The monotonically weak Lindelöf spaces

Bruno A. Pansera

joint work with M. Bonanzinga and F. Cammaroto

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3 Results
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Definitions

We assume all spaces to be *Tychonoff*.

**Definition**

The family of sets $A$ *refines* a family of sets $B$ we mean that every element of $A$ is a subset of an element of $B$.

**Definition**

A space $X$ is *weakly Lindelöf* (wL) [Frolik, 1959] if for every open cover $U$ of $X$ there is a countable subfamily $U_0 \subseteq U$ with the union dense in $X$.

**Definition**

A space $X$ is *monotonically Lindelöf* (mL) [Matveev, 1994] if there is a function $r$, henceforth called an mL-operator, that assigns to every open cover $U$ of $X$ a *countable open cover* $r(U)$ which refines $U$ in such a way that $r(U)$ refines $r(V)$ whenever $U$ refines $V$. 

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We will see that in some aspects mwL is similar to mL, but in some others it behaves quite differently. This is not surprising because wL follows not only from Lindelöfness, but also from c.c.c.
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Examples and counterexamples

The class of mwL spaces contains: all second countable spaces, the one-point Lindelöfication of discrete space of cardinality $\omega_1$.

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The class of mwL spaces is much broader than the class of mL spaces: it contains many non-mL, sometimes even non-Lindelöf spaces.
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Theorem 1

Let $X$ be a space and $D$ be a countable dense subspace of $X$ consisting of isolated points. Then $X$ is mwL.

Example

All $\Psi$-spaces are mwL.

$\Psi(\mathcal{A}) = \omega \cup \mathcal{A}$: Isbell and Mrowka’s space, where $\mathcal{A}$ is an infinite maximal almost disjoint family of infinite subsets of $\omega$. 
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Recall that $L(\kappa)$, the one-point Lindelöfication of the discrete space of cardinality $\kappa$ is the set $X = \kappa \cup \{p\}$ equipped with the topology in which the points of $\kappa$ are isolated and every neighborhood of $p$ has the form $\{p\} \cup (\kappa \setminus A)$ where $|A| \leq \omega$.

Theorem (Levy-Matveev)
$L(\kappa)$ is mL iff $\kappa \leq \omega_1$.
Thus $L(\omega_1)$ is mwL.

Problem
For what $\kappa > \omega_1$ is $L(\kappa)$ mwL?
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Recall that the *Alexandroff duplicate* $AD(X)$ of the topological space $X$ is the set $X \times 2$ where the points of $X \times \{1\}$ are isolated while a basic neighborhood of a point $(x, 0) \in X \times \{0\}$ takes the form $(U \times 2) \setminus \{(x, 1)\}$ where $U$ is a neighborhood of $x$ in $X$.

**Theorem**

If $X$ is a second countable space, then $AD(X)$ is mwL.
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**Theorem**

*If $X$ is a second countable space, then $AD(X)$ is mwL.*
Examples and counterexamples

Levy and Matveev proved that the one-point compactification of the discrete space of cardinality \( \geq \omega_1 \) is not mL.

**Theorem**

Let \( \kappa \leq c \). Then the one-point compactification of the discrete space of cardinality \( \kappa \) is mwL.

**Problem**

For what cardinals \( \kappa > c \) is the one-point compactification of the discrete space of cardinality \( \kappa \) mwL?
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For what cardinals $\kappa > c$ is the one-point compactification of the discrete space of cardinality $\kappa$ mwL?
Recall that any hereditarily Lindelöf space having a base $\sigma$-(linearly ordered by $\supset$) is mL (see Levy and Matveev).

**Theorem**

Any space with $\sigma$-(linearly-ordered by $\supset$) $\pi$-base is mwL.

**Corollary**

Every space $X$ with a dense countable set $D$ of points of countable character is mwL.

**Remark**

In particular, every separable first countable space is mwL.
Subspaces

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Recall that mL is preserved by closed subspaces. That mwL is not preserved by closed subspaces because a Ψ-space contains an uncountable closed discrete subspace which of course is not mwL.

**Theorem**

Let $X$ be an mwL space and $Y$ a regular closed subset of $X$. Then $Y$ is mwL.
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Let $X$ be a space and $Y$ be an open dense mwL subspace of $X$. Then $X$ is mwL.

**Theorem**

Let $X$ be a $T_3$ space and $Y$ be a dense mwL subspace of $X$. Then $X$ is mwL.

**Example**

There is a Hausdorff space $X$ and a dense subspace $Y \subseteq X$ such that $Y$ is mwL but $X$ is not.
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Uncountable products

Levy and Matveev shown that if $X$ is a dense subspace in the product of uncountably many nontrivial (that is $T_1$ and consisting of at least two points) factors, then $X$ is not mL at any point.

**Theorem**

If $X$ is a dense subspace in the product $Y = \prod_{a \in A} Y_a$, where $|A| > \omega$ and for each $a$, $Y_a$ is a regular space and $|Y_a| \geq 2$, then $X$ is not mwL at any point.

The following are immediate corollaries from previous Theorem:

- $2^\kappa$ is not mwL whenever $\kappa > \omega$
- A dense countable subspace in $2^\kappa$ (where $\omega_1 \leq \kappa \leq c$) is an example of a countable space which is not mwL (at any point).
- $C_p(X)$ is mwL iff $X$ is countable.
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If $X$ is a dense subspace in the product $Y = \prod_{a \in A} Y_a$, where $|A| > \omega$ and for each $a$, $Y_a$ is a regular space and $|Y_a| \geq 2$, then $X$ is not mwL at any point.

The following are immediate corollaries from previous Theorem:

- $2^\kappa$ is not mwL whenever $\kappa > \omega$
- A dense countable subspace in $2^\kappa$ (where $\omega_1 \leq \kappa \leq \mathfrak{c}$) is an example of a countable space which is not mwL (at any point).
- $C_p(X)$ is mwL iff $X$ is countable.
Cardinal Functions

Theorem

Every Tychonoff space of weight $\leq c$ can be embedded in an mwL Tychonoff space as a closed subspace.

So we see that an mwL space may have cardinality $\geq 2^c$, extent $\geq c$ and character $\geq c$.

Levy and Matveev asked if the weight of every mL space is not greater than $c$ and noted that a similar question makes sense apparently for any other ”reasonable” cardinal function.
Cardinal Functions

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*Every Tychonoff space of weight \( \leq c \) can be embedded in an mwL Tychonoff space as a closed subspace.*

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Further problems

Problem
If $X$ is an mwL space, does it follow that $w(X) \leq c$? What can one say about other cardinal invariants of $X$?

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Is $\omega^* = \beta\omega \setminus \omega$ mwL?
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Is mwL preserved by closed irreducible maps?

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Suppose all continuous images of $X$ are mwL. What can be concluded about $X$?

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Is every mwL LOTS or GO-space mL? In particular, is $[0, \omega_1]$ mwL?
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References


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Thank you!!!