t-symmetrizable quasi-uniformities

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**Definition 1** Let $X$ be a set and let $d : X \times X \to [0, \infty)$ be a function mapping into the set $[0, \infty)$ of the nonnegative reals. Then $d$ is called a quasi-pseudometric on $X$ if

(a) $d(x, x) = 0$ whenever $x \in X$,

(b) $d(x, z) \leq d(x, y) + d(y, z)$ whenever $x, y, z \in X$.

We shall say that $d$ is a $T_0$-quasi-(pseudo)metric provided that $d$ also satisfies the following condition: For each $x, y \in X$,

$d(x, y) = 0 = d(y, x)$ implies that $x = y$. 
For each positive $\epsilon$ we shall set $U_{d,\epsilon} := \{(x, y) \in X \times X : d(x, y) < \epsilon\}$.

The quasi-pseudometric quasi-uniformity induced by $d$ on $X$ will be denoted by $U_d$.

As usual, we say that a topological space $(X, \tau)$ has a compatible quasi-uniformity if there is a quasi-uniformity $\mathcal{U}$ on $X$ such that $\tau = \tau\mathcal{U}$, where by $\tau\mathcal{U}$ we denote the topology induced by $\mathcal{U}$, and we say that $(X, \tau)$ has a compatible quasi-pseudometric if there is a quasi-pseudometric $d$ on $X$ such that $\tau = \tau U_d$. 
Remark 1 Let $d$ be a quasi-pseudometric on a set $X$, then $d^{-1} : X \times X \to [0, \infty)$ defined by $d^{-1}(x, y) = d(y, x)$ whenever $x, y \in X$ is also a quasi-pseudometric, called the conjugate quasi-pseudometric of $d$.

As usual, a quasi-pseudometric $d$ on $X$ such that $d = d^{-1}$ is called a pseudometric.

Note that for any ($T_0$-)quasi-pseudometric $d$, $d^s = \sup \{d, d^{-1}\} = d \vee d^{-1}$ is a pseudometric (metric).
According to Matthews a quasi-pseudometric $q$ on a set $X$ is called *weightable* provided that there is a function $|\cdot| : X \to [0, \infty)$ such that

$$q(x, y) + |x| = q(y, x) + |y|$$

whenever $x, y \in X$.

The function $|\cdot|$ will be called a *weight* for $q$. 
More generally we shall call a quasi-uniformity $\mathcal{U}$ on a set $X$ weightable if $\mathcal{U} = \bigvee_{d \in \mathcal{E}} \mathcal{U}_d$ where $\mathcal{E}$ is a family of weightable quasi-pseudometrics on $X$.

We shall call a quasi-uniform space $(X, \mathcal{U})$ weightable if $\mathcal{U}$ is a weightable quasi-uniformity on $X$. 
For later use, we recall that a quasi-uniformity $\mathcal{U}$ is called *precompact* provided that for each $U \in \mathcal{U}$ there is a finite $F \subseteq X$ such that $U(F') = X$.

Furthermore a quasi-uniformity $\mathcal{U}$ is called *totally bounded* provided that the uniformity $\mathcal{U}^s = \mathcal{U} \lor \mathcal{U}^{-1}$ is precompact.

A quasi-uniform space $(X, \mathcal{U})$ is said to be *hereditarily precompact* provided that each subspace of $(X, \mathcal{U})$ is precompact.

For any quasi-uniformity $\mathcal{U}$, by $\mathcal{U}_\omega$ we shall denote the finest totally bounded quasi-uniformity coarser than $\mathcal{U}$.
A quasi-uniformity is called *transitive* provided that it has a base consisting of transitive entourages.

Throughout $\mathcal{D}$ will denote the discrete uniformity on a set $X$.

Moreover for two real numbers $a, b$ we shall set $a \cdot b = \max\{a - b, 0\}$.

**Example 1** Equip the set $[0, \infty)$ with the quasi-pseudometric $u(x, y) = y \cdot x$ whenever $x, y \in [0, \infty)$, and set $|x| = x$ whenever $x \in [0, \infty)$. Then $u$ is a $T_0$-quasi-pseudometric on $[0, \infty)$ that is weighted by the usual norm $| \cdot |$ on $\mathbb{R}$ restricted to $[0, \infty)$.

