads us to search for the "language of π" in the biblical period. Petr Beckmann[12] and others incorrectly assume that only the Bible gives a value of 3 for π. Contrary to this misunderstanding, there are many sources of evidence that other cultures also figured π as 3. Radha Charan Gupta[13] claims that a second-millennium cuneiform text shows a circumference as equaling exactly three diameters. He cites also Indian Vedic literature (Mehta),[14] where the value of 3 for π is used in the Bandhayana Sulba Sutra (500 BCE or earlier). Buddhist cosmography before the common era uses 3 for the perimeter calculation of Godaniag Island (Vasubandhu).[15] Egyptian papyri in the Hellenistic period use 3 for the value of π. The Han period (202 BCE to 220 CE) Chinese text *Chou Pei Suan Ching* (Nine Chapters on Mathematical Art) uses a ratio of exactly 3. ("At the winter solstice the sun's orbit has a diameter of 476,000 miles, the circumference of the orbit being 1428,000 miles.")[16]

John Pottage[17] reports that in the first century, Roman architect Vitruvius used 3 as the wheel circumference-to-diameter ratio in his book *De Architectura*.

All of the above evidence is summed up by Jan Gullberg:[18] "Nearly all peoples of the ancient world used the number 3 for the ratio of circle's circumference to its diameter as an approximation sufficient for everyday needs….The early Greeks also began with $\pi = 3$ for everyday use."

Was more accurate knowledge of the value of π, then, unavailable when the Book of Kings was written? Kim Jonas[19] discusses a 4000-year-old cuneiform tablet demonstrating that the Sumerians knew the ratio of an inscribed hexagon to its circular perimeter accurate enough for an approximation of 3.1065. Likewise, some French researchers, like F. Thureau-Dangin,[20] maintain that the Susa manuscript implies Babylonian knowledge of π sufficient for the approximation of 3.125. But did the Sumerians actually understand the connection with π? Kazuo Muroi[21] challenges the idea that the Babylonians ever had the 3.125 approximation in the first place. Likewise, Jens Høyrup writes: "in spite of widespread assertions, $\pi = 3^1/_8$ was probably not used."[22]

Likewise, in the Egyptian Rhind Papyrus (circa 1650 BCE) the scribe Ahmes calculates the area of a circular field as to imply an approximation of $\pi = 3.16$. However, Jonas opines that Ahmes simply received a good empirical result, without knowing the concept of π. In 1930, the Moscow Papyrus from 1890 BCE was assumed to contain a calculation of a hemispherical surface indicating an advanced three-dimensional application of π. Carl B. Boyer,[23] however, shows later analysis indicating that this is a calculation for a much more simple problem, and again, there is no proof that the concept of π is involved.

According to Dario Castellanos,[24] the late-fourth-century BCE mathematician Euclid managed to prove only that π is larger than 3 and smaller than 4.[25] Archimedes made his breakthrough calculation of $3.140845\ldots < \pi < 3.142857$ $(= 3^1/_7)$ only in the next century.

A circumference-to-diameter ratio (the meaning of this term will be clarified in Section 2.3) "better" than 3 may have been unknown until 300 BCE! Even if some isolated individuals had made the breakthrough in an earlier age, they didn't have professional journals or the Internet to help get the word out. It is clear that 3 was the everyday value of antiquity. Therefore, the Book of Kings, using human language, would report that a 10-cubit diameter had a 30-cubit circumference.

2.2 *The Approximation of $\pi = 3$ as a Fence against Physical and Moral Failure*

I shall now offer some observations that to the best of my knowledge are not found elsewhere in the literature on π.

Consider a straight rod of a circular cross-section with radius a and cross-sectional area πa^2. The rod is subjected to a tensile force, F. Assuming uniformity far from the ends, the axial stress (pressure) upon the cross-sectional area is $S = F/\pi a^2$, according to Saint Venant's principle. In order for the rod not to break, it must not be "overstressed." This implies that the stress must be less than some critical value, S_{cr}—dependent upon the material composing the rod. This means that the following inequality must hold: $S < S_{cr}$. Note that Maimonides (1135-1204) states that the commandment, "If you will build a new house, you shall make a fence for your roof, so that you will not place blood in your house if a fallen one falls from it," (Deuteronomy 22:8) applies to any dangerous situation.[26]

