INFINITESIMAL ANALYSIS   88-503   MOED A   14 JULY '22

All answers must be fully justified by giving complete proofs.
Each problem is worth 22 points.

1. Consider the full relational structure $Rel_{\mathcal{R}}$ including all relations over $\mathbb{R}$ with finitely many unknowns. Consider the corresponding relational structure $Rel_{^*\mathcal{R}}$ over $^*\mathbb{R}$ obtained by "starring" all the relations of $Rel_{\mathcal{R}}$. Prove that the set $\mathbb{N}$ does not belong to $Rel_{^*\mathcal{R}}$.

2. Use the hyperreal characterisation of convergence to show that if a sequence converges in $\mathbb{R}$ then it has exactly one cluster point (nekudat hitztabrut).

3. Use the hyperreal characterisation of uniform continuity to show that $f(x) = \frac{1}{x}$ is not uniformly continuous on $(0, 1)$.

4. Let f be a real function that is continuous on some interval $A \subseteq \mathbb{R}$. If $f(x)$ is real for all $x \in {}^*A$, show that f is constant on A.

5. Use countable saturation to infer the existence of positive infinite and negative infinite members of $^*\mathbb{R}$.

Good Luck!