Furthermore the quasi-uniformity $\mathcal{U}_{u^{-1}}$ is not weightable.
Example 2 The fine quasi-uniformity $\mathcal{F}$ of the topological space of the rationals $\mathbb{Q}$ (equipped with its usual topology) is not weightable.

Proposition 1 A topological space has a compatible weightable quasi-pseudometric if and only if it has a compatible weightable quasi-uniformity with a countable base.

Proposition 2 Let $(X,\mathcal{U})$ be a $T_0$-quasi-uniform space. Then its bi-completion $(\tilde{X},\tilde{\mathcal{U}})$ is a weightable $T_0$-quasi-uniform space, too.
In the following we discuss some analogues of uniform hyperspaces which yield some important examples of weightable quasi-uniformities.

Let \((X, d)\) be a quasi-pseudometric space and let \(\mathcal{P}_0(X)\) be the set of nonempty subsets of \(X\). Moreover let \(A, B \in \mathcal{P}_0(X)\). We set

\[
H^+_d(A, B) = \sup_{b \in B} d(A, b)
\]

and

\[
H^-_d(A, B) = \sup_{a \in A} d(a, B).
\]

(As usual, here for instance \(d(A, b) = \inf\{d(a, b) : a \in A\}\).

Furthermore

\[
H_d(A, B) = H^+_d(A, B) \lor H^-_d(A, B).
\]
Then $H_d^+$ is the extended upper Hausdorff quasi-pseudometric, $H_d^-$ is the extended lower Hausdorff quasi-pseudometric and $H_d$ is the extended Hausdorff quasi-pseudometric on $\mathcal{P}_0(X)$.

Similarly for each $x \in X$, set

$$(W_d^+)x(A, B) = d(A, x) - d(B, x)$$

and

$$(W_d^-)x(A, B) = d(x, B) - d(x, A).$$

Moreover for each $x \in X$, let

$$(W_d)x(A, B) = (W_d^+)x(A, B) \lor (W_d^-)x(A, B).$$
Then for each $x \in X$, $(W^+_d)_x$ is the upper Wijsman quasi-pseudometric at $x$, $(W^-_d)_x$ is the lower Wijsman quasi-pseudometric at $x$, and $(W_d)_x$ is the Wijsman quasi-pseudometric at $x$. 
For each \( x \in X \), \((W_d^-)_x\) is a weightable quasi-pseudometric with weight \((w_d^-)_x(A) = d(x, A)\) whenever \( A \in \mathcal{P}_0(X) \).

Let \( A, B \in \mathcal{P}_0(X) \). It is known that

\[
H^+_d(A, B) = \sup_{x \in X} (W^+_d)_x(A, B) = \sup_{x \in X} (d(A, x) - d(B, x)).
\]

Similarly we have

\[
H^-_d(A, B) = \sup_{x \in X} (W^-_d)_x(A, B) = \sup_{x \in X} (d(x, B) - d(x, A)).
\]

Hence also \( H_d(A, B) = \sup_{x \in X} (W_d)_x(A, B) \).
Proposition 3 Each totally bounded quasi-uniformity $\mathcal{U}$ on a set $X$ is weightable.

Given any quasi-pseudometric space $(X, d)$, we can define quasi-uniformities

$$\mathcal{U}_{W_d^+} = \bigvee_{x \in X} \mathcal{U}_{(W_d^+)x}$$

and similarly

$$\mathcal{U}_{W_d^-} = \bigvee_{x \in X} \mathcal{U}_{(W_d^-)x}$$

and

$$\mathcal{U}_{W_d} = \bigvee_{x \in X} \mathcal{U}_{(W_d)x}$$
on $\mathcal{P}_0(X)$. 
Corollary 1 Let \((X, d)\) be a quasi-pseudometric space. When restricting the Wijsman quasi-uniformities to \(X\) (where we identify the points with singletons), we have

\[ U_{W^+} \subseteq U_d \]

and

\[ U_{W^-} \subseteq U_d. \]

consequently

\[ U_{W_d} \subseteq U_d. \]

Equality holds in the three inclusions provided that \(U_d\) is totally bounded.
Remark 2 There are obvious connections between the upper resp. lower constructions considered above and the operation of conjugation.