In mechanics there is a fence concept called the "required safety factor." The brinkmanship inequality $S < S_{cr}$ becomes the buffered equivalent $S < S_{cr}/S.F.$, where $S.F.$ is the required safety factor chosen through experience and insight. It must be greater than unity to distance the dangerous level of critical stress S_{cr} (in the case at hand, yield stress—the ultimate stress before our rod gives way). To build the rod reliably, the expression for stress and safety requirements must be combined as $F/\pi a^2 \leq S_{cr}/S.F.$ This provides the design value of the cross-section radius:

$$a_{design} = \sqrt{\{F \cdot S.f. / (\pi\, S_{cr})\}}.$$

Now, if the value of 3 instead of 3.14… is used for π, the design value of the radius will be increased by the factor of $\sqrt{\{\pi/3\}} = 1.02….$ Alternatively, the safety factor will be increased by the factor $\pi/3 = 1.047…$, or about 4.7 percent. It seems reasonable to posit that the rounding off to the nearest smaller integer in assessing the diameter was strengthened by the consideration of introducing a protective "fence." Vitruvius should not be blamed for using $\pi = 3$ in his architectural treatise![27] It is remarkable, as Henry Petroski writes in his book, *To Engineer Is Human*, that "the analysis of the many piping systems in nuclear plants seems to be especially prone to gremlins, and one computer program used for calculating the stresses in pipes was reportedly using the wrong value of pi." This remark was made between the late 1970s and early 1980s.[28]

Likewise, consider round *matsah* (unleavened Passover bread), purchased ideally by weight, in practice by piece count. Weight = W, such that $W = N\pi a^2 h\gamma$, where N is the number of pieces, a is the matsah radius, h is the thickness per matsah, and γ is the material density. For a given transaction weight, W_t, we may express the target radius as

$$a = \sqrt{\{W_t / (N\pi h\gamma)\}}.$$

However, if π is approximated as 3, then the target radius will have a built-in margin of approximately 5 percent. This is a fair compromise in order to protect the buyer from being overcharged.

This conjecture correlates well with the *Mishnah*:[29]

> The rabbis taught us as follows: The verse Leviticus 19:15, "you should do no unrighteousness in judgment," applies to mensuration of land, as well as to the weighing and measuring of solids and fluids….

I do not know of any direct talmudic or post-talmudic discussion on this mishnaic ruling and would be pleased to hear about any from readers.

We conclude that the value of 3.16 for π associated with the Rhind Papyrus is not "better" than 3, although it is closer to the "exact" value of π. The implied Babylonian value of 3.125 in the highly debated Susa manuscript—while less than π and numerically closer to the exact value—would also be a better approximation than that by Ahmes. Scripture leads us to a universally known lower bound, apparently with a practical margin for error.

Upper-bound, "better" approximations, such as the early Common Era $\sqrt{10} = 3.162\ldots$, the debatable Ahmes 3.16, or the popular (and often wrongly assumed perfect) Archimedes value of $3^1/_7 = 3.142857\ldots$, appear to be morally "worse" than 3 because they would destroy the physical and moral fences required by the Torah. In discussing the required dimensions of a *sukkah* booth, the Talmud provides an important clue about when and why approximations are used:

> But is it not to be maintained that one may be assumed to give approximate figures only when the law is thereby restricted, but could such an assumption be made where a law is thereby relaxed? … that is what was meant

> that he only gave an approximate figure: and in this case it is in the direction of stringency.[30]

We learn from the above discussion that Jewish law assigns a *purpose* to approximation. Approximation is a permissible form of simplification in cases where the error is known to favor *stringency*. Approximation is allowed to be used as a fence to prohibit violation of the law. This consideration will be visited in greater detail in Part Three.

2.3 Value of 3 Is the First Approximation

Maimonides states that geometers have proven it impossible to know the exact circumference-to-diameter ratio. Furthermore, the Sages "took the nearest integer and said that every circle whose circumference is three fists is one fist wide, and they contented themselves with this for their needs on religious law."[31]

Regarding King Solomon's "molten sea," Rabbi Menahem Mendel Schneerson (the Lubavitcher Rebbe) observed,

> It would seem that even the rounded number should have read 31. The answer to this query is that the actual circumference was exactly 30 cubits and the diameter was less than 10, with the latter number rounded off to 10.[32]

According to this interpretation, the exact diameter was 9.549…, which, when approximated to the nearest integer, becomes 10.

