Infinitesimal analysis 88-503 homework set 4

Due Date: 6 june '22

1. Prove that a set $A \subseteq \mathbb{R}$ is open if and only if for every point $x \in A$ one has $hal(x) \subseteq {}^*A$.

2. Show that each S-open set in ${}^*\mathbb{R}$ is a union of halos, but a union of halos need not be S-open.

3. Show that overflow is equivalent to the following statement: If an internal subset $X \subseteq {}^*\mathbb{N}$ contains arbitrarily small infinite members, then it is unbounded in $\mathbb{N}$, i.e., contains arbitrarily large finite members.

4. Use countable saturation to infer the existence of positive infinite and negative infinite members of ${}^*\mathbb{R}$.