For instance, for any quasi-pseudometric $d$ on a set $X$ we have that for any $x \in X$,

$$((W^+_{d^{-1}}x)^{-1} = (W^-_{d})x,$$

and thus

$$(U_{W^+_{d^{-1}}})^{-1} = U_{W^-_{d}}$$

on $\mathcal{P}_0(X)$. 
Proposition 4 Let $d$ be a totally bounded quasi-pseudometric on a set $X$. Then

$$U_{W_d}^+ = U_{H_d}^+,$$
$$U_{W_d}^- = U_{H_d}^-$$

and

$$U_{W_d} = U_{H_d}$$

on $\mathcal{P}_0(X)$. 
Given a quasi-pseudometric space \((X, d)\), for each \(x \in X\) set

\[ r_x(a) = d(a, x) \]

and

\[ l_x(a) = d(x, a) \]

whenever \(a \in X\).

Similarly let us define for each \(x \in X\),

\[ R_x(A) = d(A, x) \]

and

\[ L_x(A) = d(x, A) \]

whenever \(A \in \mathcal{P}_0(X)\).
The following definition is well known.

A quasi-pseudometric \( d \) on a set \( X \) is called \textit{bounded} provided that there is a constant \( M > 0 \) such that \( d(x, y) \leq M \) whenever \( x, y \in X \).

Let us observe that for any \( x \in X \), \( r_x \) is bounded on \( X \) if and only if \( R_x \) is bounded on \( \mathcal{P}_0(X) \).

Analogously for any \( x \in X \), \( l_x \) is bounded on \( X \) if and only if \( L_x \) is bounded on \( \mathcal{P}_0(X) \).
Lemma 1 A quasi-pseudometric $d$ on a set $X$ is bounded if and only if there is an $x \in X$ such that both $r_x$ and $l_x$ are bounded.

Boundedness conditions imply hereditary precompactness properties of Wijsman type quasi-uniformities:

Remark 3 The quasi-uniformity $U_{u^{-1}}$ is hereditarily precompact on $[0, \infty)$. Hence given any set $X$, for any function $f : X \to [0, \infty)$, the initial quasi-uniformity $(f \times f)^{-1}U_{u^{-1}}$ is hereditarily precompact.
Suppose now that $d$ is a quasi-pseudometric on $X$. Observe that on $\mathcal{P}_0(X)$ we have
\[ \mathcal{U}_{(W^+_d)x} = (R_x \times R_x)^{-1} \mathcal{U}_{u^{-1}} \]
and
\[ \mathcal{U}_{(W^-_d)x} = (L_x \times L_x)^{-1} \mathcal{U}_u \]
whenever $x \in X$. Therefore for any quasi-pseudometric space $(X, d)$ and $x \in X$, $\mathcal{U}_{(W^+_d)x}$ is hereditarily precompact on $\mathcal{P}_0(X)$ and $(\mathcal{U}_{(W^-_d)x})^{-1}$ is hereditarily precompact on $\mathcal{P}_0(X)$.

Consequently $\mathcal{U}_{W^+_d}$ and $(\mathcal{U}_{W^-_d})^{-1}$ are hereditarily precompact on $\mathcal{P}_0(X)$, since hereditary precompactness is preserved under arbitrary suprema of quasi-uniformities.
Given $x \in X$, $r_x$ is bounded on $X$ if and only if $\mathcal{U}(W_d^+)_x$ is totally bounded on $\mathcal{P}_0(X)$.

Similarly, given $x \in X$, $l_x$ is bounded on $X$ if and only if $\mathcal{U}(W_d^-)_x$ is totally bounded on $\mathcal{P}_0(X)$.

Given $x \in X$, $\mathcal{U}(W_d)_x$ is totally bounded on $\mathcal{P}_0(X)$ if and only if $d$ is bounded on $X$. 
**Corollary 2** Let \((X, d)\) be a quasi-pseudometric space.

Then \(U_{W^+} \) is totally bounded on \(P_0(X)\) if and only if for each \(x \in X\), \(r_x\) is bounded on \(X\).

Similarly \(U_{W^-}\) is totally bounded on \(P_0(X)\) if and only if for each \(x \in X\), \(l_x\) is bounded on \(X\).