Similarly, Peter Stevenson[33] notes,

> …only approximate values are used, much as current authors use in speaking of the distance to the sun as 93,000,000 miles. Obviously, the thought here is not to state that the earth travels in a circular orbit of this radius. Likewise, the Biblical writer is not intending anything other than a general description of the "molten sea"… .It is difficult to see how the Hebrews had failed to have had knowledge of such a fundamental ratio.

The *Mishnat Ha'Middot*, a work that the *Universal Jewish Encyclopedia* maintains to be "the oldest Hebrew mathematical treatise known," demonstrates a clear knowledge of the $\pi = 3^1/_7$ approximation. In fact, it asks the natural question of why the Bible didn't use this value:

> Nehemiah says, since the people of the world say that the circumference of a circle contains three times and a seventh of the diameter, take off from that one-seventh for the thickness of the sea on the two brims, then there remain thirty cubits [that compass it round about].

If *Mishnat Ha'Middot* was contemporaneous with the early Mishnaic sage

Ne<u>h</u>emiah, as Solomon Gandz upholds,[34] then this would be definitive evidence that the sages of the talmudic period knew the $3^1/_7$ approximation. Victor Katz, however, presents evidence that *Mishnat Ha'Middot* might actually have been composed as late as the ninth century CE.[35]

Part Three
Evidence of a More Precise Traditional Knowledge of π

Isaac Elishakoff and Elliot Pines

3.1 The Implications of a Circular Sukkah

Boaz Tsaban and David Garber[36] consider another important point from the larger discussion in the Talmud on the sukkah brought up in Part Two. This discussion concerns the religious validity of a circular sukkah. A 4-cubit by 4-cubit square must be circumscribed. Rabbi Yo<u>h</u>anan implies that a circular sukkah is valid if twenty-four men can sit around the circumference. Yet this provides an 18-cubit circumference, while $16^4/_5$ should suffice. While permission for approximation in support of stringency was granted, Rabbi Yo<u>h</u>anan was known for exactness. Tsaban and Garber explain:

> If indeed Rabbi Yo<u>h</u>anan used the inexact values [of π and $\sqrt{2}$], he could have said that 23 persons suffice. This would give $(23/\pi_0 - 2)\,\pi_0 = 17$ cubits for the circumference of the booth, which is much closer to $16^4/_5$ and yet more than the minimum requirement….The solution to this problem is to be found in Rabbi Shimon Ben Tsema<u>h</u>'s explanation, which follows. Rabbi Yo<u>h</u>anan's statement is quite precise, if we assume that he used more precise values for π and $\sqrt{2}$. For this, he takes $3^1/_7$ and for π and [diagonal] d slightly greater than $1^2/_5$, for $\sqrt{2}$. The minimum circumference is…4 $d \cdot 3^1/_7$ which is a little more than $17^3/_5$. The circumference of the booth is… $(24 / 3^1/_7 - 2)\, 3^1/_7 = 17^5/_7$, which is more than the minimum of $17^3/_5$ and the difference is not more than $^4/_{35}$ cubits.

This would correspond to a knowledge of π and $\sqrt{2}$ to a combined error not exceeding $(^4/_{35}/18) \cdot 100\% = 0.6\%$, that is, at least 8 times better than allowed by the approximation $\pi = 3$. Rabbi Tzvi Inbal[37] argues the point of the Sages' true knowledge even more strikingly. Seating men outside the sukkah seems a strange way to approximate, especially for Rabbi Yo<u>h</u>anan. Presume rather that exact value is being sought. That is, $(2 + (4 \cdot d))\, \pi = (2$

$+ (4\sqrt{2}))\,\pi = 24.055$ or simply 24 "men" fit by placing them outside the sukkah, a leeway of only 0.055—demonstrating an actual knowledge of both π and $\sqrt{2}$ to a combined error not exceeding $(0.055 / 24 \cdot 100\% = 0.23\%$, that is, an estimate of π at least twenty times better than $\pi = 3$.

Clearly, the Sages had at least a feel for the errors that they were dealing with. Given this, the weight of Maimonides' opinion, and the possibly contemporaneous evidence offered by *Mishnat Ha'Middot*, the preponderance of the evidence does suggest that they had knowledge at least on the order of the approximation of $\pi = 3^1/_7$ (0.04% accuracy).

We also see "the exception that proves the rule," a rare instance where a circumference is specified, rather than a radius or diameter, and an upper-bound estimate on π is required. Why were the Sages more exacting on this upper bound than they had been in lower-bound cases? As neither danger nor theft apply here, the Sages perhaps sought greater precision to minimize people's financial burden.