Finally \(U_{W^d}\) is totally bounded on \(P_0(X)\) if and only if \(d\) is bounded on \(X\).
Proposition 5 Let \( q \) be a weightable quasi-pseudometric with weight function \( f \) on a set \( X \). Then

\[
[(f \times f)^{-1} \mathcal{U}_u] \subseteq \mathcal{U}_q
\]

and

\[
\mathcal{U}_q \subseteq [(f \times f)^{-1} \mathcal{U}_u] \lor \mathcal{U}_{q^{-1}}.
\]

In the following we replace the quasi-metric theory of weightability by a quasi-uniform approach to weightability.
Let $\mathcal{U}$ be a quasi-uniformity on a set $X$. Then $\mathcal{U}$ contains a finest symmetric quasi-uniformity coarser than $\mathcal{U}$, namely $\mathcal{U} \wedge \mathcal{U}^{-1}$, and $\mathcal{U}$ is contained in a coarsest symmetric quasi-uniformity finer than $\mathcal{U}$, namely $\mathcal{U}^s$.

A quasi-uniformity $\mathcal{A}$ on $X$ will be called a *symmetrizer* of $\mathcal{U}$ provided that

$$\mathcal{U} \vee \mathcal{A}$$

is symmetric, that is, $\mathcal{U} \vee \mathcal{A}$ is a uniformity.
A symmetrizer $\mathcal{A}$ of $\mathcal{U}$ will be called *adequate* provided that $\mathcal{A} \subseteq \mathcal{U}^{-1}$.

**Example 3** Let $\mathcal{U}$ and $\mathcal{Z}$ be quasi-uniformities on a set $X$. Then $\mathcal{Z}$ satisfies both $\mathcal{Z} \subseteq \mathcal{U}^{-1}$ and $\mathcal{U}^{-1} \subseteq \mathcal{U} \lor \mathcal{Z}$ if and only if $\mathcal{Z}$ on $X$ is an adequate symmetrizer of $\mathcal{U}$. 
Remark 4 Let $d$ be a weightable quasi-pseudometric with weight function $f$ on a set $X$. We set $A = (f \times f)^{-1}(U_{u^{-1}})$.

We have $A \subseteq U_{d^{-1}}$ and $U_{d^{-1}} \subseteq U_d \lor A$.

Hence $A$ is an adequate symmetrizer for $U_d$. It follows that

$$U_d^s = U_d \lor A.$$

For any weight $f$, $A$ is hereditarily precompact, since $f$ is bounded below by 0.

For any weight $f$, $A^s$ is preLindelöf.

For a bounded weight $f$, $A$ is totally bounded.
If $A$ is a(n adequate) symmetrizer for a quasi-uniformity $U$, then $A^{-1}$ is a(n adequate) symmetrizer for $U^{-1}$.

For each quasi-uniformity $U$, the conjugate quasi-uniformity $U^{-1}$ is an adequate symmetrizer of $U$.

Consider a quasi-uniformity $U_T$ generated by a partial order $T$ on a set $X$ (that is, $U_T$ has the base $\{T\}$) and let $L$ by a linear extension of $T$ on $X$.

Then in general the quasi-uniformity $U_{L^{-1}}$ generated by $L^{-1}$ is strictly coarser than the quasi-uniformity $U_{T^{-1}}$ generated by $T^{-1}$ on $X$, but obviously both are adequate symmetrizers of $U_T$. 
For each uniformity $\mathcal{U}$, any quasi-uniformity coarser than $\mathcal{U}$ is an adequate symmetrizer of $\mathcal{U}$.

Trivially, each totally bounded quasi-uniformity $\mathcal{U}$ on a set $X$ can be made transitive by taking the supremum with the finest possible (transitive) totally bounded quasi-uniformity $\mathcal{D}_\omega$ on $X$.

On the other hand there are quasi-uniformities that cannot be made transitive by taking the supremum with any totally bounded quasi-uniformity.
Similarly there are quasi-uniformities that cannot be made symmetric by taking the supremum with any totally bounded quasi-uniformity.

Obviously each quasi-uniformity on a set $X$ can be made symmetric by taking the supremum with the discrete uniformity $\mathcal{D}$ on $X$. 
Lemma 2 Suppose that $\mathcal{U}$ is a quasi-uniformity on a set $X$. Let $\{A_i : i = 1, \ldots, n\}$ be a finite cover of $X$ and let $V \in \mathcal{U}$.