Why not seek a tighter lower bound? It would seem that 3 was deemed to provide a minimally sufficient fence, with the added benefit of simplicity of calculation (a significant advantage in the pre-calculator age). After all, 3 is 95 percent of π, providing for reasonable generosity to builder or seller. (For the talmudic approximation of $\sqrt{2}$ as $1^2/_5$, accurate to 99 percent, an approximation of 95 percent would likely have been deemed just as acceptable as it was for π.) Interestingly, one shouldn't imagine that there wasn't a less accurate lower-bound approximation to π used in even later history. Michael Constantine Bellus (1020-1110) approximated π as $\sqrt{8}$.[38] It is enlightening to note that most books criticizing the biblical use of 3 don't mention this or other numerically (if not morally) much "worse" approximations made over "1500 years later."

We suggest that, even if by nothing else, the biblical and rabbinic use of $\pi \approx 3$ has been thoroughly justified in terms of what today would be termed good engineering practice.

3.2 A Hidden Value of π?

Several authors comment upon a deep insight by Rabbi Max Munk,[39] seemingly misattributed to the Vilna Gaon.[40] The correct attribution is provided by Belaga,[41] who associates it with his meeting Rabbi Professor Zecharia Dor-Shav of Bar-Ilan University. Happily, Professor Dor-Shav attended the Fifth Miami International Conference on Torah and Science in 2003, when we presented this paper on π.

Rabbi Munk discovered a hidden second value of π through a comparative reading in depth of the relevant passages in I Kings and II Chronicles. The two verses match when read out loud, but differ in their written versions. Rabbi Munk compared the gematria, or numerology, of the two different verses and found that the numerical value of the written form of the term "line measure" in I Kings equals 111, while in II Chronicles both its written and read-aloud form equals 106.

I KINGS 7:23

ויעש את-הים מוצק עשר באמה משפתו עד-שפתו עגל סביב וחמש באמה קומתו

וקוה [read aloud as וקו]

ושלשים באמה יסב אתו סביב

He made the "sea" of cast [metal] ten cubits from its one lip to its [other] lip, two circular all around, five cubits its height; a thirty-cubit line could encircle it all around.

II CHRONICLES 4:2

ויעש את-הים מוצק עשר באמה משפתו אל-שפתו עגול סביב וחמש באמה קומתו

וקו

שלשים באמה יסב אתו סביב

He made the "sea" of cast [metal], ten cubits in diameter, circular in shape, five cubits high; a thirty-cubit line could go around it.
(English translation adapted from the ArtScroll Tanach, Stone 1st Edition, 1996)

Numerical values of the letters comprising the two written variations for "line measure"		
5 HEH ה	6 VAV ו	100 KUF ק

Rabbi Inbal[42] explains that the written modality of scripture reflects an ideal concept, while the read-aloud modality reflects the loss of dimensions and precision as a result of imperfect actualization. That is, written scripture reflects true reality, and read-aloud scripture reflects human reality. Since 3 is the value of π representing imperfection, the correction of this value would have to be multiplication by a ratio of the written (ideal) length of the line over the spoken (non-ideal) length. The result is 3.141509….

$$\pi_{\text{First-approximation}} = 3 \text{ (representing the practical world)}$$

$$^{\text{True}}/_{\text{Practical}} = {}^{\text{Written}}/_{\text{Read-aloud}} = {}^{\text{Kuf-vav-heh}}/_{\text{Kuf-vav}} = {}^{100+6+5}/_{100+6} = {}^{111}/_{106}$$

$$\pi_{\text{True}} = \pi_{\text{First-approximation}} \cdot {}^{\text{True}}/_{\text{Practical}} = 3 \cdot {}^{111}/_{106} = \mathbf{3.1415}\mathit{09}\ldots$$

Belaga[43] points out:

> It should be stressed that the proposed two-level semantical structure of a biblical verse (in our case, I Kings 7:23), one level for legal purposes and another for "connoisseurs," is not only a typical phenomenon in the rabbinic tradition—in a sense, such a multi-level approach to texts is the main methodological legacy of this tradition. As Ramban [Naẖmanides] [1195-1270] writes: "Everything that was transmitted to Moses our teacher through the forty-nine gates of understanding was written in the Torah explicitly or by implication in words, in the numerical value of the letter, or in the form of the letter, that is, whether written normally or with some change in form, such as bent or crooked letters, and other deviations."[44]

Arndt and Haenel[45] agree: "This value [$\pi_{\text{Bible}} = {}^{333}/_{106} = 3.141509 = \pi - 0.000083\ldots$] is accurate to four decimal places, and if it could only be confirmed, it would certainly silence the mirth at the apparent inaccuracy of the Bible."