Then

$$\bigcup_{i=1}^{n} (V^{-1}(A_i) \times V(A_i))$$

belongs to $\mathcal{U}_\omega$.

Theorem 1 Let $\mathcal{U}$ and $\mathcal{V}$ be quasi-uniformities on a set $X$ such that there is a totally bounded quasi-uniformity $\mathcal{Z}$ on $X$ with

$$\mathcal{U} \subseteq \mathcal{V} \vee \mathcal{Z}.$$ 

Then

$$\mathcal{U} \subseteq \mathcal{V} \vee (\mathcal{U} \vee \mathcal{V}^{-1})_\omega.$$
Let us call a quasi-uniformity $\mathcal{U}$ on a set $X$ \textit{t-symmetrizable} provided that there is a totally bounded quasi-uniformity $\mathcal{Z}$ on $X$ such that $\mathcal{U} \lor \mathcal{Z}$ is a uniformity, that is, $\mathcal{U}$ possesses a totally bounded symmetrizer $\mathcal{Z}$. 
Remark 5 (a) The supremum of any family of $t$-symmetrizable quasi-uniformities on a set $X$ is $t$-symmetrizable.

(b) The conjugate of a $t$-symmetrizable quasi-uniformity is $t$-symmetrizable.

(c) Each totally bounded quasi-uniformity is $t$-symmetrizable.

(d) The restriction to a subspace of a $t$-symmetrizable quasi-uniformity is $t$-symmetrizable.

(e) The product quasi-uniformity of any family of $t$-symmetrizable quasi-uniformities is $t$-symmetrizable.
Corollary 3 (a) Let \((q_i)_{i \in I}\) be a family of quasi-pseudometrics on a set \(X\) where \(q_i\) is weightable by a bounded weight \(f_i\) \((i \in I)\). Then \(\bigvee_{i \in I} U_{q_i}\) is \(t\)-symmetrizable.

(b) Each weightable quasi-uniformity inducing a countably compact topology is \(t\)-symmetrizable.

Proposition 6 Let \(U\) be a \(t\)-symmetrizable quasi-uniformity on a set \(X\). Then \(U\) can be adequately symmetrized by \((U^{-1})_\omega\) to \(U^s\).
Corollary 4 The following statements for a quasi-uniformity \( U \) on a set \( X \) are equivalent:

(a) \( U \) is \( t \)-symmetrizable.
(b) \( U \) has a totally bounded adequate symmetrizer.

Remark 6 Let \( U \) be a quasi-uniformity on a set \( X \). Note that the existence of a quasi-uniformity \( Z \) on \( X \) such that \( U = U^{-1} \lor Z \) implies that \( U^s = U^{-1} \lor Z = U \), hence that \( U \) is a uniformity.
Let $\mathcal{U}$ be a $t$-symmetrizable quasi-uniformity.

Then according to the preceding results $\mathcal{U}^s$ is clearly the smallest uniformity that we can obtain by $t$-symmetrization of $\mathcal{U}$.

$$\mathcal{U}^s \lor \mathcal{D}_\omega =$$
$$\mathcal{U} \lor (\mathcal{U}^{-1})_\omega \lor \mathcal{D}_\omega = \mathcal{U} \lor \mathcal{D}_\omega$$

is obviously the finest uniformity that we can reach by $t$-symmetrization of $\mathcal{U}$. 
Indeed each uniformity in between these two extreme cases can be obtained by $t$-symmetrization of $U$:

**Proposition 7** Let $U$ be a $t$-symmetrizable quasi-uniformity on a set $X$. Then any uniformity $V$ on $X$ such that $U^s \subseteq V \subseteq U^s \lor D_\omega$ satisfies $V = U \lor V_\omega$.

**Remark 7** Suppose that $U$ is a quasi-uniformity on a set $X$ that has a countable base and is $t$-symmetrizable. Then $U$ can be symmetrized by some totally bounded quasi-uniformity on $X$ having a countable base.
Theorem 2 Let $U$ be a $t$-symmetrizable quasi-uniformity on a set $X$. Then $U$ can be written as the supremum of a family of quasi-pseudometric quasi-uniformities $U_q$ where $q$ is a quasi-pseudometric on $X$ such that the quasi-uniformity $U_q$ is $t$-symmetrizable.