Note that according to Belaga's analysis the value chosen for religious law is 3, and the encoded value is 3.141509, both representing *lower-bound* approximations of π. This fact would seem to match the authors' earlier contention that the *lower bound* of 3 likely was used specifically to produce an acceptable safety fence for purposes of engineering and/or interpersonal transactions

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Part Four: The Esoteric Approach

Elliot Pines

4.1 Five Questions

Rabbi Munk's observation that ${}^{111}/_{106}$, the numerical value of kuf-vav-heh (111) divided by the numerical value of kuf-vav (106), begs five questions:

1. Could this "encoding" be just a coincidence?
2. Alternatively, could the "encoding" be the product of ancient genius?
3. To what end is the far tighter lower bound compared to the practical one of $\pi = 3$ discussed above by Professor Elishakoff?
4. What eternal significance is there in a few significant figures?
5. What makes a value "true"?

4.2 From the Perspective of Information Theory

I began my search for answers to these questions by assuming that most single-word gematrias (numerological sums of letters) are less than or equal to 1000. Therefore, all possible pairs of numbers greater than or equal to 1, and less than or equal to 1000—that is, one million cases (1000 denominators times 1000 numerators)—were studied. Thirty cases (including ${}^{111}/_{106}$ itself and its unitary multiples, ${}^{222}/_{212,}$ ${}^{333}/_{318}$ and so on) provided π transformation of 3 of equal or smaller magnitude error. (The most extreme of these was the upper bound, $^{355}/_{339}$ with $^{1}/_{312}{}^{th}$ the magnitude error, and the second most extreme was $^{954}/_{911}$ with $^{1}/_{8}{}^{th}$ the magnitude error.) In other words, only about 0.003% did as well or better than Rabbi Munk's gematria ratio!

Suppose we claim that the Israelites pre-dating or contemporaneous with I Kings estimated π to at least three orders of magnitude greater accuracy than contemporary cultures and were the true originators of this tradition. Could they have been expected to encode it so compactly? They would have needed to ratio a three-letter string to a two-letter string with:

a. a single letter not common to both strings;
b. one string that is a validly spelled word;
c. semantic sense in context;
d. direct logical relationship that, given traditional rules, justify the ratio as a multiplier of 3.

The requirements of (a) and (d) would seem to imply some word corresponding to either line (or line-measure), diameter, radius, circumference, perimeter, or boundary. While certain options of ancient Hebrew for these words might be lost, it is reasonable to assume that newer ones of at least equal number exist in modern Hebrew. Using Lazar's and Ben Yehuda's dictionaries,[46] eliminating all equivalents with more than three letters, and ignoring the proper forms of final letters, I was left with:

kamat קמט	*meshekh* משכ	*hoog* חוג	*hevel* חבל
in addition to, of course, *kav* (line) קו.			

•The last four words in this list allow for a match by dropping any of three letters, so we have 4 • 3 = 12 possible pairs. Examples are:

מש/משכ מכ/משכ שכ/משכ

Ignoring any Masoretic-rule-based-limitations, to be on the conservative side, another twenty-two possible pairs can be made by adding any letter of the Hebrew alphabet to the first word on this list. For example:

קו/קוא, קו/קוב, קו/קוה

for a total of *12 + 22 = 34* possible pairs. (The third example above, *kuf-vav-heh,* is the actual case in the text.) In other words, there appear to be about thirty-four "finalists."

From the expectation perspective of information theory, these thirty-four unique possibilities would allow the ancients to produce approximately the base-2 logarithm of 34, $Log_2\{34\} = 5.1$ *BITs* of information. (BIT = *bi*nary *t*erm, as used in measuring computer memory). One information content measure for the gematria ratio in BITs considers the absolute value of the inverse of the fractional error relative to $\pi/3$, $Log_2[\ (^{\pi}/_{3}) / ((^{\pi}/_{3}) - (^{111}/_{106}))\ ] = 15.2$ *BITs*.

Another such measure looks at the inverse of the fractional probability of obtaining that error or better by random numerators and denominators in the range of 1 to 1000, $Log_2\{\ ^{1}/_{0.00003}\ \} = 15.0$ *BITs*—a fairly close match. This leaves a shortfall to the ancients of about $(15.2 + 15.0) / 2 - 5.1 = 10.0$ *BITs*, or a factor of $2^{10} = 1024$. Even if the ancients had encoded the hidden value as cleverly as possible, they still would have required a great amount of "luck."

In fact, if any three letters could be given for numerator gematria, and any two of those for the denominator gematria, this provides for

$$Log_2\{\ 22 \bullet 22 \bullet 22 \bullet 3\ \} = 15.0\ BITs.$$

In other words, $^{\text{kuf-vav-heh}}/_{\text{kuf-vav}}$ is likely the only string ratio that works, even without semantic or syntactic limitations.

If this encoding is divine in origin, what does a mere finite improvement in accuracy accomplish? To shed more light on this question, let us consider it from a mystical perspective.

4.3 The Transcendence of π

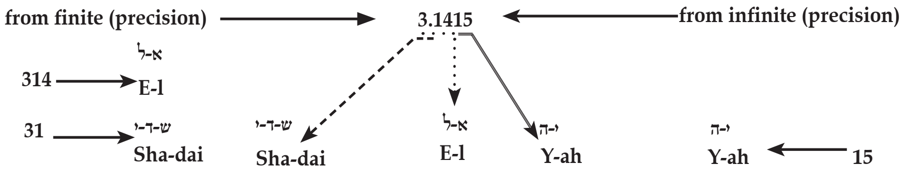
Consider the digit string *31415*.

Earlier writers have already noted that the first two digits taken together as 31 form the numerical equivalent of E-l, a name of G-d implying strength. Even more significantly, it has already been noted by others that the first three digits, 314, form the numerical equivalent of Sha-dai, another name of G-d, implying limit.

I wish to build upon this foundation. Firstly, we note that 314 can also be considered as the most compact possible representation of these two Divine names taken together in their order of appearance, E-l Sha-dai. This compound is itself a third name of G-d, implying the A-lmighty, or according to Saadia Gaon, the Omnipotent A-lmighty.[47] Next, take together the remaining fourth and fifth digits—1 and 5—to form *yud-heh*, yet another name of G-d, Y-ah, which means the Eternal, according to Saadia Gaon.[48]

The ancient kabbalistic text *Sefer Yetsirah* states that "With thirty-two mystical paths of Wisdom engraved Y-ah…E-l Sha-dai….And He created His universe with three books, with text, with number, and *with communication*."[49] Rabbi Aryeh Kaplan comments,

> The first five designations [which begin with Y-ah] represented the downward process from G-d to the universe, through which the creative force is channeled. The author, however, is now [beginning with E-l Sha-dai] designating the names that relate to the upward process through which man approaches the Divine.[50]



I propose that "15" in the scheme above could represent G-d's approach to us from infinite precision (i.e., the side of 3.14|15 finitely summarizing the places out towards infinite precision), while "314" represents the concept of our approach to G-d from finite precision. This also leads to the question, would such a representation be a totally abstract symbolism of G-d's interaction with the world, or could it be manifest in physical reality? A brief speculation on this subject is offered in Appendix 1.

Part Five: Conclusions

Isaac Elishakoff and Elliot Pines

Blanter notes in his book, "Every imaginable explanation of the discrepancy has been proposed, from 'This is proof that the Bible is false,' to 'This is proof that pi really does equal 3, and scientists are lying to us.'"[51] We have attempted to offer here something beyond this simple linear spectrum, rendering Blanter's statement imprecise.

Beckmann states, "The inaccuracy of the Biblical value of π is, of course, no more than amusing curiosity. Nevertheless, with the hindsight of what happened afterwards, it is interesting to note this little pebble on the road to the confrontation between science and religion."[52] Certainly, the issue of π is a mere *hors d'oeuvre* sampling of the great banquet of Torah and modern science interaction. We feel it to be a sampling of possible harmony rather than contradiction.

Two important pedagogical lessons may be drawn from our study:

1. How good is good enough? Even the 1.2 trillion digit approximation of π made by Professor Yasumasa Kanada[53] of Tokyo University in 2002 is still only an approximation. It is humbling to realize that there is something that we can never really know, and π provides us with this experience.

2. Sound research and teaching are multi-level. One-dimensional glosses can mislead, while multidimensionality makes for a more complete and trustworthy study. It also provides students with depth and direction for integrating a subject. Along these lines, we suggest that our findings be modified and included in mesivtah and yeshivah high school curricula. "Train a child according to his way; even when he is old, he will not depart from it,"[54] advises King Solomon, the maker of the "molten sea."