A quasi-uniformity $U$ on a set $X$ is called *proximally symmetric* provided that the finest totally bounded quasi-uniformity $U_\omega$ coarser than $U$ is a uniformity on $X$. 
Proposition 8 Each $t$-symmetrizable proximally symmetric quasi-uniformity $U$ on a set $X$ is a uniformity.

Proposition 9 Each $t$-symmetrizable quasi-uniformity $U$ on a set $X$ satisfies $(U^s)_\omega = (U^{-1})_\omega \lor U_\omega$. 
Several results that were originally proved for weightable quasi-pseudometrics (with bounded weights) indeed hold for $t$-symmetrizable quasi-uniformities:

In our next result $|X|$ will denote the (infinite) cardinality of a set $X$.

**Proposition 10** For each $t$-symmetrizable quasi-uniformity $\mathcal{U}$ on a set $X$, if $D$ is an infinite $\mathcal{U}^s$-discrete subset of $X$, then there is $B \subseteq D$ such that $|B| = |D|$ and $B$ is $\mathcal{U}$-discrete.
A base $\mathcal{B}$ is a $\theta$-base for a topological space $X$ if $\mathcal{B}$ can be written as $\bigcup_{n \in \mathbb{N}} \mathcal{B}_n$ in such a way that given any open set $U$ of $X$ and any point $x \in U$ there is $n_x \in \mathbb{N}$ such that ord($x, \mathcal{B}_{n_x}$) (that is, the order of $\mathcal{B}_{n_x}$ at $x$) is finite and some member $B$ of $\mathcal{B}_{n_x}$ has $x \in B \subseteq U$.

It is known that a topological space has a $\theta$-base if and only if it is quasi-developable.
The following result generalizes the fact that for each weightable quasi-pseudometric $d$ on a set $X$ the topology $\tau_{U_d}$ has a $\theta$-base.

**Proposition 11** Let $\mathcal{U}$ be a quasi-uniformity with a countable base on a set $X$ possessing the property that $\mathcal{U}^s = \mathcal{U} \vee \mathcal{A}$ where $\mathcal{A}$ is a quasi-uniformity on $X$ such that $\mathcal{A}^s$ is preLindelöf.

Then $\tau_{\mathcal{U}}$ has a $\theta$-base.
A quasi-uniformity $\mathcal{U}$ on a set $X$ is called *Smyth completable* provided that each left $K$-Cauchy filter on $(X, \mathcal{U})$ is a $\mathcal{U}^s$-Cauchy filter.

A filter $\mathcal{F}$ on a quasi-uniform space $(X, \mathcal{U})$ is called a *left $K$-Cauchy filter* (resp. *right $K$-Cauchy filter*) provided that for each $U \in \mathcal{U}$ there is $\mathcal{F}_U \in \mathcal{F}$ such that $U(x) \in \mathcal{F}$ (resp. $U^{-1}(x) \in \mathcal{F}$) whenever $x \in \mathcal{F}_U$.

A filter $\mathcal{F}$ on a quasi-uniform space $(X, \mathcal{U})$ is called *$\mathcal{U}$-stable* provided that for each $U \in \mathcal{U}$ we have that

$$\bigcap_{F \in \mathcal{F}} U(F) \in \mathcal{F}.$$
Remark 8 Let $(X,\mathcal{U})$ be a quasi-uniform space such that each left $K$-Cauchy filter on $(X,\mathcal{U})$ is contained in a $\mathcal{U}^s$-Cauchy filter. Then each left $K$-Cauchy filter $\mathcal{F}$ on $X$ is in fact a $\mathcal{U}^s$-Cauchy filter.

The following result generalizes the fact that for each weightable quasi-pseudometric $d$ on a set $X$ with a weight $|\cdot|$ the quasi-uniformity $\mathcal{U}_d$ is Smyth completable.
Proposition 12 Each quasi-uniformity \( U \) on a set \( X \) possessing an adequate symmetrizer \( A \) where \( A \) is hereditarily precompact is Smyth completable.

Problem 1 Let \( q \) be a quasi-pseudometric on a set \( X \) such that the quasi-uniformity \( U_q \) is \( t \)-symmetrizable.

Is \( U_q = U_d \) where \( d \) is a quasi-pseudometric on \( X \) that is weightable by a bounded weight?
References