Appendix 1

A Boundary on Physical Reality?

Elliot Pines

Rabbi Munk's finding is connected only with one object, the "molten sea." Thus, the remaining difference with ideal π constitutes an absolute error in length—of circumference, diameter, or some combination. If this error is attributed completely to the circumference (having exactly a 10-cubit diameter), it is $-1.71 \cdot 10^{-5}$ cubits in length. Rabbinical opinion on the length of the cubit ranges from 18 to 24 inches, which at 25,400 microns per inch translates to a range of circumference variation from -380 to -500 microns. If attributed to a diameter of an exactly thirty-cubit circumference, this means a range of diameter variation of +130 to +170 microns. Could this total range of magnitude variation of about 100 to 500 microns represent a minimum unit of size? Let us consider some interesting speculations.

Below this range (in all three dimensions), an object like a single grain of dust is too small for detection by the unaided eye. Jewish law considers a minuscule insect in a salad or bread crumb during Passover of this dimension *bitul*—nullified or nonexistent.

Indeed, do such tiny objects actually exist? Roger Penrose[55] suggests that the alternative possibilities allowed by the Heisenberg uncertainty principle of quantum mechanics collapse into one objective reality. This would happen once one of the parallel histories interacted with about one particle of gravity (a "graviton")—a minimum unit of curved space/time produced by 22 micrograms, called "Planck mass."[56] Taking water as having a typical mass density on Earth, we note that 22 micrograms occupy 2.2 nanoliters, spanning about *300* microns.

Pharaoh's sorcerers could not duplicate the third plague of *kinnim* (tiny insects) as they did the first two plagues of blood and frogs. The Talmud *Sanhedrin* 67a explains that the spiritually impure forces of the sorcerers could not (secretly) gather such small creatures in order to make the illusion of transforming them from dust. Perhaps these forces of impurity, being without mass themselves, so dependent on the spiritual vacuum possible only in our (curved) space/time, simply cannot deal with a mass too small to produce its own (curved) space/time.

If 100 to 500 microns is a divine "lower-bound approximation," then (3 times) $^{111}/_{106}$ has already been shown to be the most compact fraction guaranteeing it. Reflecting Professor Elishakoff's approach to an ultimate abstract, why might the Divine Engineer require a boundary with safety factors? We may infer an answer to this from Rabbi Moshe H̲ayyim Luzzatto (the Ramh̲al, 1707-1747), who writes that without boundaries life would be overwhelmed by Divine Light. Living creatures need leeway so that the channels of spiritual sustenance will not be cut off.[57]

Why might such channels be represented as having their source in what is vanishingly small? Regarding the Lurianic terminology of "a very small spark, which is G-dliness that extends from [the Creator]," the early twentieth-century kabbalist Rabbi Yehudah Ashlag comments, "…the unattainable is called very small…".[58] Consider, too, that Nahmanides (1195-1270) explains primordial creation as being "like a very small point."[59]

Safety factors themselves have limits. The Ramhal quotes Proverbs 22: 28 and 23:10, "Do not move the boundary of the universe." This is a warning from King Solomon, who specified the dimensions of the "molten sea," the spiritual pool of spiritual pools, in a boundary of boundaries.

Appendix 2

Isaac Elishakoff

Does anyone today use 3 as the value for π? During my sabbatical in Japan from December 2006 to February 2007, I learned that the answer is yes. The Japanese Ministry of Education, Culture, Sports, Science, and Technology has instituted minimum requirements, in order to reduce the amount of material that students must memorize. Accordingly, the "circular constant" π is taught in Japan as 3. Additionally, Kazuo Muroi wrote to me, "I agree with you that the Hebrew sages used the value 3 for convenience's sake as the Babylonian scribes did so."[60]

Notes

1 David Blanter, *The Joy of π* (Allen Lane, 1997).

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5 Petr Beckmann, *A History of π* (New York: St. Martin's Press, 1971) p.15.

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8 *Encyclopedia Judaica* (New York: The Macmillan Company, 1971) vol. 11, pp. 1122-1123.

9 Isaac Landman, ed., *Universal Jewish Encyclopedia* (New York: Ktav Publishing, 1969) vol. 7, pp. 408-411.

10 Shlomo Edward G. Belaga, "On the Rabbinical Exegesis of an Enhanced Biblical Value of π," Proceedings, XVIIth Canadian Congress: History, Philosophy and Mathematics (1991) pp. 93-101.

11 Talmud *Bava Metsia* 31b and elsewhere.

12 Beckmann, op. cit., p.15.

13 Radha Charan Gupta, "On the Values of π from the Bible," *Ganita Bharati*, vol. 10, nos. 1-4 (1988) pp.

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[19] Kim Jonas, "Why Did the Ancients Invent Increasingly Subtle and Ingenious Methods to Arrive at an Exact Value of π? Human Curiosity," *Archeology Odyssey* (Mar/Apr 2000) pp. 10-11. This paper was kindly provided by Professor Nachum Sarna.

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[22] Jens Høyrup, "Babylonian Mathematics," in *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, ed. I. Grattan-Guiness, ed., (London: Routledge, 1994) vol. 1, p. 24; Jonas, op. cit.

[23] C.B. Boyer, *A History of Mathematics*, 2nd ed. (New York: Wiley, 1991).

[24] Dario Castellanos, "The Ubiquitous π," *Mathematics Magazine*, vol. 61 (1988) pp. 148-163.

[25] See Castellanos's reference on F. Peyrard (1819).

[26] Maimonides, *Mishneh Torah*, *Hilkhot Rotseah*; Y.D. Kappah, *Mishnah with Maimonides' Commentary* (Jerusalem: Mosad Harav Kook, 1963) *Moed* , pp. 63-64 (in Hebrew); Blanter, op. cit.

[27] See T.L. Heath, *A History of Greek Mathematics* (Elibron Classics, 2006) vol. 2, p. 302.

[28] Henry Petroski, *To Engineer Is Human* (2005) p. 197. In a personal communication in 2005, Professor Petroski informed I.E. that "To the best of my recollection, the anecdote related to the nuclear industry, [is] probably [correct for the period] between the late 1970s and the early 1980s."

[29] Mishnah *Bava Metsia* 61b.

[30] Talmud *Sukkah* 8a (Soncino translation).

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[48] Ibid., p.15.
[49] Ibid., p.5.
[50] Ibid., pp. 17, 18.
[51] Blanter, op. cit.
[52] Beckmann, op. cit.
[53] Audrey McAvoy, "Professor Breaks Own Record—for Thrill of Pi," (Associated Press) *Seattle Post-Intelligencer*, 7 Dec 2002, http://seattlepi.nwsource.com/national/98912_pi07.shtml. It would consume a forest to print out the π-expansion digits. To learn more about them, Professor Kanada's site www.super-computing.org is a good read.
[54] Proverbs 22:6.
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Postscript

If we have missed any important information available in other sources, we would be most grateful to receive your comments. Please send them to the e-mail addresses on the title page of this article.

Dr. Israel Gilat has informed us that Professor Bernard Pinchuk of the Netanya Academic College has presented a paper on π in Scripture. Likewise, Dr. Shlomo Yanez of Bar-Ilan University has informed us that he is preparing a paper on this topic, which we look forward to reading.

Isaac Elishakoff

Elliot M. Pines

Acknowledgments

We wish to thank Professor Herman Branover for recommending that the contents of this paper be taught at yeshivah high schools and other places of religious study. We thank Professor Roald Hoffman of Cornell University and Professors Nathan Aviezer and Zecharia Dor-Shav of Bar-Ilan University, and Dr. Boaz Tsaban of the Weizmann Institute for their thought-provocative questions and Rabbi Tzvi Inbal of Arachim for his correspondence. We thank Professor Kiyohiro Ikeda for inviting I.E. to the Tokyo University of Japan and providing him the opportunity to meet Mr. Kazuo Muroi of the Bunri Preparatory College. Mr. Muroi was most helpful in preparing this paper. We are grateful to Professor Makoto Ohsaki for inviting I.E. to Kyoto University, where Mr. Takao Hagishita informed us and translated documents on how π should be taught in Japan. We thank Dr. Rivka Gilat for inviting I.E. to the College of Judea and Samaria; in these last two institutions the final touches were made to the manuscript. We thank our three anonymous peer-reviewers for their constructive comments and the OU Israel Center in Jerusalem for inviting I.E. to present the paper in June 2007. The work was partially supported by the J.M. Rubin Foundation of Florida Atlantic University. I.E. intends to extend this work into a book to help students appreciate the interaction between Scripture and mathematics. Funding for this project is welcome.